Outline:

- Magnetic Materials.
- Faraday’s Law.
- Sign Convention: Lenz’s Law

The 3rd Maxwell’s Equation (in combination with the 4th one): the basis for modern technology.
Sources of local magnetic fields inside any material:
1. Magnetic moments associated with the electrons orbiting around nuclei.
2. The magnetic moments associated with spins of both electrons and nuclei.

**Weak** interactions between internal magnetic moments:
- **Diamagnets**: No permanent magnetic moments (cf. nonpolar dielectrics)
- **Paramagnets**: Randomly oriented moments - the average magnetic field is zero (cf. polar dielectrics)

**Strong** interactions between internal moments: the moments ordered in space.
- **Ferromagnets**: parallel ordering (iron, nickel, etc.)
- **Antiferromagnets**: antiparallel ordering (chromium, etc.).

In ferromagnets, there is a strong local magnetic field in the material. Large number of elementary moments line up in the magnetic domains. The global field depends on the orientation of magnetic domains. In permanent magnets the domains are preferentially oriented.
Dia-, Para-, and Ferromagnets in Non-zero External $B$

$B_{\text{ext}} \neq 0$

$\vec{B} = \vec{B}_{\text{ext}} + \mu_0 \vec{M}$

$\vec{M} = \frac{\chi_m}{\mu_0} \vec{B}_{\text{ext}} = \frac{K_m - 1}{\mu_0} \vec{B}_{\text{ext}}$

$\vec{B} = \vec{B}_{\text{ext}} + \mu_0 \vec{M} = K_m \vec{B}_{\text{ext}}$

- $\vec{M}$ - the magnetic moment per unit volume
- $\vec{B}$ - the total field inside the material
- $\chi_m$ - the magnetic susceptibility
- $K_m$ - the relative permeability
  - Unit for $M = \text{A/m}$
  - Unit for $\mu_0 = \text{Tm/A}$
- “linear” magnetic materials

**Paramagnets and Diamagnets:**

In the external field, there is a (very small) preferential orientation of individual moments (along the field in paramagnetics, against $-$ in diamagnetics). The “orientation” effect of $B_{\text{ext}}$ is weak (thermal “disorientation” is much stronger), and, as a result, $K_m$ is very close to unity, $K_m - 1 = \pm (10^{-5} - 10^{-4})$.

**Ferromagnets:**

External $B$ results in motion of domain walls in such a way that the domains oriented along the field grow while the domains oriented against the field shrink. The global magnetic field can be very strong, leading to $K_m$ up to $10^6$ in *weak* external magnetic fields.

$B_{\text{ext}} = 0$
Levitation condition \( m - \) the induced-by-\( B_{\text{ext}} \) magnetic moment per unit volume, \( \rho - \) the substance density).

\[
m = \frac{\chi m}{\mu_0} B_{\text{ext}}
\]

\[
\chi_m \equiv K_m - 1 \approx -10^{-5} \quad \text{for water}
\]

\[
\frac{\chi m}{\mu_0} B_{\text{ext}} \frac{dB_{\text{ext}}}{dz} = \rho g
\]

\[
B_{\text{ext}} \frac{dB_{\text{ext}}}{dz} = \frac{\rho g \mu_0}{\chi_m} \approx \frac{10^3 \cdot 10 \cdot 10^{-6}}{10^{-5}} = 10^3 T^2/m
\]

For graphite
\( \chi_m \approx -10^{-3} \)

Magnetic levitation has no connection with the electromagnetic induction!
Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$

Reasonable guess: by moving a wire in a magnetic field, we can generate a current.

Each set-up [(a) – (d)] produces an EMF:

(a) and (b) – EMF should be the same (only relative motion matters). However, the interpretations are different.

(b) and (c) – EMF in one case implies an EMF in the other: the loop cannot tell whether the wire is approaching or the current is increasing.

(d) $B$ is time-independent, but an EMF is produced if the area of the loop changes.

