Homework #3 is due tonight at 11:59PM

Hour exam 1 is coming up in just under three weeks (Sunday October 11 6:10-7:30 PM)

I will start posting practice material this week. We will have an in-class exam review Thursday, October 8

If you have individual considerations (accommodation letter that you have not yet sent to me, irreconcilable schedule conflict requiring a makeup exam) contact me by email trabe@physics.rutgers.edu
potential energy – a powerful concept for solving certain kinds of problems involving the motion of a particle under a force:

review from 123H/124H
Ball of mass $m$ thrown straight up into the air with initial velocity

\[ \vec{v}_0 = (0, 0, v_0) \]

let’s put the origin at the starting point $(x_0, y_0, z_0) = (0, 0, 0)$

How high will it go?

COULD SOLVE N2 ($F = ma$) FOR $z(t)$ and find maximum value

\[ -mg \hat{k} = ma \]
\[ \frac{d^2z}{dt^2} = -g \]
\[ z(t) - z(t = 0) = v_0 t - \frac{1}{2} gt^2 \]
\[ \frac{dz}{dt} = v_0 - gt \]

\[ \text{max} \quad z(t) - z(t = 0) = \frac{v_0^2}{2g} \]
Ball of mass $m$ thrown straight up into the air with initial velocity 

$$\vec{v}_0 = (0, 0, v_0)$$

let’s put the origin at the starting point $(x_0, y_0, z_0) = (0, 0, 0)$

How high will it go?

BETTER: use conservation of energy
Kinetic energy + potential energy is same at the start and at the highest point
Kinetic energy = $(1/2)mv^2$
Gravitational force is uniform so potential energy = $mgz$
$(1/2)mv_0^2 + 0 = 0 + mgz_{\text{max}}$ so $z_{\text{max}} = v_0^2/(2g)$

*You can do it in one line!*
Solve similar problems for $1/r^2$ force $\vec{F}(r) = \frac{GMm}{r^2} \hat{r}$

If I shoot a rocket with mass $m$ and initial speed $v_0$ straight upward from the surface of a spherical planet of radius $R$ and mass $M$, how far from the center of the planet will it get before it falls back?

use conservation of energy
Potential + kinetic energy same at start and at highest point
potential energy $U(r) = -\frac{GmM}{r}$

$(1/2)mv_0^2 - \frac{GmM}{R} = 0 - \frac{GmM}{R_f}$
Solve for $R_f$
Today:
Potential energy for a point charge (rocket) feeling electric force from a fixed charge distribution (Earth)

A new concept: electric potential $V(r) = \text{potential energy per charge}$
(just like electric field is electric force per charge)
$V(r)$ is determined by the electric field.

Energy needed to take charges at infinite separation and put them in a given arrangement “electric potential energy of the charge distribution”
Chapter 24

Electric Potential
Electric field of a point charge $\vec{E}(\vec{r}) = \frac{kQ}{r^2}\hat{r}$
Solve similar problems for $1/r^2$ force

$$\vec{F}(\vec{r}) = \frac{kQq}{r^2} \hat{r}$$

If I shoot a particle with charge $q$, mass $m$ and initial speed $v_0$ straight upward from the surface of a spherical shell of radius $R$ and charge $Q$, how far from the center of the shell will it get before it falls back (neglect the gravitational force)

use conservation of energy
Potential + kinetic energy same at start and at highest point

potential energy $U(r) = -\frac{kQq}{r}$

$$(1/2)mv_0^2 - kQq/R = 0 - kQq/R_f$$
Solve for $R_f$
Uniform electric field $\vec{E}(\vec{r}) = \vec{E}$

(produced by an infinite uniform sheet of charge)
\( W_{BA} = \) work done by the electric force on a charge \( q \) that is moved from A to B

\[
\vec{E}(\vec{r}) = -E_0 \hat{j}
\]

\[
\vec{F} = q \vec{E}
\]

Choose a path

Work = product of the component of force along the motion and the displacement

\[
= 0 + (-qE_0)(y_B - y_A) = -qE_0(y_B - y_A)
\]
$W_{BA} =$ work done by the electric force on a charge $q$ that is moved from $A$ to $B$

$$\vec{E}(\vec{r}) = -E_0 \hat{j}$$

$$\vec{F} = q\vec{E}$$

Choose a path
Work = component of force along the motion x displacement

$$= 0 + (-qE_0)(y_B - y_A) = -qE_0(y_B - y_A)$$

Choose another path
Work = $(-qE_0)(y_B - y_A) + 0 = -qE_0(y_B - y_A)$

THE SAME
\( W_{BA} \) = work done by the electric force on a charge \( q \) that is moved from \( A \) to \( B \)

\[
\vec{E}(\vec{r}) = -E_0 \hat{j}
\]

\[
\vec{F} = q \vec{E}
\]

