Lecture 10. EMF and Batteries, Power in Electric Circuits

Outline:
- Parallel and in-series connection of resistors
- Batteries.
- Non-ideal batteries: internal resistance.
- Potential distribution around a complete circuit.
- Energy and power in electric circuits.

Lecture 9:
Electric current: flow of charge carriers, requires $E \neq 0$ in a conductor. To keep current running, we need to maintain a non-zero potential difference across a conductor.

Microscopic picture: electron “mosquito cloud” slowly drifting in the field.

Linear regime: Ohm’s Law = drift velocity $\propto E$.

Resistance: the coefficient of proportionality between $V$ and $I$, depends on materials parameters.
Which of the following statements is FALSE?

To run a current, we need

A. mobile charge carriers
B. electric field inside a conductor
C. non-zero net charge density inside a conductor
D. non-equipotential conductors
Iclicker Question

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Connections: In Series vs. In Parallel

**In Series:**
- current is the same through all elements
- voltage across them can be different

**In Parallel:**
- voltage is the same across all elements
- current through them can be different

(compare with capacitors: replace current with charge)
Iclicker Question

Which of the following is true about the circuit shown?

A. R1 is in parallel with R2.
B. R1 is in parallel with R3.
C. R2 is in parallel with R3.
D. R1 is in series with R2 and/or R3.
E. R2 is in series with R3.
Iclicker Question

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Resistors: Connection in Series and in Parallel

**Connection in series**

- voltage difference across $R_1, R_2, R_3$
- common current

$$R_{eq} = \frac{V_{ab}}{I}$$

$$V_{ab} = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$$

$$R_{eq} = \sum_i R_i$$

**Connection in parallel**

- common voltage difference
- total current $I$

$$I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\sum_i \frac{1}{R_i}\right)^{-1}$$

$$R_1 \parallel R_2: \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$
Three identical resistors, each of resistance $R$, are connected as shown. What is the equivalent resistance of this arrangement of three resistors?

A. $3R$
B. $2R$
C. $3R/2$
D. $2R/3$
E. $R/3$
Three identical resistors, each of resistance $R$, are connected as shown. What is the equivalent resistance of this arrangement of three resistors?

\[
R_{eq} = \frac{R \cdot R}{R + R} + R
\]

A. $3R$
B. $2R$
C. $3R/2$
D. $2R/3$
E. $R/3$
More Complex Circuits

\[ R_1 = \left( \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \right)^{-1} = 1\Omega \quad R_2 = \frac{1 \cdot 2}{1 + 2} + \frac{1}{3} = 1\Omega \quad R_2 \ldots R_5 = 1 + 1 = 2\Omega \]

\[ R_4 \parallel (R_2 \ldots R_5) = \frac{1 \cdot 2}{1 + 2} = \frac{2}{3}\Omega \]

\[ R_3 \ldots [R_4 \parallel (R_2 \ldots R_5)] = \frac{2}{3} + \frac{1}{3} = 1\Omega \]

\[ R_{eq} = R_1 \parallel \{R_3 \ldots [R_4 \parallel (R_2 \ldots R_5)]\} = \frac{1 \cdot 1}{1 + 1} = 0.5\Omega \]

\[ I = \frac{10V}{0.5\Omega} = 20A \quad P = IV = 200W \]
Ideal Batteries (no energy dissipation inside battery)

We assume that the wires have resistance much smaller than the load $R$ (all voltage difference provided by the battery is applied to the load).

Conclusion: an “external agent” inside the battery (not the conservative electrostatic $E$ !!!) forces the charge carriers to climb “the potential hill”.
Ideal Batteries (no energy dissipation inside battery)

Conclusion: an “external agent” inside the battery (not the conservative electrostatic $E$ !!) forces the charge carriers to climb “the potential hill”.
Ideal Batteries (no energy dissipation inside battery)

Conclusion: an “external agent” inside the battery (*not the conservative* electrostatic *E* !!!) forces the charge carriers to climb “the potential hill”.

