
Outline:

- Hall Effect.
- Magnetic Force on a Wire Segment.
- Torque on a Current-Carrying Loop.

Lecture 13: Magnetic Forces on Moving Charges
- we considered individual charges;
- and ignored electrostatic interactions among the charges in the electron beam.
Iclicker Question

Under what circumstances is the total magnetic flux through a closed surface *positive*?

A. if the surface encloses the north pole of a magnet, but not the south pole

B. if the surface encloses the south pole of a magnet, but not the north pole

C. if the surface encloses both the north and south poles of a magnet

D. none of the above
A particle with a positive charge moves in the $xz$-plane as shown. The magnetic field is in the positive $z$-direction. The magnetic force on the particle is in

A. the positive $x$-direction.
B. the negative $x$-direction.
C. the positive $y$-direction.
D. the negative $y$-direction.
E. none of these

$\vec{F} = q(\vec{v} \times \vec{B})$
A charged particle moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the particle to move through this region at a constant velocity,

A. the electric and magnetic fields must point in the same direction.

B. the electric and magnetic fields must point in opposite directions.

C. the electric and magnetic fields must point in perpendicular directions.

D. The answer depends on the sign of the particle’s electric charge.
Hall Effect

Current-carrying conductors in external magnetic field: two systems of charges - mobile current carriers and immobile ion lattice - and only the current carriers are affected by $B$.

In the steady state, the magnetic force on moving charges is compensated by the electrostatic force due to *uncompensated* surface charge:

$$\vec{F}_E = -\vec{F}_B \quad qE = qv_d B \quad E = v_d B$$

$$I = qnv_d W t \quad n \text{ – the density of mobile carriers}$$
$$W \quad \text{– the width of the conductor}$$
$$t \quad \text{– the thickness of the conductor}$$

$$V_H = EW = v_d BW = \frac{IBW}{qnWt} \quad V_H = \frac{IB}{qnt} \quad \text{- Hall voltage}$$

The sign of $V_H$ depends on the sign of the charge of current carriers $q$, the magnitude $\propto 1/n$. 

Edwin Hall (1855-1938)
**Example:** typical two-dimensional \((n \cdot t)\) charge density in Si field-effect transistor (in the “on” state) is \(10^{16} \text{ 1/m}^2\). Let’s run a 0.1A current and place the transistor in a 0.1T magnetic field:

\[
V_H = \frac{IB}{qnt} = \frac{0.1A \cdot 0.1T}{1.6 \cdot 10^{-19} C \cdot 1 \cdot 10^{16} m^{-2}} \approx 6V
\]

Hall effect measurements allow determination of
- charge carrier density and
- mobility
in conductors and semiconductors.
The electrons drift \textit{along} the wire because the net (el.\,+mag.) force on them in the direction normal to the wire is 0. However, positively charge ions are at rest, they feel only an \textit{uncompensated} electric force:

\[ E_H = \frac{IB}{qnWt} \quad f = qE_H = \frac{IB}{nWt} \quad \text{- force per one ion} \]

\[ F = nWtf = IB \quad \text{- force per unit length of the conductor} \]

The force on a wire segment of length $d\mathbf{l}$:

\[ \mathbf{F} = I(d\mathbf{l} \times \mathbf{B}) \]
Example: a straight horizontal length of wire has a mass of \( m/l = 10 \text{ g/m} \); it carries a current of 1A. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire in the Earth’s gravitational field?

\[
F_B = mg \quad lIB = \left( \frac{m}{l} \right) lg \quad B = \frac{(m/l)g}{I} = \frac{0.01 \text{kg/m} \cdot 10 \text{m/s}^2}{1 \text{A}} = 0.1 T
\]
Appendix I: Ampere’s Motor

Which orientation?

