Physics 227: Lecture 9
Current, Resistance, Resistivity

- Lecture 8 review:
  - Energy Stored = \( U = \frac{Q^2}{2C} = CV^2/2 \)
  - Energy stored problems can be confusing – is charge or voltage held constant?
  - For constant charge, adding a dielectric of constant \( K \) increases capacitance a factor \( K \), decreases voltage a factor of \( K \), and decreases energy stored a factor \( K \).
  - (For constant voltage, energy stored increases a factor of \( K \).)
  - Force on dielectrics: pulled into constant charge capacitor, pushed out of constant voltage capacitor.
Fields in Conductors

• Earlier we have said that there is no field inside a conductor.
• If there were a field, the electrons would be forced to move, and would do so until they arranged in a pattern that made the internal field vanish.
• But there has been a somewhat hidden assumption here.
• We have no energy source that continues to supply energy to move the electrons around. For example...

In a conductor, electric field $\leftrightarrow$ current.
Moving Charges

- In conductors and many semiconductors, the moving charges are the electrons. The positive ions (atoms missing an electron) are generally fixed.
- In some semiconductors, positive "holes" are the charge carriers.
- In solutions and gases, the charge carriers can be positive charges, or negative charges, or both.
- The motion is not the smooth cartoon pictured. Electrons scatter from the more or less fixed atoms. You might picture a small drunk person weaving through a crowd of still large people.
If there is an E field, there is continually acceleration. The electrons must then accelerate and get increasing speed, leading to a continually increasing current. Why doesn’t this happen?

A. The electric field E is only nonzero outside the conductor, so the electron velocity is constant inside.

B. When the electrons scatter, on average their velocity is ≈0, so they never get that fast between scatterings.

C. The current does not increase when the electrons move faster.

D. When the electrons scatter, some of them keep going backwards.

E. The question is wrong – it does happen.
If there is an E field, there is continually acceleration. The electrons must then accelerate and get increasing speed, leading to a continually increasing current. Why doesn’t this happen?

A. The electric field $E$ is only nonzero outside the conductor, so the electron velocity is constant inside.

B. When the electrons scatter, on average their velocity is $\approx 0$, so they never get that fast between scatterings.

C. The current does not increase when the electrons move faster.

D. When the electrons scatter, some of them keep going backwards.

E. The question is wrong – it does happen.
**Buildup of Charge?**

- Is there a buildup of charge?

- The motion of charges does not require a buildup of charge.

- When we consider the steady motion of charges, there is no buildup of charge. We have a complete path, and charges circulate around it.

- In poor analogy, like the motion of cars on a street with no traffic lights or stop signs.

- Note that we will have situations in which current is not steady and charge builds up – like the charging of a capacitor.
The light bulb is an indicator of current.

If the circuit is not complete – air is between points a and b – there is no current flow.

Put in some water, and the circuit is complete. Current flows.

Tap water contains + and - ions that are the charge carriers. E.g.: salt breaks up into Na\(^+\) Cl\(^-\).
Current

- I = ΔQ/Δt, in Amperes = Coulombs/second.
- Current I > 0 if:
  - +Q moves in + direction.
  - -Q moves in - direction
- Current I < 0 if:
  - +Q moves in - direction
  - -Q moves in + direction
- Assume a constant charge density n per unit volume, charge e, constant common velocity v_d, and wire cross sectional area A.
  - There is a current density j = nev_d, in Amperes/m^2.
  - There is a current I = neA v_d Δt/Δt = neAv_d.
What is kept constant as the current flows in wire A and wire B? There may be more than 1 right answer.

- Assume a constant charge density $n$ per unit volume, charge $e$, constant common velocity $v_d$, and wire cross sectional area $A$.
  - There is a current density $j = nev_d$, in Coulombs/(m$^2$s).
  - There is a current $I = neA v_d \Delta t / \Delta t = neA v_d$. 

Options:

A. The electric field $E$.
B. Drift velocity $V_D$.
C. Current density $j$.
D. Current $I$.
E. Charge density $n$. 

Monday, October 3, 2011
What is kept constant as the current flows in wire A and wire B? There may be more than 1 right answer.

- Charge density is a property of the material and does not change. Current is constant so there is no buildup of charge. The current density then has to change since $j = I/A$. The drift velocity then has to change since $j = nqV_d$. The electric field then has to change since the drift velocity depends on the electric field.
**Current as a vector and Drift velocity**

- There is a current $I = neA\nu_d \Delta t / \Delta t = neA\nu_d$.
- So current is a vector... but
  - as the electrons move along a wire, they regularly scatter from atoms and are re-accelerated by the electric field.
  - As the wire bends the current changes direction
  - So while the magnitude of the current is a concern the direction may be unimportant.

