I. Superposition and current-carrying wires.

The figure to the right shows cross sections of two wires carrying current $i_0$, with wire A carrying current into the page and wire B carrying current out of the page.

1. Draw a vector on the diagram to show the direction of the net magnetic field vector at point P, if one exists. Explain in words your reasoning.

We now wish to place a third wire C (carrying the same current $i_0$, directed out of the page) such that there is zero net magnetic field magnitude at point P.

2. If we consider point P the origin, in which quadrant should the wire C be located: I, II, III, or IV?

3. Is the distance from wire C to point P greater than, less than, or equal to the distance from either of the original wires to point P?

4. Indicate on the diagram the approximate location of wire C such that there is zero magnetic field magnitude at point P. Explain in words how you determined your answer.

II. Exploring Ampere’s Law.

Since the magnetic field circulates around the wire rather than following lines that are radial to it (such as electric field lines from a point charge), Ampere’s law relates the current enclosed by the loop to the “circulation” along the loop, which describes both how large the magnetic field is and how well-aligned it is with the Amperean loop.

1. Redraw the picture at the right from a top down view.
2. What quantity will contribute to increasing the value of the “circulation”, \( \mathbf{B} \cdot \Delta \mathbf{L} \) or \( |\mathbf{B} \times \Delta \mathbf{L}| \)? Explain in words your reasoning. Recall circulation of a vector field gives us an idea as to how a vector field is oriented along (NOT perpendicular) a closed curve.

Ampere’s law is written more formally (in the limit as \( \Delta \mathbf{L} \) goes to zero):

\[
\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{enc}
\]

The circle in the integral sign means the line integral is calculated around a closed Amperean loop. This integral is the circulation, and it is proportional to the enclosed current.

III. Application of Ampere’s Law.

1. The figure shows a cross-sectional view of several conductors that carry current through the plane of the figure. The currents have the magnitudes \( I_1, I_2, I_3 \) with the directions shown. Four paths labeled \( a \) through \( d \) are also shown.

What is the value of the line integral, \( \oint \mathbf{B} \cdot d\mathbf{L} \), for each path? Each integral involves going around the path in the counterclockwise direction. Explain in words your reasoning.

a. 

b. 

c. 

d.
2. Consider the cylindrical wire of infinite length shown to the right. Unlike the infinitely thin wire that we considered in the Pre-Recitation, this time the wire has a finite circular cross-section with radius \( R \), shown in the figure. The total current \( I \) is distributed over the cross-sectional area \( A \) of the wire. The amount of current passing through a given area is called the current density typically given by the letter \( j \), such that \( j = I/A \), with units of Amperes per square meter.

a. Suppose a wire carries a total current \( I \) over cross sectional area \( A \). In terms of \( a \), \( A \), and \( I \) calculate the amount of current that passes through area \( a \) of the wire, assuming that \( a < A \). [Hint: if we enclose only a piece of the conductor as shown in the dashed circle, does all of the current pass through that dashed circle or only a portion of it?]

b. In the picture above, assuming that \( r \) is the radius describing the area \( a \) from the last question, how much current is enclosed inside the dashed circle? Write your answer in terms of \( I \), \( r \), and \( R \) [Hint: what is the area of the dashed circle and how does that compare to the total cross sectional area of the wire?]

3. For the next questions, we want to use Ampere’s law to find the magnitude of magnetic field \( B(r) \) at a distance \( r \) away from the center of the cylindrical wire shown in the figure above. [Hint: The dashed circle is the Amperean loop that you should consider. Does the current distribution have translational and rotational symmetry? Can you generalize the method developed in the Pre-Recitation for an infinitely thin and infinitely long current carrying wire for this situation?]

a. What is the value of the line integral, \( \oint \vec{B} \cdot d\vec{L} \), for the dashed circle?

b. How much current is enclosed by the dashed circle \( (I_{enc}) \) in terms of the total current \( I \)?

c. Use the fact that \( \oint \vec{B} \cdot d\vec{L} = \mu_0 I_{enc} \) and solve for \( B(r) \).

4. Use Ampere’s law along with symmetry considerations to find the magnetic field outside the wire as a function of distance from the center, \( B(r) \) for \( r > R \). Explain in words your reasoning. [Hint: the process is very similar to what you have done for question 3 above.]

5. Does the field outside of the wire depend on the radius of the wire, \( R \)? Explain in words your reasoning.