Final exam is Monday morning, December 16 from 8-11 AM here (PLH)
You can use 3 (ie three) 8.5”x11” sides of self-prepared notes.
32-33 questions, same style as the hour exams
All material, with emphasis on Chapters 30 onward

Last homework is due WED 12/11 at 11:59PM
No office hours this Sunday or Monday. I will schedule an exam review for later next week.

Please fill out the CTAAR evaluation!
https://sirs.ctaar.rutgers.edu/blue
I will save a few minutes at the end of the lecture for this.
Recap/review

Maxwell’s equations in vacuum (no charges or currents) -> wave equations for each component of $E$ and $B$

$$\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0$$

$$\frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0$$

6 “independent” equations for the 6 field components

$$E_y(x, y, z, t) = E_{my} \sin(kx - \omega t + \phi_y)$$

More general

$$E_y(x, y, z, t) = E_{my} \sin((k_x x + k_y y + k_z z) - \omega t + \phi_y)$$

$$\vec{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$$
Electromagnetism in empty space

\[ \nabla \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0 \]

\[ \vec{k} \cdot \vec{E}_m = 0 \quad \vec{k} \cdot \vec{B}_m = 0 \]

E and B fields are always perpendicular to the direction of propagation.
Electromagnetism in empty space

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{k} \times \vec{E}_m = \omega \vec{B}_m \]

\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]

Choice of \( E_m \) determines \( B_m \) (or vice versa)

\((\vec{E} \times \vec{B})\) is in the same direction as the propagation
Simple “linearly polarized” electromagnetic wave

Direction of propagation
Wavelength (or frequency)
Direction of the oscillating E field – perpendicular to prop
Amplitude of the oscillating E field: intensity = power/area
Direction of the oscillating E field = “polarization”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]
\[ B_m = \frac{E_m}{c} \]
Figure 33-28 shows the electric and magnetic fields of an electromagnetic wave at a certain instant. Is the wave traveling into the page or out of it?

(a) Into
(b) Out of
(c) I don’t know how to decide
Figure 33-28 shows the electric and magnetic fields of an electromagnetic wave at a certain instant. Is the wave traveling into the page or out of it?

(a) Into
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(c) I don’t know how to decide
Electromagnetic waves transport energy in the direction of propagation

Intensity = power/area
= energy/area in one wavelength / (one period $T = 2\pi/\omega$)

\[
\int_{x=0}^{x=\lambda} \left( \frac{1}{T} \left( \varepsilon_0 E_m^2 \sin(kx-\omega t)^2/2 + B_m^2 \sin(kx-\omega t)^2/(2\mu_0) \right) \right) dx
\]

\[
= \left( \frac{\lambda}{2T} \right) \left( \varepsilon_0 E_m^2/2 + B_m^2/(2\mu_0) \right)
\]

\[
= c \varepsilon_0 E_m^2/2
\]
CHECKPOINT 2

The figure here gives the electric field of an electromagnetic wave at a certain point and a certain instant. The wave is transporting energy in the negative z direction. What is the direction of the magnetic field of the wave at that point and instant?

(a) -y  
(b) +x  
(c) -x  
(d) +z  
(e) -z
The figure here gives the electric field of an electromagnetic wave at a certain point and a certain instant. The wave is transporting energy in the negative $z$ direction. What is the direction of the magnetic field of the wave at that point and instant?

(a) $-y$
(b) $+x$
(c) $-x$
(d) $+z$
(e) $-z$
33.5: Energy Transport and the Poynting Vector:

The direction of the Poynting vector \( \vec{S} \) of an electromagnetic wave at any point gives the wave’s direction of travel and the direction of energy transport at that point.

\[
\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad \text{(Poynting vector)}.
\]

Linearly polarized wave \( E \) is perpendicular to \( B \) and to direction of propagation.

\[
S = \frac{1}{\mu_0} EB, \quad S = \frac{1}{c \mu_0} E^2
\]

\[
I = S_{\text{avg}} = \left( \frac{\text{energy/time}}{\text{area}} \right)_{\text{avg}} = \left( \frac{\text{power}}{\text{area}} \right)_{\text{avg}} = \frac{1}{c \mu_0} [E^2]_{\text{avg}} = \frac{1}{c \mu_0} [E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.
\]

\[
E_{\text{rms}} = \frac{E_m}{\sqrt{2}}.
\]

\[
I = \frac{1}{c \mu_0} E_{\text{rms}}^2 = c \varepsilon_0 E_{\text{rms}}^2 = \text{speed} \times \text{energy per volume}
\]
Force exerted by a beam of particles

Depends on the rate of momentum transfer

“impulse” \( \int_{t_i}^{t_f} \vec{F}(t) \, dt = \Delta \vec{p} \) of the object

\[ F_{\text{avg}} = \frac{J}{\Delta t} = -\frac{n}{\Delta t} \Delta p = -\frac{n}{\Delta t} m \Delta v. \]

n is the number of particles that arrive in time \( \Delta t \)
minus sign because this \( \Delta p \) is the change in
momentum of each particle
Electromagnetic waves carry momentum

Momentum per volume
**Direction** is the direction of propagation
**Magnitude** = \( \frac{u}{c} \)
where \( u \) is the energy density

This means that light shining on a surface exerts a force on the surface
Radiation pressure

Pressure = force/area
Force depends on the rate of momentum transfer
\[ F_{\text{avg}} = - \frac{\text{change in momentum of the wave in time } \Delta t}{\Delta t} \]

Flat **absorbing** surface of area A perpendicular to the propagation
change in momentum of the wave in time \( \Delta t = - \left( \frac{I A \Delta t}{c} \right) \)
Pressure = \( \frac{I}{c} \)

Flat **reflecting** surface of area A perpendicular to the propagation
change in momentum of the wave in time \( \Delta t = -2 \left( \frac{I A \Delta t}{c} \right) \)
Pressure = \( 2 \frac{I}{c} \)
Simple electromagnetic wave

Direction of propagation and wavelength (or frequency) = $\vec{k}$
Amplitude of the oscillating E field: intensity $I = \text{power/area}$
Direction of the oscillating E field = “polarization”

$E_y(x, y, z, t) = E_m \sin(kx - \omega t)$
$B_z(x, y, z, t) = B_m \sin(kx - \omega t)$
$\lambda = 2\pi/k$ ; $\omega = ck$

Vertically polarized light headed toward you—the electric fields are all vertical.
An electric field component parallel to the polarizing direction is passed (transmitted) by a polarizing sheet; a component perpendicular to it is absorbed.
A component along the polarizing direction is transmitted.

Effect is to rotate the polarization and reduce the intensity proportional to \( E_m^2 \).
“Unpolarized” light
Random mixture of directions
  e.g. light bulb

One polarizing sheet:
Unpolarized light of intensity $I \rightarrow$ linearly polarized; $I = I_0/2$

Stack another polarizing sheet on top:
Rotate: no difference ---- completely black
Two polarizing sheets are stacked with directions at right angles and placed on the projector, and transmit no light. A third sheet with direction at 45 degrees to each is inserted between the two sheets. What is then true about the intensity $I$ of the light transmitted by the 3-sheet stack?

a) No light is transmitted
b) $I = I_0$
c) $I = I_0/2$
d) $I = I_0/4$
e) $I = I_0/8$

ANSWER WITH THE DEMO!
Two polarizing sheets are stacked with directions at right angles and placed on the projector, and transmit no light. A third sheet with direction at 45 degrees to each is inserted between the two sheets. What is then true about the intensity $I$ of the light transmitted by the 3-sheet stack?

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