The second midterm will be held on Thursday NIGHT Nov 13, 9:50-11:10 PM.

Allison Road Classroom 103 (Busch)  
Physics Lecture Hall (Busch)  
Lucy Stone Hall A102 Aud (Livingston)  
Beck Hall 100 Aud (Livingston)  

Aaa-Gzz  
Haa-Lzz  
Maa-Rnn  
Roa-Zzz

The exam on Nov 13 is based on Chapters 25-29 inclusive (today’s lecture also!).

There will be optional review sessions on Thursday, November 13; details will be announced.

All exams will be no calculator, closed book exams with only 1 page of equations. Types of questions include iclickers, only formulae, pure concepts, simple numbers. For details go to http://www.physics.rutgers.edu/ugrad/227/intro.html#Examinations.

If you have a conflict you have to contact Professor Cizewski Cizewski@physics.rutgers.edu at your earliest opportunity but not later than midnight on Sunday, November 9 to request a conflict exam.
Outline:

- Ampere’s Law must be modified in dynamics!
- Maxwell’s Fix: Displacement Current.
- Maxwell’s Equations.
Faraday Law Examples

Consider a wire loop (radius $a$) and a long solenoid (radius $b$) with a uniform $B(t)=B_0(t/\tau)$ inside and $B=0$ outside of the solenoid. Find current $I$ in the wire if its resistance is $R$.

\[
\oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{\text{surf}} \vec{B} \cdot d\vec{A}
\]

\[
2\pi a \cdot E = -\pi b^2 \cdot \frac{dB(t)}{dt} = -\pi b^2 \cdot \frac{B_0}{\tau}
\]

\[
E = -\frac{b^2 B_0}{2a\tau}, \quad I = -\frac{b^2 B_0}{2a\tau R}
\]

Note that $B=0$ at the wire’s location! Still, there is an induced $E \neq 0$.

A wire loop has a total resistance $R$. If the total flux through it changes from $\Phi_i$ to $\Phi_f$, show that the magnitude of the total charge that will flow through the loop is given by

\[
Q = \frac{\Phi_i - \Phi_f}{R}
\]

\[
Q = \int_{i}^{f} I(t)dt = \int_{i}^{f} -\frac{1}{R} \frac{d\Phi(t)}{dt}dt = \frac{\Phi_i - \Phi_f}{R}
\]
What have we got so far?

- Gauss’s Law: charges are sources of electric field (and a non-zero net electric field flux through a closed surface); field lines begin and end on charges.

\[ \oint_{\text{surf}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

- No magnetic monopoles; magnetic field lines form closed loops.

\[ \oint_{\text{surf}} \vec{B} \cdot d\vec{A} = 0 \]

- Faraday’s Law of electromagnetic induction; a time-dependent \( \Phi_B \) generates \( E \).

\[ \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_{\text{surf}} \vec{B} \cdot d\vec{A} \]

- Ampere’s Law; \( B \) is generated by currents.

\[ \oint_{\text{loop}} \vec{B} \cdot d\vec{l} = \mu_0 I \]

- The force on a (moving) charge.

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]

This is not fair!!
Changing the B field with time can give rise to an E field, but the poor E field can not give rise to a B field by changing!!
It is not just a question of being fair, if this is all there is no TV, radio, cellphones ...
can exist!!!!!!!
Charging a Capacitor

Consider the capacitor on the right, which is being charged by a current $i_c$

As the charge builds up, the E-field within the capacitor is obviously changing.

Regardless of the shape of the surface spanning the Amperian loop, we should get the same result for

$$\oint \vec{B}(r) \cdot d\vec{l}$$

Now, with “old” Ampere’s law:

$$\int_{\text{loop}} \vec{B}(r) \cdot d\vec{l} = \mu_0 \int_{\text{surf}} J \cdot d\vec{A} = \text{zero on the left, } \mu_0 i_c \text{ on the right}$$

Very bad!!
Maxwell suggested generalization of Ampere’s Law by adding the quantity $\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ to $\vec{J}$. This modification resolves all paradoxes and makes the system of equations symmetric: not only the time-dependent $\vec{B}$ generates $\vec{E}$, but also the time-dependent $\vec{E}$ generates $\vec{B}$!

