Homework #2 is due Tuesday at 11:59PM

Quiz this Friday/Monday will be on Gauss’ law topics

Hour exam 1 is coming up in just under three weeks (Sunday October 6 6:10-7:30) 
If you have an accommodation letter and need testing at ODS, send email to rabe@physics.rutgers.edu
Quick review: Definition of *electric potential* $V(r)$

Electric field $\mathbf{E}(r)$ produced by some fixed charge distribution
Choose a reference point (could be the origin, infinity…)

$V(r) = -(\text{work done on charge } q \text{ by electric force on a path that starts at the reference point and ends at } r) / q$

$V(r)$ is well defined because the work done is the SAME FOR ANY PATH and is proportional to the charge $q$

If I know $V(r)$, I can find the work done by the electric force on a particle of charge $q$ on any path

Potential energy $U(r) = qV(r)$

(A new unit of energy: $\text{eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ J/C} = 1.6 \times 10^{-19} \text{ J}$)
Electric potential and potential energy

Solve problems with conservation of energy

For example:
- if a positive charge $q$ is released from rest a distance $r_0$ from another positive charge $Q$, how fast will it be moving when it’s very far away?
- If a negative charge at a distance $r_0$ from a positive charge $Q$ is shot off directly away from $Q$ at speed $v$, what is the maximum distance from $Q$ that it will reach?
Electric potential energy $U$ of a system of charges $q(V(r_B)-V(r_A))$ is minus the work done by electric force as $q$ moves from $r_A$ to $r_B$

= work done by an external agent to move $q$ from rest at $r_A$ to at rest at $r_B$

$U$ = total work done by external agent to assemble system

For a single charge $Q_1$ at $\vec{r}_1$, $U$=0, sets up potential $V_1(\vec{r})$

Bring in the next charge $Q_2$: force due to $Q_1$

$$U = Q_2 V_1(\vec{r}_2) = kQ_1Q_2 / r_{12}$$

Bring in the next charge $Q_3$: force due to $Q_1$ and $Q_2$

$$\Delta U = Q_3 (V_1(\vec{r}_3) + V_2(\vec{r}_3)) = kQ_3Q_2 / r_{32} + kQ_3Q_1 / r_{31}$$

Electric potential energy $U$ = sum over all pairs

example of three charges arranged in a triangle
Quick review: Definition of electric potential $V(r)$

Electric field $E(r)$ produced by some fixed charge distribution
Choose a reference point (could be the origin, infinity…)

$V(r) = -(\text{work done per charge by electric force on a path that starts at the reference point and ends at } r)$
$V(r)$ is well defined because the work done is the SAME FOR ANY PATH and is proportional to the charge $q$

For a charged particle $q$ on a line with electric field $E(x)$, reference point $x_0$

$$V(x) = -\int_{x'=x_0}^{x'=x} E(x') \, dx'$$

Fundamental theorem of calculus: $E(x) = -\frac{dV}{dx}$
For a charged particle $q$ on a line with electric field $E(x)$, reference point $x_0$

$$V(x) = -\int_{x' = x_0}^{x' = x} E(x) \, dx$$

Fundamental theorem of calculus: $E(x) = -\frac{dV}{dx}$
Relation between E field and potential on a line

**Calculation of potential from field (reference point at x=0)**

constant E: \( V(x) = -E \, x \)

non-constant E(x):

**Calculation of field from potential**

\( E(x) = -\frac{dV(x)}{dx} \)
In three dimensional space \((\vec{r} = x\hat{i} + y\hat{j} + z\hat{k})\)

\[
E_x(\vec{r}) = -\frac{\partial V(\vec{r})}{\partial x}
\]

\[
E_y(\vec{r}) = -\frac{\partial V(\vec{r})}{\partial y}
\]

\[
E_z(\vec{r}) = -\frac{\partial V(\vec{r})}{\partial z}
\]

If you know \(V(x,y,z)\) then the electric field \(\vec{E}(x, y, z)\) is obtained by partial derivatives

If you know the electric field \(\vec{E}(x, y, z)\) then you are looking for a single function \(V(x,y,z)\) that satisfies all three equations
If one exists, \(E(x, y, z)\) is called a “conservative” vector field

