Part I: The parallel plate capacitor

Two large thin conducting plates are a distance $D$ apart and the space between them is filled with air. The surface area of the face of each plate is $A$. A side view of a small portion near the center of the plates is shown. The plates are connected to opposite terminals of a battery in order to charge them up. After being disconnected from the battery, the inner surface of the right plate has a uniform charge density of $+\sigma_o$; the left plate, $-\sigma_o$. The charge density on the outer surface of each plate is zero.

A. For the following questions, recall that the magnitude of the electric field created by an infinite sheet of charge with charge density $\sigma$ is given by $E = \sigma/2\varepsilon_0$.

1. Draw and label two vectors at each marked point on the diagram, one, $E_+$, to represent the electric field created by the positive surface charge and another, $E_-$, to represent the field created by the negative charge.

2. Now apply the principle of superposition to determine the magnitude and direction of the net electric field at each point. Show your work and/or explain.

3. Is the magnitude of the net electric field at point 2 greater than, less than, or equal to the magnitude of the field at point 3? Explain in words your selection.

4. Use your results to find the voltage of the battery that was used to charge up the plates. Express your answer in terms of the given quantities $A$, $D$, and $\sigma_o$. Show your work. (HINT: Think about the relationship between the potential difference between two locations and the electric field in the space between those locations.)
B. The right plate is moved to the left as shown. (Both plates are kept insulated, and are not connected to the battery.) Describe how each of the following quantities will change (if at all). Explain.

- the charge density on each plate

- the electric field both outside and between the plates

- the potential difference between the plates

C. As you found in the pre-recitation assignment, the capacitance of two separated conductors is defined as the ratio of the charge Q on each conductor and the potential difference $\Delta V$ between the conductors (i.e., $C = \frac{Q}{\Delta V}$).

1. What is the capacitance of the system in part A.4? Express your answer in terms of the given quantities ($D$, $A$, and $\epsilon_0$).

2. What is the capacitance of these plates when they are separated by the new distance $d$. (part B.) Express your answer in terms of the given quantities ($d$, $A$, and $\epsilon_0$).

3. How, if at all, would the capacitance change if the charge densities on the plates were doubled? Explain in words.

D. Summary questions. For a parallel plate capacitor, does the capacitance depend on:

- the distance between the plates?

- the size of the plates?

- the amount of charge on the plates?
Part II: Application question

A parallel plate capacitor of capacitance $C$ is charged so that it has a potential difference $V$ between the plates and then is disconnected from the battery that charged it. The capacitor is then connected in an electric circuit to a motor and a switch as shown. When the switch is closed, the motor turns (infinitely slowly, such that the block moves infinitely slowly as well and has no kinetic energy), raising a mass $m$ a vertical distance $L$. In this problem, make the simplifying assumption that no energy is lost to friction or heating in the motor.

The right is a graph of energy vs the vertical position of the block, $y$.

1. Label the line on the graph that represents the gravitational potential energy, $U_G(y)$, associated with mass $m$.

2. Describe what the remaining line on the energy graph represents. What object(s) are included in that system?

3. Add a line to the graph to represent the electrical energy stored in the capacitor, $U_E(y)$.

4. Write an equation that shows how the three energies, $E_{tot}(y)$, $U_G(y)$, and $U_E(y)$, depicted in the graph are related to each other.

5. Suppose that the capacitor has capacitance $C = 0.10 \text{ F}$ and is charged to a potential difference of 10 V. Find the vertical distance that a mass $m = 1.0 \text{ kg}$ could be lifted. Show your work and remember to use correct units and 2 significant figures. You can assume $g = 10. \text{ m/s}^2$. 