Faraday (and Henry) realized that all these situations have one thing in common: the flux of the magnetic field through the loop changes with time!
Faraday’s Law

The e.m.f. is induced in a loop if the magnetic flux changes, and is proportional to the rate of change of the magnetic flux.

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- the induced e.m.f.

The flux can change for a variety of reasons:

a) \( B \) may be changing in time;

b) The loop may be altering in shape;

c) The orientation of the loop with respect to \( B \) is changing;

d) Any combination of these effects.
Faraday’s Law (cont’d)

\[
\oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{\text{surface}} \vec{B} \cdot d\vec{a} = -\frac{d}{dt} \Phi_m
\]

Choose the direction of circulation around the loop \((d\vec{l})\). This also gives the direction of \(d\vec{a}\).

In order to have negative \(\oint_{\text{loop}} \vec{E} \cdot d\vec{l} < 0\), the electric field must be directed against \(d\vec{l}\) \(\Rightarrow\) current flows clockwise.

Direction of the \(B\) field induced by the current – into the board.
Lenz’s “Law” (better Rule)

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_{\text{surface}} \mathbf{B} \cdot d\mathbf{a} \]

1. Choose the direction of circulation – e.g., clockwise.
2. Corresponding $d\mathbf{a}$ - into the board (right-hand rule).
3. The flux decreases $\rightarrow$ the induced-by-current $B$ field has to be into the board.
4. Thus, in order to have $\oint \mathbf{E} \cdot d\mathbf{l} > 0$, $\mathbf{E}$ should be oriented along $d\mathbf{l}$.

**Lenz:** An induced current has a direction such that the magnetic field due to the induced current *opposes the change* in the magnetic flux that induces the current.

At home, analyze the problem by choosing the direction of circulation counterclockwise.
Multi-turn Coils

If we have a coil with $N$ identical turns and the flux is the same through each turn, the net e.m.f. will be $N$ times the e.m.f. across a single turn:

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

**Example:** A 2-cm-radius loop of $N=100$ turns of wire lies in the plane of the paper. A magnetic field of 0.15 T is introduced, pointing into the paper, in time 0.3 s. Find the e.m.f. Does current tend to flow clockwise or counterclockwise?

$$\mathcal{E} = -\pi r^2 N \frac{dB}{dt} = -3.14 \times (0.02m)^2 \times 100 \times \frac{0.15T}{0.3s} = -0.063V$$

counterclockwise
Experiment with a magnet falling inside a copper pipe

Please provide your explanation of the experiment.
Next time: Lecture 18. Motional EMF
§§ 29.4-6
Magnetically Induced Non-Conservative Electric Field

The magnetically induced e.m.f., similar to the e.m.f. in a battery, is the result of *non-conservative* forces:

\[ \mathcal{E} = \oint_{\text{loop}} \vec{E}_{NC} \cdot d\vec{l} \neq 0 \]

The total electric field is the sum of conservative and non-conservative terms:

\[ \vec{E} = \vec{E}_C + \vec{E}_{NC} \]

conservative (due to charges at rest)

\[ \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = \oint_{\text{loop}} \vec{E}_C \cdot d\vec{l} + \oint_{\text{loop}} \vec{E}_{NC} \cdot d\vec{l} = \oint_{\text{loop}} \vec{E}_{NC} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_{\text{surface}} \vec{B} \cdot d\vec{A} \]

- the integral form of Faraday’s Law.

**Holds for any closed path, doesn’t require presence of the conducting loop**

(the wire loop is just a “detector” of the electric field generated by \( \frac{d\Phi}{dt} \)).

**Units.** \( E \text{[N/C] \cdot dl} \text{[m]} = (N \cdot m=J) = J/C. \) Thus, the units of the e.m.f. are \([J/C]\), which we define as volt \([V]\). The right-hand side: \([T \cdot m^2/s] = (T=N \cdot s/(C \cdot m)) = J/C. \)