Homework #1 is due tonight at 11:59PM

Hour exam 1 is coming up in just under three weeks (Sunday October 6 6:10-7:30)
If you have an accommodation letter and need testing at ODS, send email to rabe@physics.rutgers.edu
GAUSS’ LAW RECAP

• Relates charge distribution and electric field; equivalent to Coulomb’s law

• New concept: electric flux through a closed surface

• You can find the electric flux through a closed surface if you know the charge distribution (just integrate the charge inside)

• You can find the charge inside a closed surface if you know the electric flux through the surface

• Very simple way to find the electric field of a symmetrical charge distribution: spherical shell, cylindrical shell, plane
potential energy: review from 123H/124H
Ball of mass m thrown straight up into the air with initial velocity

$$\vec{v}_0 = (0, 0, v_0)$$

let's put the origin at the starting point \((x_0, y_0, z_0) = (0, 0, 0)\)

How high will it go?

COULD SOLVE N2 (\(\mathbf{F} = m\mathbf{a}\)) FOR \(z(t)\) and find maximum value
Ball of mass m thrown straight up into the air with initial velocity

\[ \vec{v}_0 = (0, 0, v_0) \]

let’s put the origin at the starting point \((x_0, y_0, z_0) = (0, 0, 0)\) 
How high will it go?

BETTER: use conservation of energy
Kinetic energy + potential energy is same at the start and at the highest point
Kinetic energy = \( \frac{1}{2}mv^2 \)
Gravitational force is uniform so potential energy = \( mgz \)
\( \frac{1}{2}mv_0^2 + 0 = 0 + mgz_{\text{max}} \) so \( z_{\text{max}} = \frac{v_0^2}{2g} \)
Solve similar problems for $1/r^2$ force \[ \vec{F}(r) = \frac{GMm}{r^2} \hat{r} \]

If I shoot a rocket with mass $m$ and initial speed $v_0$ directly outward from the surface of a spherical planet of radius $R$ and mass $M$, how far from the center of the planet will it get before it falls back?

use conservation of energy
Potential + kinetic energy same at start and at highest point
potential energy $U(r) = -\frac{GmM}{r}$

\[(1/2)mv_0^2 - \frac{GmM}{R} = 0 - \frac{GmM}{R_f}\]

*(note – if not directly outward, rocket will have a nonzero tangential velocity at highest point. Use conservation of angular momentum to get this velocity as a function of $r$)*
Chapter 24

Electric Potential
Electric field of a point charge $\vec{E}(\vec{r}) = kQ / r^2 \hat{r}$
Solve similar problems for $1/r^2$ force  \[ \vec{F}(\vec{r}) = \frac{kQq}{r^2} \hat{r} \]

If I shoot a particle with charge $q$, mass $m$ and initial speed $v_0$ outward from the surface of a spherical shell of radius $R$ and charge $Q$, how far from the center of the shell will it get before it falls back (neglect the gravitational force)?

use conservation of energy
Potential + kinetic energy same at start and at highest point
potential energy $U(r) = -\frac{kQq}{r}$

\[
(1/2)mv_0^2 - kQq/R = 0 - kQq/R_f
\]
Uniform electric field \( \vec{E}(\vec{r}) = \vec{E} \)
(produced by an infinite uniform sheet of charge)
\( W_{BA} \) = work done by the electric force on a charge \( q \) that is moved from A to B

\[
\vec{E}(\vec{r}) = -E_0 \hat{j}
\]

\[
\vec{F} = q \vec{E}
\]

Choose a path
Work = product of the component of force along the motion and the displacement
\[
= 0 + (-qE_0)(y_B - y_A) = -qE_0(y_B - y_A)
\]
\( W_{BA} \) = work done by the electric force on a charge \( q \) that is moved from \( A \) to \( B \)

\[
\vec{E}(\vec{r}) = -E_0 \hat{j}
\]

\[
\vec{F} = q\vec{E}
\]

Choose a path
Work = product of the component of force along the motion and the displacement
= 0 + \((-qE_0)(y_B-y_A)\) = \(-qE_0(y_B-y_A)\)
Choose another path
Work = \((-qE_0)(y_B-y_A)\) + 0 = \(-qE_0(y_B-y_A)\)
$W_{BA} =$ work done by the electric force on a charge $q$ that is moved from A to B

$\vec{E}(\vec{r}) = -E_0 \hat{j}$

$\vec{F} = q \vec{E}$

Choose a path
Work = product of the component of force along the motion and the displacement

$= 0 + (-qE_0)(y_B-y_A) = -qE_0(y_B-y_A)$

Choose another path
Work $= (-qE_0 \cos \theta)L = (-qE_0) \frac{(y_B - y_A)}{L} L = -qE_0(y_B - y_A)$
$W_{BA} =$ work done by the electric force on a charge $q$ that is moved from $A$ to $B$

$\vec{E}(\vec{r}) = -E_0 \hat{j}$

$\vec{F} = q\vec{E}$

Choose a path
Work = product of the component of force along the motion and the displacement
$Work = 0 + (-qE_0)(y_B-y_A) = -qE_0(y_B-y_A)$