Choose a path
Work = component of force along the motion x displacement
= \( 0 + (-qE_0)(y_B-y_A) = -qE_0(y_B-y_A) \)

Choose another path
Work = \((-qE_0 \cos \theta)L = (-qE_0) \frac{(y_B - y_A)}{L}L = -qE_0(y_B - y_A) \)

THE SAME
\[ W_{BA} = \text{work done by the electric force on a charge } q \text{ that is moved from } A \text{ to } B \]

\[ \vec{E}(\vec{r}) = -E_0 \hat{j} \]

\[ \vec{F} = q \vec{E} \]

Choose any path

Work = component of force along the motion \( x \) displacement

\[ \text{Work} = -qE_0(y_B - y_A) \]

For uniform field, work will be the same
\[ W_{BA} = \text{work done by the electric force on a charge } q \text{ that is moved from } A \text{ to } B \]

\[ \vec{E}(\vec{r}) = -E_0 \hat{j} \]

\[ \vec{F} = q \vec{E} \]

For a given path \( C \), \( W_{BA} \) is the line integral of \( \vec{F}(\vec{r}) \) along the path

\[ W_{BA} = \oint_C \vec{F} \cdot d\vec{l} \]
Define change in potential energy $U_{BA} = -W_{BA}$

define potential energy function $U(x,y)$ by choosing a reference pt
$U(x,y) = -$ work done when particle is moved from the reference point to the point $(x,y)$
$U_{BA} = U(x_B,y_B) - U(x_A,y_A)$
$= U_B - U_A$ (note: $U_B$ is shorthand for $U(x_B,y_B)$)
Change in potential energy $U_{BA} = - W_{BA}$

define potential energy function $U(x,y)$ by choosing a reference pt
$U(x,y) = -$ work done when particle is moved from the reference point to the point $(x,y)$
$U_{BA} = U_B - U_A$ (note: $U_B$ is shorthand for $U(x_B, y_B)$)

Why the (confusing) minus sign in front of $W_{BA}$?

Recall the work-kinetic energy theorem
Work done on a particle changes the kinetic energy $W_{BA} = K_B - K_A$
Positive work increases the kinetic energy
Negative work decreases the kinetic energy
With the minus sign, $W_{BA} = -U_{BA} = -(U_B - U_A) = U_A - U_B = K_B - K_A$
$K_A + U_A = K_B + U_B$
energy = kinetic energy + potential energy does not change
‘conservation of energy’
Change in potential energy $U_B - U_A = U_{BA} = - W_{BA}$

define potential energy function $U(x,y)$ by choosing a reference pt $U(x,y) = -$ work done when particle is moved from the reference point to the point $(x,y)$

$U_{BA} = U_B - U_A$

For uniform E field in the -y direction

$W_{BA} = -qE_0(y_B-y_A)$

choose reference point at the origin

$U(x,y) = -(-qE_0(y-0)) = +qE_0y$

(check: $U = 0$ at the origin $(0,0)$ as it should be)
For uniform $E$ field in the $-y$ direction, choose reference point at the origin
$U(x,y) = +qE_0 y$
$U = 0$ at the origin $(0,0)$

Solve certain kinds of problems by conservation of energy $K + U$

Example:
proton starts at $(0,1 \text{ m})$ with velocity $(0, 0.5 \text{ m/s})$ in uniform field

$\vec{E}(\vec{r}) = -4 N / C \hat{j}$

What is its velocity when it reaches the point $(0,0)$?
Where does it stop and turn around?
$W_{BA} = \text{work done by the electric force on a charge } q \text{ that is moved from A to B}$

$\vec{E}(r) = \frac{kQ}{r^2}\hat{r}$

$\vec{F} = q\vec{E}$

Work = component of force along the motion x displacement
Only displacements in the radial direction contribute
$W_{BA} = -kqQ(1/r_B - 1/r_A)$ independent of path
Reference point is at infinity (large r)
$U(r) = -(kqQ(1/r-0)) = +kqQ/r$
General case for electric field produced by N point charges
\[ E = E_1 + E_2 + \cdots + E_N \]
\[ W = W_1 + W_2 + \cdots + W_N \]
\( W_1, W_2, \cdots W_N \) are independent of path
Therefore \( W \) is independent of path

The work done by the electric force is path independent for the electric field produced by any distribution of charges

can define a potential energy function \( U(\vec{r}) \)
Uniform field in the $-y$ direction: $U(x,y) = +qE_0y$
Field of point charge: $U(r) = +kqQ/r$
Field of set of point charges: $U(r) = +kq\sum Q_i/r_i$

Electric force is proportional to the charge $q$
$U$ depends linearly on the force on $q$ (doubling $E$ doubles $U$)

