Outline:

- Intro to Magnetostatics.
- Magnetic Field Flux, Absence of Magnetic Monopoles.
- Force on charges moving in magnetic field.
Our goal: to describe the magnetostatic field the same way we’ve described the electrostatic field.

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \text{- always true} \]
\[ \oint \vec{E} \cdot d\vec{l} = 0 \quad \text{- true only if the fields are static} \]
\[ \vec{F} = q\vec{E} \]

\[ \oint \vec{B} \cdot d\vec{A} = ? \]
\[ \oint \vec{B} \cdot d\vec{l} = ? \]
\[ \vec{F} = ? \]

Do you know a vector field \( \vec{a} \) that has both \( \oint \vec{a} \cdot d\vec{A} = 0 \) and \( \oint \vec{a} \cdot d\vec{l} = 0 \)?
Charges at rest do not generate $B$. What would be our “best bet” for the source of the magnetic field?

“magnetic point charge” (a magnetic monopole): have not been observed yet (though its existence doesn’t contradict anything)

a moving single charge: the field is non-stationary, not good for magnetostatics

steady (time-independent) current: our best bet

**Sources of $B$:**

- Charges in motion: currents, orbital motion of electrons in atoms.
- Ferromagnetic materials (electron spins – purely quantum phenomenon).
- Time-dependent electric fields (we’ll consider this source later in Electrodynamics).

**Units:** tesla, T
Characteristic Magnetic Fields

\[ B \approx 10^{-4} T \]

rare-earth magnets

\[ B \text{ up to } 1.4 T \]

\[ B \approx 1 - 2 T \]

Superconducting solenoids in LHC

\[ B \text{ up to } 8 T \]

Superconducting solenoids

\[ B \text{ up to } 30 T \]
No “magnetic point charges” (magnetic monopoles): have not been observed yet.

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]

compare with

\[ \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} \]

**Breaking a magnet in two ...**

... yields two magnets, not two isolated poles.

**Consequence:** for ANY closed surface

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

- always true, not only in electrostatics but also in electrodynamics
Magnetic field is a **non-conservative** vector field. In general

\[ \oint \mathbf{B} \cdot d\mathbf{l} \neq 0 \]

to be specified later

Magnetic field lines are **closed loops** (no mag. monopoles):
Force on a Charge Moving in Magnetic Field

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]

\[ F = qvB\sin\phi \]

\( \vec{F} = 0 \) for charges moving along \( \vec{B} \)

\( F \) is max when \( \vec{v} \perp \vec{B} \)

Magnetic field lines are not lines of force!
No Work Done by Magnetic Force!

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]  \[ \Rightarrow \quad \vec{F} \perp d\vec{l} \]  \[ \Rightarrow \quad dW = \vec{F} \cdot d\vec{l} = 0 \]

The work done by
the magnetic field
on a moving charge

\[ \text{You cannot } \textit{increase} \text{ the speed of charged particles using magnetic field. However, the } B\text{-induced acceleration is non-zero, it’s just perpendicular to the velocity } (\vec{a} \equiv \frac{d\vec{v}}{dt}). \text{ An accelerated charge (e.g. moving with constant speed along a curved trajectory) } \textit{loses} \text{ its energy by radiating electromagnetic waves.} \]

\[ \text{However, there are many situations in which this statement } \textit{appears} \text{ to be false: in a non-uniform magnetic field, a current-carrying wire loop would accelerate, two permanent magnets would accelerate towards one another, etc. In these cases, the kinetic energy of objects increases. Who does the work? For discussion, see } \]

http://van.physics.illinois.edu/qa/listing.php?id=17176
Cross Product

\[ \vec{F} = -e(\vec{v} \times \vec{B}) \]

An electron moves along the z axis, the \( B \) field is in the x-y plane.

\[ \vec{v} = 1\hat{k} \text{ m/s} \quad \vec{B} = (2\hat{i} + 3\hat{j})T \]

\[ \vec{F} = -e(\vec{v} \times \vec{B}) = -e \left( 1\hat{k} \times (2\hat{i} + 3\hat{j}) \right) \]

\[ = -e(1\hat{k} \times 2\hat{i} + 1\hat{k} \times 3\hat{j}) \]

\[ = -e(2\hat{j} - 3\hat{i}) = e(3\hat{i} - 2\hat{j}) \]
Cyclotron Motion in Magnetic Field

