Energy in Electromagnetic Waves

- $|\hat{S}|$ is the rate of energy transferred per unit area.

$$|\hat{S}| = \frac{EB}{\mu_0}$$

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

- The time average of $\cos^2(kt - \omega t)$ is $\frac{1}{2}$ then, the wave intensity $I$ is:

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{cB_{max}^2}{2\mu_0}$$
Energy in Electromagnetic Waves

- Recall the energy density associated with an electric field is:
  \[ u_E = \frac{1}{2} \varepsilon_0 E^2 \]

- And the energy density associated with a magnetic field is given by:
  \[ u_B = \frac{1}{2\mu_0} B^2 \]

- We note that since \( \frac{E}{B} = c \) and \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \):
  \[ u_E = \frac{1}{2} \varepsilon_0 B^2 c^2 = \frac{1}{2\mu_0} B^2 \]
Energy in Electromagnetic Waves

- **Observation:** The instantaneous energy density associated with the magnetic field of an electromagnetic wave equals the instantaneous energy density associated with the electric field.

- **Instantaneous energy density:**
  \[ u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0} \]

- Then the average energy density of an electromagnetic wave is:
  \[ u_{av} = \frac{1}{2} \epsilon_0 E_{max}^2 = \frac{B_{max}^2}{2\mu_0} \]
  \[ \Rightarrow I = S_{av} = cu_{av} \]
Momentum and Radiation Pressure

- Electromagnetic waves carry momentum as well as energy so as this momentum is absorbed by some surface, pressure is exerted on the surface.

- An em-wave of energy $U$ at normal incidence to a surface and being completely absorbed will carry a momentum of magnitude:

  $$p = \frac{U}{c}$$

- Then the radiation pressure is:

  $$P = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{1}{c} \left( \frac{1}{A} \frac{dU}{dt} \right)$$
Momentum and Radiation Pressure

• The energy arriving at the surface per unit area is the magnitude of the pointing vector “S”.

• Then the radiation pressure exerted on a surface is:

\[
P = \frac{S}{c} \quad [\text{Complete Absorption}]
\]

\[
P = \frac{2S}{c} \quad [\text{Complete Reflection}]
\]
ICLICKER QUESTION

What is the magnitude of $\vec{S}$ measured 1m from a 100 W lightbulb?

a) 1 W/m$^2$
b) 4 W/m$^2$
c) 2 W/m$^2$
[d)] 8 W/m$^2$
e) 12 W/m$^2$
The amplitude of the electric field 10 km from a radio transmitter is $E_{\text{max}} = 0.2 \text{ V/m}$. What is the power emitted by the transmitter?

\[
I = S_{av} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{c B_{\text{max}}^2}{2\mu_0}
\]

$\mu_0 = 1.23 \times 10^{-6} \text{ N/A}^2$ \hspace{1cm} $c = 3.0 \times 10^8 \text{ m/s}$
Consider a circuit consisting of a source of emf, a resistor and a switch.

**Observation:** When the switch is closed the current does not jump immediately to the maximum value \( I_{\text{max}} = \frac{\varepsilon}{R} \). There is an induced (back) emf.

This effect is called self-induction because it arises from the circuit itself.
Self Inductance

- From Faraday’s law we can deduce that a self-induced emf $\mathcal{E}_L$ is always proportional to the time rate of change of the current.

\[ \mathcal{E}_L = -L \frac{dI}{dt} \]

- The proportionality constant $L$ is called the inductance of the coil.

- Inductance depends on the geometry of the coil and other physical characteristics.

- For an N turn coil: $L = N \frac{\Phi_B}{I}$
Unit of Inductance

- The SI unit for inductance is the henry (H):

\[ 1H = 1 \frac{V \cdot s}{A} \]

Joseph Henry (1797-1878)
Determine the inductance of a solenoid of \( N \) turns and length \( \ell \).

\[
L = N \frac{\Phi_B}{I}
\]
RL Circuits

• If a circuit contains a coil, such as a solenoid. The self-inductance prevents the current in the circuit from increasing or decreasing instantaneously.

• A circuit element that has a large self-inductance is called an inductor. The symbol for an inductor in a circuit is 🍀

• The inductance of the inductor results in a back emf so an inductor in a circuit opposes changes in the current in that circuit.
Consider a circuit with an emf, a resistor of resistance $R$ and an inductor of inductance $L$.

When the switch is closed (at time $t=0$), the current in the circuit begins to increase.

Since the current is increasing over time, $\mathcal{E}_L$ is negative. This indicates a decrease in potential from $a$ to $b$.

Then applying Kirchhoff loop rule:

$$\mathcal{E} - I(t)R - L \frac{dI}{dt} = 0$$
RL Circuits

• As the current increases though an inductor of inductance $L$:
  $$I(t) = \frac{\mathcal{E}}{R} \left( 1 - e^{-tR/L} \right)$$
  $$I(t) = I_{max} \left( 1 - e^{-t/\tau} \right)$$
  Time Constant $\tau = \frac{L}{R}$

• Considering a decrease in current over time (e.g. switch moving from a to b):
  $$IR + L \frac{dI}{dt} = 0$$
  $$I(t) = I_0 e^{-t/\tau}$$
ICLICKER QUESTION

A steady current flows through an inductor L as shown.

a) The potential is greater at point a than b.
b) The potential is less at point a than at point b.
[c) The potential at point a is the same to that at point b.
d) The answer depends on the inductance L.
Energy in B-Field

\[ \varepsilon - I(t)R - L \frac{dI}{dt} = 0 \]

- The rate at which the battery supplies energy is \( I \varepsilon \)
- The rate at which energy is being delivered to the resistor is \( I^2 R \).
- Then, the rate at which energy is being stored in the inductor:

\[ \frac{dU}{dt} = IL \frac{dI}{dt} = 0 \]
Energy in B-Field

• The total energy stored in an inductor of inductance $L$ is:

$$U_L = \frac{1}{2} LI^2$$

• If we consider a solenoid ($L = \mu_0 n^2 A \ell, B = \mu_0 nI$):

$$U_L = \frac{1}{2} \mu_0 n^2 I^2 A \ell$$

• The energy density ($u = U_L/\text{Volume}$) is then:

$$u_B = \frac{1}{2\mu_0} B^2$$
ICLICKER QUESTION

A current flows through an inductor L as shown. If the current is decreasing.

a) The potential is greater at point a than at point b.
b) The potential is less at point a than at point b.
c) The potential at point a is the same to that at point b.
d) The answer depends on the inductance L.