The first common hour midterm exam will be held on Thursday October 1, 9:50 to 11:10 PM TODAY (at night) on the Busch campus. You should go to the room corresponding to the first 3 letters of your last name. If you have a conflict with the exam time, please contact Prof. Cizewski Cizewski@physics.rutgers.edu with your entire schedule for the week of September 28 at your earliest convenience but not later than 5:00 pm on Wednesday, September 23.

Aaa – Hoz      ARC 103
Hua – Moz      Hill 114
Mua – Shz      PHY LH
Sia – Zzz      SEC 111
Uncompensated charges reside on the surfaces, the total field is the sum of the external field and the field due to uncompensated charges.

Has the field outside been changed?

Dielectric constant: \( K \equiv \frac{E_0}{E_{in}} > 1 \)

\[
\frac{\sigma_i}{\varepsilon_0} = E_0 - E_{in} = E_0 \left(1 - \frac{1}{K}\right)
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0000</td>
</tr>
<tr>
<td>Air (1 atm)</td>
<td>1.0006</td>
</tr>
<tr>
<td>Paraffin</td>
<td>2.2</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.6</td>
</tr>
<tr>
<td>Vinyl (plastic)</td>
<td>2–4</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
</tr>
<tr>
<td>Quartz</td>
<td>4.3</td>
</tr>
<tr>
<td>Oil</td>
<td>4</td>
</tr>
<tr>
<td>Glass, Pyrex</td>
<td>5</td>
</tr>
<tr>
<td>Rubber, neoprene</td>
<td>6.7</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6–8</td>
</tr>
<tr>
<td>Mica</td>
<td>7</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>80</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>300</td>
</tr>
</tbody>
</table>

What would be \( K \) for metal?
Dielectric-filled Capacitors

The voltage source ($V=\text{const}$) is connected to (1) a “vacuum” capacitor and (2) the same capacitor filled with dielectric.

$$\Delta V = V$$

(charge on metallic electrodes)

(that’s what we measure/control)

charge on metallic electrodes

(1) $E_1 = \frac{V}{d} = E$

$\sigma_1 = \sigma_m = \varepsilon_0 E = \varepsilon_0 \frac{V}{d}$

$C_1 = \frac{Q}{V} = \frac{\sigma_m A}{V} = \varepsilon_0 \frac{A}{d}$

(2) $E_2 = \frac{V}{d} = E$

$\sigma_2 = \sigma_m - \sigma_i = \varepsilon_0 E$

$\sigma_i = \varepsilon_0 E \sin(K-1) = \varepsilon_0 E (K-1)$

$\sigma_m = \sigma_2 + \sigma_i = \varepsilon_0 E K$

$C_2 = \frac{Q}{V} = \frac{\sigma_m A}{V} = \varepsilon_0 K \frac{A}{d} = K C_1$

$K$ times greater than that for a vacuum capacitor

$C = K \varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$

$\varepsilon \equiv K \varepsilon_0$ - permittivity of a dielectric
Energy Stored in a Dielectric-filled Capacitor

We want to calculate the total reversible work done on the free charges in the charging circuit (these are the charges that we control); this work can be retrieved from the capacitor.

\[ \delta W = \delta q_m \cdot V(q_m, q_i) = \delta q_m \cdot \frac{\sigma_m - \sigma_i}{\varepsilon_0} \cdot d \]

\[ \sigma_i = \sigma_m \left( 1 - \frac{1}{K} \right) \quad \sigma_m - \sigma_i = \frac{\sigma_m}{K} = \frac{q_m}{KA} \]

We took into account the reaction of bound (polarization) charges: \( V \) depends on \( \sigma_i \).

\[ W = \int_{q_m=0}^{q_m=Q} \delta q_m \cdot \frac{q_m}{\varepsilon_0 KA} \cdot d = \frac{d}{\varepsilon_0 KA} \cdot \frac{Q^2}{2} = \frac{1}{2} C_{filled} V^2 \]

\[ C_{filled} = \frac{\varepsilon_0 KA}{d} \]

Thus, if we take two identical capacitors - one empty, another one filled with a dielectric – and charge them from the same voltage source (= to the same voltage), the energy stored in the filled one will be greater (extra reversible work on polarization of the dielectric).
**Different experiment**: we charge an “empty” capacitor to charge \( Q \), *disconnect it from the voltage source*, and insert dielectric.

\[
U_{filled} / U_{empty} = ?
\]

Now \( Q \) is fixed (rather than \( V \)).

