Lecture

- Review: Conductors
- Electric Field from Electric Potential
- Parallel plates
- Capacitance
An **equipotential** is a line or surface over which the potential is constant.

- Electric field lines are perpendicular to equipotentials.
- The surface of a conductor is an equipotential.
ICLICKER QUESTION

Which point corresponds to the greatest magnitude of the electric field?

\[ \Delta V_{ba} = - \int_a^b \vec{E} \cdot d\vec{\ell} \]

a) A
b) B

c) C
d) D
e) They are all the same.
Electric Field From Electric Potential

• The potential difference between two points a distance $d\ell$ apart is:

$$dV = -\vec{E} \cdot d\ell$$

• Consider a field entirely in the x-direction, then:

$$dV = -Edx$$

$$\Rightarrow E = -\frac{dV}{dx}$$

• That is, the $x$-component of the electric field is:

$$\vec{E}_x = -\frac{dV}{dx} \hat{i}$$
Determining \( \vec{E} \) from \( V \)

- In general for any field \( \vec{E}(x, y, z) \) and \( d\vec{\ell} \):

\[
\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}
\]

\[
d\vec{\ell} = \hat{i} dz + \hat{j} dy + \hat{k} dz
\]

\[
dV = -\vec{E} \cdot d\vec{\ell}
\]

\[
-dV = E_x dx + E_y dy + E_z dz
\]

\[
\vec{E} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)
\]

\[
\vec{v} = \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \text{[Gradient Operator]}
\]
ICLICKER QUESTION

Consider two concentric spherical conducting shells. One shell of radius “b” and charge “-Q” and one shell of radius “a” and charge “+Q”. What is the potential difference between the shells?

a) \( k \frac{Q}{b} \)

b) \( k \frac{Q}{a} \)

c) \( k \frac{Q}{b-a} \)

d) \( kQ \left[ \frac{1}{b} - \frac{1}{a} \right] \)

e) None of the other answers.
E-Field Between Parallel Plates

- A battery produces a specified potential difference $\Delta V$ between its terminals.
- Consider a 12V battery connected by conducting wires to two parallel conducting plates separated by a distance $d=0.3$ mm.

- The potential difference between the plates is equal to the potential difference between the terminals of the battery (12V).
- The electric field points from A to B and its magnitude is $E = \frac{|V_A - V_B|}{d} = 4 \times 10^3 \frac{V}{m}$.
Capacitance

A capacitor consists of two conductors that are close but not touching.

A capacitor has the ability to store electric charge.
(a) Parallel-plate capacitor connected to a battery.

(b) is a circuit diagram.
Capacitance

• When a capacitor is connected to a battery, the charge on its plates is proportional to the voltage: $Q \propto \Delta V$.

\[ Q = C\Delta V \]

• The proportionality constant $C$ is called the capacitance: (Capacitance $C = \frac{Q}{\Delta V}$).

• By definition, the capacitance “$C$”, is always a positive quantity (i.e. $Q$ and $\Delta V$ in the equation above are always expressed as positive quantities).
Units of Capacitance

Unit of capacitance: the farad (F)

$$1 \, F = 1 \frac{C}{V}$$

Michael Faraday
ICLICKER QUESTION

Capacitor $C_1$ is connected across a battery of 5 V. An identical capacitor $C_2$ is connected across a battery of 10 V. Which one has the most charge?

a) $C_1$

b) $C_2$

c) Both have the same charge

d) It depends on other factors

$Q = C \Delta V$
Find the capacitance of the spherical capacitor consisting of two concentric spherical shells one of radius “a” and one of radius “b”.

\[ Q = C \Delta V \]
\[ \Delta V = kQ \left[ \frac{1}{b} - \frac{1}{a} \right] = kQ \left[ \frac{b-a}{ab} \right] \]

a) \( kQ \left[ \frac{b-a}{ab} \right] \)

b) \( \frac{ab}{k(b-a)} \)

c) \( \frac{ab}{kQ(b-a)} \)

d) None of the other answers.
Capacitance

Note that the capacitance does not depend on the voltage “ΔV” or charge “Q”;

• it is a function of the geometry and the distance “d” between the conductors.
Capacitance

For a parallel-plate capacitor:

\[ |\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \]

\[ |\Delta V| = E d = \frac{Q d}{A\varepsilon_0} \]

\[ C = \varepsilon_0 \frac{A}{d} \quad [Parallel\,-\,plate\,Capacitor] \]

- The capacitance of a parallel plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.
ICLICKER QUESTION

A parallel-plate capacitor initially has a voltage of 400 V and stays connected to the battery. If the plate spacing is now doubled, what happens?

a) the voltage decreases  
b) the voltage increases  
**c) the charge decreases**  
d) the charge increases  
e) both voltage and charge change

\[ Q = C \Delta V \quad \quad C = \varepsilon_0 \frac{A}{d} \]