**Diagram:**
- Air resistance leads to terminal velocity

**Diagram:**
- Atomic scattering / resistance leads to terminal velocity for electrons: $v_D$. 

Monday, October 3, 2011
Ohm’s Law

- As the electrons scatter from the atoms recoil - they gain kinetic energy
- The wire heats up.
- The distance between collisions depends on the structure of the material. The drift velocity is proportional to the electric field.

Atomic scattering / resistance leads to terminal velocity for electrons: \( v_D \).

- Bigger \( E \) → larger \( a \) → bigger \( v \) before next collision in same distance

- We define a resistivity by relating the current density \( j \) to the electric field \( E \): current density \( j = nev_d = E/\rho \)

- Units of \( \rho = E/j \): (V/m) / (A/m^2) = Vm/A = \Omega m, with 1\( \Omega = 1V/A \)
Resistivity and conductivity

Resistivity is a measure of how difficult it is to accelerate electrons / how much they scatter off atoms / what the drift velocity is.

- low resistivity: large current for applied $E$
- large resistivity = small current for applied $E$

One can also use conductivity $\sigma = 1/\rho$, the same symbol but NOT the same meaning as surface charge density.
Temperature Dependence

Resistivity generally depends on temperature. A simple picture: when conductors are warmer, the atoms vibrate back and forth more, so they appear bigger and cause more scattering of electrons. So typically resistivity increases with temperature.

For semiconductors, the main effect is that as the temperature increases, the density of charge carries increases (more electrons are free to move), so the resistivity decreases.
Typical resistivities

<table>
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<th>( \rho(\Omega \text{m}) )</th>
<th>( \alpha(\degree \text{C})^{-1} )</th>
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<tbody>
<tr>
<td>metals</td>
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<tr>
<td>Cu</td>
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<tr>
<td>insulator</td>
<td>S</td>
<td>\approx 10^{15}</td>
</tr>
</tbody>
</table>

Coefficient for temperature dependence or resistivity:

\[
\rho(T) = \rho_0 [1 + \alpha(T - T_0)]
\]
Superconductivity

In 1911, Onnes discovered superconductivity—below some temperature around a few -20 K, depending on material, resistivity = 0 (for some materials). Need to be cooled with liquid He (about 4 K).

Explained (Bardeen, Cooper, Schreiffer) as from Cooper pairs - interactions lead to pairs of electrons that do not scatter from atoms in the material.

High temperature superconductors first discovered in 1987. Now known up to \( \approx 200 \) K. Can be cooled with cheap liquid N (77 K).

With the vanishing resistivity, electrons move to eliminate magnetic fields from conductors.
Ohm’s Law

Use $E = \rho j$ and $V = Ed$ to obtain:

$$V = \rho jd$$

Use $j = I/A$ to obtain:

$$V = I \rho d/A$$

Use resistance $R = \rho d/A$ to obtain:

$$V = IR$$

Units of $\rho$ are $\Omega m$, so units of $R$ are $\Omega$. Wires can follow arbitrary paths and in principle vary in thickness, so one might have to do an integral to calculate the resistance, or one can simply do a measurement and use $R_{ab} = V_{ab}/I$.

Since $R = \rho d/A$, they have the same temperature dependence:

$$\rho(T) = \rho_0 [1+\alpha(T-T_0)] \Rightarrow R(T) = R_0 [1+\alpha(T-T_0)]$$
Ohm’s Law

For an “Ohmic” resistor:
\[ V = IR \]

For a semiconductor diode, \( I = f(V) \), as shown.

For other circuit elements, we will see other relationships between current and voltage. E.g. for a capacitor, \( Q = CV \) and \( I = dQ/dt \Leftrightarrow I = C \frac{dV}{dt} \).

Since \( R = \rho d/A \), they have the same temperature dependence:
\[ \rho(T) = \rho_0 [1+\alpha(T-T_0)] \Rightarrow R(T) = R_0 [1+\alpha(T-T_0)] \]
What happens to the brightness of the bulb when the resistor is dipped into the coolant?

A. It gets brighter.
B. Nothing.
C. It gets dimmer.
D. It gets so dim it goes out.
E. It gets so bright it blows up!
Current iClicker

What happens to the brightness of the bulb when the resistor is dipped into the coolant? You saw the demo!

A. It gets brighter.
B. Nothing.
C. It gets dimmer.
D. It gets so dim it goes out.
E. It gets so bright it blows up!
Thank you.

Thursday, Oct 6, 9:40 - 11:00 PM EXAM:
Morning lectures are replaced by optional morning review session.

If you have a conflict (or ≥2 other exams that day) e-mail to Jolie Cizewski
cizewski@physics.rutgers.edu ASAP

Evening exam 9:40 - 11:00 PM.
If your last name starts with:

A-I       Go to:       ARC103
J-M       SEC 111
N-R       Physics Lecture Hall
S-Z       HILL 114