So the potential energy function $U$ is proportional to charge $q$

Define electric potential $V(\vec{r}) = U(\vec{r}) / q$

UNITs: Joules/Coulomb = N m/C = V (volt)
Uniform field in the $-y$ direction: $V(x,y) = +E_0y$
Field of point charge: $V(r) = +kQ/r$
Field of set of point charges: $V(r) = +k\sum Q_i/r_i$

Graphical representation of the electric potential
Contour plot = topographical map
 equipotential lines at equal intervals in $V$
Important electric field configurations

Electric field of a point charge

Uniform electric field
Equipotential surfaces

On field line diagram, equipotential lines are perpendicular to field lines – no work done

Lines are drawn for values of $V(r)$ at equal intervals 0, 20 V, 40 V, 60 V…

Compare two points on same diagram:
Spacing is smaller where magnitude of field is larger
Poll question

The three diagrams show equipotentials for three different electric fields covering the same size region of space. In which is the magnitude of the electric field the greatest?
Poll question

The three diagrams show equipotentials for three different electric fields covering the same size region of space. In which is the magnitude of the electric field the greatest?
The electric potential $V(r)$ provides a completely equivalent alternative specification of the electric field

$$\vec{E}(\vec{r}) \rightarrow V(\vec{r})$$

$$V(\vec{r}) \rightarrow \vec{E}(\vec{r})$$

One advantage of $V$ that we will see in the next lecture: easier to do the integral for the potential, then get the field from $V$ than to do the integral for $E$ directly
Relation between E field and potential on a line

Calculation of potential from field (reference point at x=0)

constant E:

\[ V(x) = V(x = 0) - \int_{x'=0}^{x'=x} E(x') \, dx' = 0 - Ex = -Ex \]

non-constant E(x):

\[ V(x) = V(x = 0) - \int_{x'=0}^{x'=x} E(x') \, dx' \]

Calculation of field from potential

E(x) = \(-dV(x)/dx\) (fundamental theorem of calculus)
What about in three dimensional space?

The electric field $\vec{E}(\vec{r})$ – a vector at each point $\vec{r}$
The electric potential – a number at each point $\vec{r}$

Isn’t there a lot more information in $\vec{E}(\vec{r})$? How can $V(r)$ determine $E(r)$?
In electrostatics, the electric field is a conservative vector field.

Definition: a vector field \( F(x,y,z) \) is conservative if there exists a function \( \phi(x,y,z) \) such that

\[
F_x = -\frac{\partial \phi}{\partial x} \\
F_y = -\frac{\partial \phi}{\partial y} \\
F_z = -\frac{\partial \phi}{\partial z}
\]

(partial derivatives – wrt \( x \), hold \( y \) and \( z \) constant⋯)
If $\mathbf{F}(x,y,z)$ is conservative, then the line integral of $\mathbf{F}$ over a curve $C$ from point $\mathbf{r}_A$ to point $\mathbf{r}_B$ is equal to $\phi(\mathbf{r}_A) - \phi(\mathbf{r}_B)$.

(3D generalization of Fundamental Theorem of Calculus)

Depends only on the endpoints, not the path!

PATH INDEPENDENT

If curve $C$ is a closed loop from $\mathbf{r}_A$ back to $\mathbf{r}_A$, the line integral is ZERO.
How to test if a vector field is conservative?

**YES** if you can find a function $\phi$ that works

**YES** if the cross derivatives are all equal.

**NO** if there is any closed loop path along which the line integral is not zero.
Uniform electric field

\[ \vec{E}(\vec{r}) = \vec{E}_0 \]

\[ V(\vec{r}) = -\vec{E}_0 \cdot \vec{r} \]

\[ = -E_{0x}x - E_{0y}y - E_{0z}z \]

\[ E_x(\vec{r}) = -\frac{\partial V}{\partial x} = E_{0x} \]

\[ E_y(\vec{r}) = -\frac{\partial V}{\partial y} = E_{0y} \]

\[ E_z(\vec{r}) = -\frac{\partial V}{\partial z} = E_{0z} \]

Check! Electric field of point charge passes test to be a conservative vector field
Electric field of point charge $Q$

\[ \vec{E}(\vec{r}) = \frac{kQ}{r^2}\hat{r} \]

\[ V(\vec{r}) = \frac{kQ}{r} \]

\[ = \frac{kQ}{\sqrt{x^2 + y^2 + z^2}} \]

\[ E_x(\vec{r}) = -\frac{\partial V}{\partial x} \]

\[ E_y(\vec{r}) = -\frac{\partial V}{\partial y} \]

\[ E_z(\vec{r}) = -\frac{\partial V}{\partial z} \]

Check! Electric field of point charge passes test to be a conservative vector field
Clicker:

The vector field shown in the figure is
(a) conservative
(b) nonconservative
(c) not enough information to determine