Motion along a circular orbit:

\[ \vec{F} = q(\vec{v} \times \vec{B}) = qvB = m \frac{v^2}{R} \]

\[ R = \frac{mv}{qB} \]

Alternatively, by measuring \( R \), one can determine the ratio \( m/q \) if \( v \) (the kinetic energy) is known.

\[ T = \frac{2\pi R}{v} = \frac{2\pi m}{qB} \] - the period \( T \) is independent of \( v \)

If a charge has a velocity component along \( \vec{B} \):

[Diagram of cyclotron motion in magnetic field]
What’s wrong? In this form, the equation works only for non-relativistic motion. To correct, you need to replace the “classical” momentum $mv$ with the relativistic one $mv/\sqrt{1 - (v/c)^2}$. 

![Diagram of Large Hadron Collider](image)

$$R = \frac{mv}{qB}$$

$$m \approx 1.6 \cdot 10^{-27} \text{kg}$$

$$v = c$$

$$B = 8 \text{T}$$

$$R = \frac{1.6 \cdot 10^{-27} \text{kg} \cdot 3 \cdot 10^8 \text{m/s}}{1.6 \cdot 10^{-19} \text{C} \cdot 8 \text{T}} \approx 0.38 \text{m}$$

8.6 km

![Image of the Large Hadron Collider](image)
Mass Spectrometer

\[
\frac{mv^2}{2} = qV \\
\frac{2qV}{m} = \rho v \\
qE = qvB \\
v = \frac{E}{B}
\]

Ionization \hspace{1cm} \text{Accelerating voltage applied} \hspace{1cm} \text{Velocity selector} \hspace{1cm} \text{Magnetic field region}

\[ R = \frac{mv}{qB} \]
Magnetostatics: $B \neq B(t)$

Sources of $B$: motion of charges (currents), orbital motion of electrons in atoms, electron spins, time-dependent electric fields.

Absence of Magnetic Monopoles: \[ \oint \vec{B} \cdot d\vec{A} = 0 \]

Force on charges moving in magnetic field: \[ \vec{F} = q(\vec{v} \times \vec{B}) \]

Next time: Lecture 14: Magnetic Forces on Currents.

§§ 27.6-27.9
Structure of the Course

Electromagnetism

$E$ - electric fields
$B$ – magnetic fields
$Q$ – charges
$I$ – currents
$\Phi$ - fluxes

$d \frac{d}{dt} = 0$

Electrostatics

Electric fields generated by charges at rest
Ch. 21 - 26

Magnetostatics

Magnetic fields generated by time-independent currents
Ch. 27 - 31

Electromagnetic Waves

$\frac{d}{dt} \neq 0$  Ch. 32

- covered in detail in more advanced courses on electrodynamics and optics.
Cosmic Rays in the Earth’s Magnetic Field

Protons with energy 1 MeV move \( \perp \) earth B field of 0.5 gauss or \( 5 \times 10^{-5} \) T. Find radius & frequency of orbit.

\[
K = \frac{1}{2} mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2K}{m}}
\]

\[
K = \left(10^6\right)\left(1.6 \times 10^{-19}\right) = 1.6 \times 10^{-13} \text{ J}
\]

\[
m = 1.67 \times 10^{-27} \text{ kg}
\]

\[
R = \frac{mv}{eB} = \frac{\sqrt{2mK}}{eB}
\]

\[
R = 2900 \text{ m}
\]

\[
f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{v}{2\pi \left(\frac{mv}{eB}\right)} = \frac{eB}{2\pi m}
\]

\[
f = 760 \text{ Hz}
\]

Frequency is independent of \( v \)!
Experiment with the vacuum electron tube

7. What is the relative magnitude of the electric and magnetic forces between two equal charges?

a) The electric force is always greater.
b) The magnetic force is always greater.
c) The two forces are always equal.
d) The electric force is never less than the magnetic force.
e) The magnetic force is never less than the electric force.