In electrostatics, the electric field is always a conservative vector field. How do we know this?
Electric field of point charge $Q$

$$\vec{E}(\vec{r}) = kQ / r^2 \hat{r}$$

$$V(\vec{r}) = kQ / r$$

$$= kQ / \sqrt{x^2 + y^2 + z^2}$$

$$E_x(\vec{r}) = -\frac{\partial V}{\partial x}$$

$$E_y(\vec{r}) = -\frac{\partial V}{\partial y}$$

$$E_z(\vec{r}) = -\frac{\partial V}{\partial z}$$

Check! Electric field of point charge passes test to be a conservative vector field
All electric field configurations in electrostatics are produced by fixed charge distributions

Sum of fields of teeny-tiny point charges

BUT the sum of conservative vector fields is conservative

So any electric field produced by a fixed charge distribution is conservative
How to know if a vector field is conservative?

**YES** if you can find a function $\phi$ that works

Two other tests you learn in multivariable calculus

**YES** if the cross derivatives are all equal.

**NO** if there is any closed loop path along which the line integral is not zero (line integral depends on path)
Clicker:

The vector field shown in the figure is
(a) conservative
(b) nonconservative
(c) not enough information to determine
Clicker:

The vector field shown in the figure is
(a) conservative
(b) nonconservative
(c) not enough information to determine
The electric potential $V(r)$ provides a completely equivalent alternative specification of the electric field

$$\vec{E}(\vec{r}) \rightarrow V(\vec{r})$$

$$V(\vec{r}) \rightarrow \vec{E}(\vec{r})$$

One advantage of $V$: if you are given the charge distribution, it is easier to do the integral for the potential $V$ and then get the field from $V$, than to do the integral for $\vec{E}$ directly
Potential of a point charge at the origin

\[ \vec{E}(\vec{r}) = \frac{kQ}{r^2} \hat{r} \]
\[ V(\vec{r}) = \frac{kQ}{r} \]

(reference point is at \( r = \infty \))
Potential of a point charge not at the origin

\[ V(\vec{r}) = \frac{kQ}{|\vec{r} - \vec{r}_Q|} \]

Simpler than the expression for electric field

\[ \vec{E}(\vec{r}) = \frac{kQ}{|\vec{r} - \vec{r}_Q|^3} (\vec{r} - \vec{r}_Q) \]
Potential of a collection of point charges

Add up the contributions from each charge

$$V(\vec{r}) = \sum_i kQ_i / |\vec{r} - \vec{r}_i|$$

**CHECKPOINT 4**

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point $P$ by the protons, greatest first.
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $dV = (k/r) \, dQ$

Uniform wire on the line

\[ V(x) = \int_{s=x_L}^{s=x_R} \frac{k\lambda ds}{|s-x|} \]

For points to the left of the wire

\[ V(x) = k\lambda \ln\left(\frac{x-x_R}{x-x_L}\right) \]
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $dV = (k/r) \, dQ$

Uniform wire in circular arc
$V$ at center of the circle $P$

$r=R$ for ALL SEGMENTS

$$V = \int \frac{kdQ}{R} = \frac{k}{R} \int dQ = \frac{kQ}{R}$$
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $dV = (k/r)\,dQ$

**Uniform ring**
V at points on the axis

$$r = \sqrt{z^2 + R^2} \text{ for all segments}$$

$$V(z) = \int \frac{k\,dQ}{\sqrt{z^2 + R^2}} = \frac{k}{\sqrt{z^2 + R^2}} \int dQ = \frac{kQ}{\sqrt{z^2 + R^2}}$$
Potential for general radial electric field \( \vec{E}(\vec{r}) = E(r)\hat{r} \)

\[
V(r) = -\int_{r'=\infty}^{r=r} E(r') \, dr' = \int_{r'=r}^{r'=\infty} E(r') \, dr'
\]

Works for point charge at the origin
Use for any system with radial electric field:
unform charged sphere, hollow sphere …
Potential of a spherical shell (charge Q, radius R)

For r > R

$$\vec{E}(\vec{r}) = \frac{kQ}{r^2} \hat{r}$$

$$V(\vec{r}) = \int_{r'\rightarrow\infty} \frac{kQ}{r'^2} \, dr' = \frac{kQ}{r}$$

Same as for point charge Q

(reference point is at $r = \infty$)

For r < R

E is zero -> V is constant

$$V(\vec{r}) = \frac{kQ}{R}$$