Lecture 18. Motional EMF

Outline:

- More on Faraday’s Law.
- Motional EMF.
Consider a loop of area $A$ in a constant $B$ field. The loop rotates at a frequency $\omega$ about the axis shown. What is the emf around the loop?

$$\Phi_m = \int \vec{B} \cdot d\vec{A} = BA \cos(\omega t)$$

$$\mathcal{E} = -\frac{d\Phi_m}{dt} = \omega BA \sin(\omega t)$$
Alternator: generator of AC current.

$\mathcal{E} = -\frac{d\Phi_B}{dt}$

DC generator: generator of DC current (DC motor in reverse).
The flux of the magnetic field in the wire loop varies with time as shown in the Figure. Which dependence(s) $\Phi_B(t)$ correspond(s) to the largest magnitude of the induced current?

A. 1
B. 2 & 3
C. 5
D. 4 & 5
Two identical concentric loops are arranged as shown in the Figure. One loop has a steady current flowing through it (provided by the power supply). When the power is turned off, will the two loops briefly attract or repel each other?

A. They will attract each other.

B. They will repel each other.

C. There is no induced current.

D. They will neither repel nor attract each other.

E. The answer depends on the direction of current in loop 1.
Experiment with a coca can in a pulse solenoid. The can is centered in the solenoid.

What happens to the can?

A. Moves to the left
B. Moves to the right
C. Contracts
D. Expands
Experiment with a coca can in a pulse solenoid. The can is centered in the solenoid.

What happens to the can?

A. Moves to the left
B. Moves to the right
C. Contracts - but only because the initial contraction is not compensated by later expansion
D. Expands
Motional EMF

**Primitive model of a DC generator.**

Part of the loop is in a uniform $B$ field. The loop is pulled to the right. Find the induced current $I$.

1. *Faraday’s Law approach:*

   \[ |\mathcal{E}| = \frac{d\Phi}{dt} = \frac{d}{dt}(BLx) = BLv \quad \text{Current:} \quad I = \frac{\mathcal{E}}{R} = \frac{BLv}{R} \]

   Thermal power (Joule “heat”) released in the wire:

   \[ P = \frac{\mathcal{E}^2}{R} = \frac{(BLv)^2}{R} \]

   Who does the corresponding work? Of course, not the magnetic field! We do, by pulling the loop with constant force:

   \[ F_{us} = ILB = \frac{B^2L^2v}{R} \quad P = Fv = \frac{(BLv)^2}{R} \]
Motional EMF (cont’d)

**Primitive model of a DC generator.**

Part of the loop is in a uniform $B$ field. The loop is pulled to the right. Find the induced current $I$.

2. **Lorentz force approach** (we consider only electrons, assuming that the force on the lattice is compensated by us):

- the drift velocity can be ignored because the corresponding component of Lorentz force is perpendicular to the wire.

$$
\mathcal{E} = \oint \vec{E}_{NC} \cdot d\vec{l} = \oint \frac{\vec{F}_L}{q} \cdot d\vec{l} = \int_{\text{vert.segment in } B \text{ field}} \frac{q \vec{v} \times \vec{B}}{q} \cdot d\vec{l} = vBL
$$

For this kind of problems, the Lorentz force provides the so-called **motional e.m.f.**, and the Faraday’s law is just a short-cut which doesn’t contain any new physics.
The rectangular loop of wire is being moved to the right at constant velocity. A constant current \( I \) flows in the long straight wire in the direction shown. The current induced in the loop is zero.

\[ \mathcal{E}_1 + \mathcal{E}_2 = 0 \]
Sometimes the motional e.m.f. can be successfully described using both the Faraday’s Law and the approach based on the Lorentz force.

General expression for the motional e.m.f.:

$$\mathcal{E} = \int_{\text{loop}} (\vec{v} \times \vec{B}) \cdot d\vec{l}$$
Example

A metal disc rotates about a horizontal axel; a uniform magnetic field $B$ is directed parallel to the axel. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact which touches the outer edge of the disc. Find the current through the resistor.

$$F_L = qvB \quad \text{- directed along the radius}$$

$$\mathcal{E} = \oint_{\text{loop}} \vec{E}_{NC} \cdot d\vec{l} = \oint_{\text{loop}} \frac{\vec{F}_L}{q} \cdot d\vec{l} = \int_0^a vBdr = \int_0^a \omega rBdr = \omega B \frac{a^2}{2}$$

$$I = \frac{\mathcal{E}}{R} = \frac{\omega Ba^2}{2R}$$
Eddy Current (Foucault) Brake

Linear eddy current brake
(useless at low speed, efficient at high speed)

Demonstrations:
brakes for *linear* and *rotational* motion
In what order do the disks race through a magnetic field?

1. Insulator
2. Conductor
3. Conductor with holes
4. Conductor with slots

A. 1, 2, 3, 4
B. 4, 3, 2, 1
C. 1, 4, 3, 2
D. 2, 3, 4, 1

Ignore changes to the moment of inertia. Assume the B field is constant and the same size as the disks.
Conclusion

\[ \mathcal{E} = \oint_{\text{loop}} \vec{E}_{NC} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]  

Faraday’s Law (the 3d Maxwell’s Eq.)

Motional e.m.f. – due to Lorentz force. Depending on the experiment set-up, can also sometimes be treated on the basis of Faraday’s law.

Explanations are reference-frame dependent!

Next time: Lecture 19. Maxwell’s Equations

29.7
A flexible loop of wire lies in a uniform magnetic field $B$ directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current

A. is zero because the magnetic field is time-independent.

B. flows upward through resistor $R$.

C. flows downward through resistor $R$.

D. does not flow through resistor $R$. 
Consider a wire loop (radius $a$) and a long solenoid (radius $b$) with a uniform $B(t)=B_0(t/\tau)$ inside and $B=0$ outside of the solenoid. Find current $I$ in the wire if its resistance is $R$.

\[
\oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{\text{surf}} \vec{B} \cdot d\vec{A}
\]

\[
2\pi a \cdot E = -\pi b^2 \cdot \frac{dB(t)}{dt} = -\pi b^2 \cdot \frac{B_0}{\tau}
\]

\[
E = -\frac{b^2 B_0}{2a\tau} \quad I = -\frac{b^2 B_0}{2a\tau R}
\]

A wire loop has a total resistance $R$. If the total flux through it changes from $\Phi_i$ to $\Phi_f$, show that the magnitude of the total charge that will flow through the loop is given by

\[
Q = \frac{\Phi_i - \Phi_f}{R}
\]

\[
Q = \int_{i}^{f} I(t) dt = \int_{i}^{f} -\frac{1}{R} \frac{d\Phi(t)}{dt} dt = \frac{\Phi_i - \Phi_f}{R}
\]