**ARCHIMEDES SCREW**
External Forces (not Electrostatic Field) separate Charges

**Fictitious** field $\vec{E}_{NC}$ inside the battery: a source of an additional force on charge carriers. This field is **non-conservative**: $\oint \vec{E}_{NC} \cdot d\vec{l} \neq 0$ ($\vec{E}_{NC}$ is zero outside the battery). Inside the battery, electrons are driven by the total electric field $\vec{E}_{net} = \vec{E} + \vec{E}_{NC}$.

Demonstrations: “lemon” battery, dynamo

Mechanical analogy: van de Graaff generator

Mr. Bean: School Open Day
Electromotive Force

- the electromotive force (the emf)

Units: volts

Ideal battery: no energy dissipation inside the battery (the internal resistance = 0)

\[ \mathcal{E} = V = IR \]

\[ \mathcal{E} \equiv \int_{-}^{+} \vec{E}_{NC} \cdot d\vec{l} \]

\[ \vec{E}_{net} = \vec{E} + \vec{E}_{NC} \]

\[ V \] - voltage difference between the battery’s electrodes
Non-ideal Battery

$r$ – the *internal resistance* of the battery

What is the output voltage $V$?

$$R_{eq} = R + r$$

$$I = \frac{\mathcal{E}}{r + R}$$

$$V = \mathcal{E} - Ir = \mathcal{E} \frac{R}{r + R}$$

$$V = \mathcal{E} \frac{R}{r + R}$$
Battery Discharge

The internal resistance of a battery increases in the process of work: it is small \((in\ comparison\ with\ a\ typical\ load\ resistor)\) for a “fresh” battery, and becomes large for an “old” (discharged) one.

\[
V = \varepsilon \frac{R}{r + R} = \varepsilon \frac{1}{r/R + 1}
\]

**Figure 2.** Single Cell 300mA Discharge Curve.
Can we use a voltmeter (very high $r_{in}$) to test the “freshness” of a battery?

fresh battery = low $r$

The voltmeter will measure $V \approx \mathcal{E}$ provided $r \ll r_{in} V$. But $r_{in} V$ is at least as high as $10^7 \ \Omega$, and even if $r \sim 10^3 \ \Omega$, we won’t notice the battery aging.
Each charge carrier going “downhill” from a higher \( V_a \) to a lower \( V_b \) dissipates the energy \( eV_{ab} \) in its environment: the gained kinetic energy is transformed into thermal energy (“heat”) due to inelastic scattering.

If charge \( dq \) passes a circuit element in \( dt \), the power generated is:

\[
P = \frac{dqV_{ab}}{dt} = V_{ab}I
\]

Joule/s (resistive) heating power

For a resistor \( R \):

\[
P = VI = RI^2 = \frac{V^2}{R}
\]

Units of power: 1J/1s = 1 Watt (W)
What is the maximum power one can get (dissipate in the load) from a given \((\mathcal{E}, r)\) battery?

\[ \mathcal{E} = I(r + R) \]

\[ P = RI^2 = \frac{R}{(r + R)^2} \mathcal{E}^2 \]

\[
\frac{dP}{dR} \propto \frac{1}{(r + R)^2} - 2 \frac{R}{(r + R)^3} = 0
\]

\[ r + R - 2R = 0 \quad \Rightarrow \quad R = r \]

\[ P_{\text{max}} \rightarrow \quad @ \quad R = r \]
Conclusion

Batteries: the potential energy of charge carriers is increased by non-electrostatic (non-conservative) forces.

Non-ideal batteries: internal resistance.

Potential distribution around a complete circuit.

Energy and power in electric circuits.

Next time: Lecture 11: Connection of Resistors, Kirchhoff’s Rules
§§ 26.1 - 26.3
Voltage Source

**The goal:** to provide output voltage that is independent of the load resistance.

**An ideal voltage source:**

- A. has “zero” internal resistance ("zero" means that $r \ll$ all possible values of load $R$).
- B. has “infinite” internal resistance ("infinite" means that $r \gg$ all possible values of load $R$).
Current Source

The goal: to provide output current that is independent of the load resistance.

An ideal current source:

A. has “zero” internal resistance (“zero” means that $r << \text{all possible values of load } R$).

B. has “infinite” internal resistance (“infinite” means that $r >> \text{all possible values of load } R$).

$$I = \frac{\mathcal{E}}{r + R}$$