Switch

Conducting bar

Conducting rails

\[ \vec{F} \]

\[ \vec{B} \]
Dipoles in Uniform Fields

Net force = 0, 
torque ≠ 0

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

Electric

\[ \vec{\tau} = \vec{p} \times \vec{E} \]

\[ U(\phi) = -pE \cos(\phi) \]

Magnetic

\[ F = ILB \]
Torque on a Current Loop

Consider \( \vec{A} \perp \vec{B} \) (\( \vec{B} \) in the loop’s plane): \( F_1 = F_2 = aIB \quad F_3 = F_4 = 0 \)

\[
\vec{\tau} = \frac{b}{2} \hat{z} \times \vec{F}_1 + \frac{b}{2} (-\hat{z}) \times \vec{F}_2 = \frac{b}{2} aIB (-\hat{y}) + \frac{b}{2} aIB (-\hat{y}) = ablB (-\hat{y})
\]

**Magnetic dipole moment:**

\( \mu = abl = Al \) \hspace{1cm} (A – the loop’s area)

Direction of \( \vec{\mu} \): the right-hand rule.

In general:

\[
\vec{\tau} = \vec{\mu} \times \vec{B} \quad \tau = \mu B \sin \phi
\]

(compare with \( \vec{\tau} = \vec{p} \times \vec{E} \))

If there are \( N \) turns, the total magnetic dipole moment is \( \mu = NAl \)

\( \tau = 0 \) if \( \phi = 0^0, 180^0 \)
Potential Energy of a Current Loop in Magnetic Field

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \mu B \sin(\phi) \]

\[ U(\phi) = -\mu B \cos(\phi) \]

- unstable equilibrium
- stable equilibrium

\[ \vec{\tau} = -\text{grad} \ U(\phi) \]

\[ = - \frac{dU(\phi)}{d\phi} \hat{\phi} \]
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic torque on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.

B. tends to orient the loop so that its plane is parallel to the direction of the magnetic field.

C. tends to make the loop rotate around its axis.

D. is zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
**Magnetic Dipole vs. Electric Dipole**

**Similarity:**
- a magnet’s magnetic field is very similar to a dipole’s electric field *at points far from the dipoles*;
- both repel/attract each other;
- both align along the field lines.

**Difference:**
- unlike electric dipoles, magnetic poles cannot be separated;
- magnets have no effect on stationary charges.
The Earth's North Magnetic Pole is actually a magnetic *south* pole and the Earth's South Magnetic Pole is a magnetic *north* pole.
Next time: Lecture 15: Magnetic Fields of Currents.
§§ 28.1- 28.5
Pressure on Solenoid Walls (see L15)

Pressure on the solenoid walls due to the interaction of currents with $B_{\text{ext}}$ produced by all other elements of the solenoid:

$$\vec{F} = lI\vec{l} \times \vec{B}_{\text{ext}}$$

- force per one wire of length $dl$

Total force per unit area $1m^2$ (pressure) on a current-carrying sheet:

$$P = nIB_{\text{ext}} = KB_{\text{ext}}$$

$$= K \frac{\mu_0 K}{2} = \frac{1}{2\mu_0}B_{\text{in}}^2$$

Thus, the pressure equals to the energy density in the solenoid.

Problem of ultra-strong mag. fields = problem of mechanical stability of solenoids

The LANL pulsed magnet: delivers $\sim 100 T$ for about 15 milliseconds. The magnet consists of an outer coil set (not shown), and a smaller coil inserted into the high field region of the outer coil set.

The LANL 80 Tesla pulsed test magnet before and after 10 pulses. The magnetic pressure acting on the walls of the coil is $\sim 4 \cdot 10^4 \text{ bar}$. 

http://www.gizmag.com/100-tesla-pulsed-magnet/21946/
Appendix II: Magnetic Dipole in Non-Uniform Magnetic Field

http://www.ru.nl/hfml/research/levitation/diamagnetic/

\[ F = \mu \cdot \frac{dB}{dz} \propto B \cdot \frac{dB}{dz} \]

Paramagnetic response: the induced \( \vec{\mu} \) is parallel to \( \vec{B} \)

Diamagnetic response: the induced \( \vec{\mu} \) is anti-parallel to \( \vec{B} \)

Levitation of a diamagnetic object

In contrast, the induced electric dipoles are oriented along the electric field, they are always **attracted** to the region of a stronger field.

Andre Geim
Nobel 2010
IgNobel 2000