Choose ANY path – for uniform field, work will be the same
$W_{BA} = \text{work done by the electric force on a charge } q \text{ that is moved from } A \text{ to } B$

$\vec{E}(\vec{r}) = -E_0 \hat{j}$

$\vec{F} = q\vec{E}$

For a given path $C$, $W_{BA}$ is the line integral of $\vec{F}(\vec{r})$ along the path

$W_{BA} = \oint_C \vec{F} \cdot d\vec{l}$
Define change in potential energy $U_{BA} = - W_{BA}$
define potential energy function $U(x,y)$ by choosing a reference pt
$U(x,y) = -$ work done when particle is moved from the reference point to the point ($x,y$)
$U_{BA} = U_B - U_A$ (note: $U_B$ is shorthand for $U(x_B,y_B)$)
Change in potential energy $U_{BA} = - W_{BA}$

define potential energy function $U(x,y)$ by choosing a reference pt
$U(x,y) = -$ work done when particle is moved from the reference
point to the point $(x,y)$
$U_{BA} = U_B - U_A$ (note: $U_B$ is shorthand for $U(x_B, y_B)$)

Why the (confusing) minus sign in front of $W_{BA}$?

Recall the work-kinetic energy theorem
Work done on a particle changes the kinetic energy $W_{BA} = K_B - K_A$
Positive work increases the kinetic energy
Negative work decreases the kinetic energy
With the minus sign, $W_{BA} = U_A - U_B = K_B - K_A$
$K_A + U_A = K_B + U_B$
energy = kinetic energy + potential energy does not change
Change in potential energy $U_B - U_A = U_{BA} = - W_{BA}$

define potential energy function $U(x,y)$ by choosing a reference pt $U(x,y) = -$ work done when particle is moved from the reference point to the point $(x,y)$

$U_{BA} = U_B - U_A$

For uniform E field

$W_{BA} = -qE_0(y_B - y_A)$

choose reference point at the origin

$U(x,y) = qE_0y$

(check: $U = 0$ at the origin $(0,0)$)
For uniform E field, choose reference point at the origin
\( U(x,y) = qE_0 y \)
\( U = 0 \) at the origin \((0,0)\)

Solve certain kinds of problems by conservation of energy
\( K + U \)

Example:
proton starts at \((0,1\ m)\) with velocity \((0, 0.5\ m/s)\) in uniform field
\( \vec{E}(\vec{r}) = -4 N / C \hat{j} \)

What is its velocity when it reaches the point \((0,0)\)?
Where does it stop and turn around?
$W_{BA} =$ work done by the electric force on a charge $q$ that is moved from A to B

$$\vec{E}(\vec{r}) = \frac{kQ}{r^2}\hat{r}$$

$$\vec{F} = q\vec{E}$$

Work = component of force along the motion x displacement
Only displacements in the radial direction contribute

$W_{BA} = -kqQ(1/r_B - 1/r_A)$ independent of path
Reference point is at infinity (large $r$)
$U(r) = +kqQ/r$
General case:
\[ E = E_1 + E_2 + \cdots + E_N \]
\[ W = W_1 + W_2 + \cdots + W_N \]

\( W_1, W_2, \ldots W_N \) are independent of path
Therefore \( W \) is independent of path

The work done by the electric force is path independent for the electric field produced by any distribution of charges

can define a potential energy function \( U(\vec{r}) \)
Uniform field $U(x,y) = +qE_0y$
Field of point charge $U(r) = +kqQ/r$
Field of set of point charges $U(r) = +kq\Sigma Q_i/r_i$

Electric force is proportional to the charge $q$
$U$ depends linearly on the force on $q$ (doubling $E$ doubles $U$)
So the potential energy function $U$ is proportional to charge $q$

Define electric potential $V(\vec{r}) = U(\vec{r}) / q$

UNITs: Joules/Coulomb = N m/C = V (volt)
Uniform field $V(x,y) = +E_0y$
Field of point charge $V(r) = +kQ/r$
Field of set of point charges $V(r) = +k\Sigma Q_i/r_i$

Graphical representation of the electric potential
Contour plot = topographical map
equipotential lines at equal intervals in $V$
Important electric field configurations

Electric field of a point charge

Uniform electric field
Equipotential surfaces

On field line diagram, equipotential lines are perpendicular to field lines – no work done

Lines are drawn for values of $V(r)$ at equal intervals $0, 20 \text{ V}, 40 \text{ V}, 60 \text{ V} \cdots$

Compare two points on same diagram:
Spacing is smaller where magnitude of field is larger
Clicker

The three diagrams show equipotentials for three different electric fields covering the same size region of space. In which is the magnitude of the electric field the greatest?

(1) 20 V 40 60 80 100
(2) -140 V -120 -100
(3) -10 V -30 -50
The three diagrams show equipotentials for three different electric fields covering the same size region of space. In which is the magnitude of the electric field the greatest?
The electric potential $V(r)$ provides a completely equivalent alternative specification of the electric field

$$\vec{E}(\vec{r}) \rightarrow V(\vec{r})$$

$$V(\vec{r}) \rightarrow \vec{E}(\vec{r})$$

One advantage of $V$ that we will see in the next lecture: easier to do the integral for the potential, then get the field from $V$ than to do the integral for $E$ directly