**Displacement current:**

$$\vec{J}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

**Generalized Ampere’s Law:**

(The 4th Maxwell’s Eq.)

$$\oint_{\text{loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{net encl}} = \mu_0 \int_{\text{surf}} \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot d\vec{A} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \int_{\text{surf}} \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A} = \mu_0 \left(I_{\text{encl}} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

**Maxwell:** “One of the chief peculiarities of this thesis is the doctrine which asserts that the true electric current, that upon which electromagnetic phenomena depend, is not the same thing as the current of conduction but that the time derivative of the electric displacement must be taken into account.”
Why was not the displacement current discovered experimentally prior to Maxwell?

Because in the “old-fashioned” experiments with highly-conducting wires and slowly-varying fields the displacement current is much smaller than the “conduction” current:

Let’s estimate how quickly the electric field should be changed in order to make the current density and displacement current density of the same order of magnitude:

\[ J \sim J_D \quad J \sim \epsilon_0 \frac{\partial E}{\partial t} \quad \sigma E \sim \epsilon_0 \frac{E}{\tau} \quad \tau \sim \frac{\epsilon_0}{\sigma} \sim \frac{10^{-11} C^2/N \cdot m}{10^8 1/\Omega \cdot m} \sim 10^{-19} \text{s} \ (!!!) \]

For the first time the magnetic field generated by the displacement current inside a capacitor was directly measured in 1985 (!!!) and this was the subject of a paper in one of the most prestigious physics journals, Physical Review Letters 55, 59 (1985).
Charging a Capacitor

Regardless of the shape of the surface over which we apply Ampere’s law, we should get the same result for $\oint \vec{B}(r) \cdot d\vec{l}$

$$\oint_{loop} \vec{B}(r) \cdot d\vec{l} = \mu_0 \int_{surf} \left( \vec{j} + \epsilon_0 \frac{d\vec{E}}{dt} \right) \cdot d\vec{A} = \mu_0 \int_{surf} \epsilon_0 \frac{d\vec{E}}{dt} \cdot d\vec{A}$$

$$B(r)2\pi r = \mu_0 \epsilon_0 \int_{surf} \frac{d\vec{E}}{dt} \cdot d\vec{A} = \mu_0 \epsilon_0 \frac{1}{\epsilon_0 A} IA = \mu_0 I$$

Now it works:

$$\frac{d\vec{E}}{dt} = \frac{1}{d} \frac{dV}{dt} = \frac{1}{Cd} \frac{dQ}{dt} = \frac{1}{\epsilon_0 A} I$$

$$\frac{dQ}{dt} = I, \ C = \epsilon_0 A/d$$

- the same as for the plane surface intersecting the current-carrying wire
Maxwell’s Equations

- Gauss’s Law: charges are sources of electric field (and a non-zero net electric field flux through a closed surface); field lines begin and end on charges.
  \[ \oint_{\text{surf}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

- No magnetic monopoles; magnetic field lines form closed loops.
  \[ \oint_{\text{surf}} \vec{B} \cdot d\vec{A} = 0 \]

- Faraday’s Law of electromagnetic induction; a time-dependent \( \Phi_B \) generates \( \vec{E} \).
  \[ \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_{\text{surf}} \vec{B} \cdot d\vec{A} \]

- Generalized Ampere’s Law; \( \vec{B} \) is produced by both currents and time-dependent \( \Phi_E \).
  \[ \oint_{\text{loop}} \vec{B} \cdot d\vec{l} = \mu_0 \oint_{\text{surf}} \left( \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{A} \]

- The force on a (moving) charge.
  \[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]
Video

Really understanding the displacement current is probably the hardest thing in this course!

But no one told you this course would be easy!!!
Maxwell’s Equations

**Electrostatics**
Electric fields generated by charges at rest

**Magnetostatics**
Magnetic fields generated by time-independent currents

**Electromagnetism**

\[ \oint E \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \]
\[ \oint B \cdot d\mathbf{A} = 0 \]
\[ \oint E \cdot ds = 0 \]
\[ \oint B \cdot ds = \mu_0 I \]
\[ \frac{d}{dt} \neq 0 \]
Next time: Review for Midterm II.
Please email me your questions!