\[
\begin{align*}
U_{empty} &= \frac{d}{\varepsilon_0 A} \cdot \frac{Q^2}{2} = \frac{Q^2}{2C_{empty}} \\
U_{filled} &= \frac{d}{\varepsilon_0 K A} \cdot \frac{Q^2}{2} = \frac{Q^2}{2C_{filled}}
\end{align*}
\]

Now the final energy is *smaller* than the initial one. The energy was wasted: we have to do some “negative” work in order to counteract the force on dielectric (which is pulled in the region of stronger field).
Dielectric-Filled Capacitors

\[ \Delta V = \Delta V = V \]

\[ k_1 = 1 \quad k_2 \]

\[ + \quad d \quad - \]

\[ C_\Sigma = C_1 + C_2 = \frac{\varepsilon_0}{d} (A_1 + k_2 A_2) \]

\[ C_\Sigma = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0 k_2 A} \right)^{-1} \]
A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $k$ is inserted between the plates. The charge stored in the capacitor

1. Increases
2. Decreases
3. Stays the Same
Concept Question Answer: Dielectric

Answer: 3. Charge stays the same

Since the capacitor is disconnected from a battery there is no way for the amount of charge on it to change.
A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $k$ is inserted between the plates. The energy stored in the capacitor

1. Increases
2. Decreases
3. Stays the Same
Concept Question Answer: Dielectric

Answer: 2. Energy stored decreases

The dielectric reduces the electric field and hence reduces the amount of energy stored in the field.

The easiest way to think about this is that the capacitance is increased while the charge remains the same so

Also from energy density:

\[ U = \frac{Q^2}{2C} \]

Also from energy density:

\[ u_{E,0} = \frac{1}{2} \varepsilon_0 E^2 \Rightarrow \frac{1}{2} \left( \kappa \varepsilon_0 \right) \left( \frac{E}{\kappa} \right)^2 \leq u_{E,0} \]
Concept Question: Dielectric

A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $k$ is inserted between the plates. The force on the dielectric

1. pulls in the dielectric
2. pushes out the dielectric
3. is zero
Concept Question Answer: Dielectric

Answer: 1. The dielectric is pulled in

We just saw that the energy is reduced by the introduction of a dielectric. Since systems want to reduce their energy, the dielectric will be sucked into the capacitor.
Consider the two concentric spheres shown. The inner sphere is a conductor with total charge \(-Q\), inner radius \(a\), and outer radius \(b\). The outer sphere is a conductor, with total charge \(+Q\), inner radius \(c\), and outer radius \(d\). Taking the potential to be 0 at \(r = \infty\), at which of the following radii is the magnitude of the potential largest?

a) \(r = a\)
b) \(r = b\)
c) \(r = c\)
d) \(r = d\)
e) at both \(r = a\) and \(r = b\)
Consider the two concentric spheres shown. The inner sphere is a conductor with total charge $-Q$, inner radius $a$, and outer radius $b$. The outer sphere is a conductor, with total charge $+Q$, inner radius $c$, and outer radius $d$. Taking the potential to be 0 at $r = \infty$, at which of the following radii is the magnitude of the potential largest?

a) $r = a$
b) $r = b$
c) $r = c$
d) $r = d$

**e) at both $r = a$ and $r = b$**

\[
E(x) = \begin{cases} 
\frac{-Q}{4\pi \varepsilon_0 b^2} & \text{for } r = a \\
\frac{-Q}{4\pi \varepsilon_0 c^2} & \text{for } r = b
\end{cases}
\]

\[
V(x) = \begin{cases} 
0 & \text{for } r = a \\
\frac{-Q}{4\pi \varepsilon_0 \left( \frac{1}{b} - \frac{1}{c} \right)} & \text{for } r = b
\end{cases}
\]
8. The electric potential $V(x)$ depends on $x$ in the fashion shown in the first panel (I) below. Which of the figures shown below most precisely describes the electric field $E_x$ in the $x$-direction?

\[ E(x) = -\frac{dV(x)}{dx} \hat{x} \]
Dipoles in a Uniform External Electric Field

The net force on a dipole is zero; however, there is a non-zero torque:

\[
\tau = 2 \times E q \frac{d}{2} \sin(\phi) = E q d \sin(\phi) = E p \sin(\phi)
\]

\(p\) – the dipole moment

In the vector form: \(\vec{\tau} = \vec{p} \times \vec{E}\) (\(\vec{p}\) directed from – to +)

Potential energy of a dipole in an electric field:

\[
U(\phi) = -pE\cos(\phi)
\]
15. In the figure, an electric dipole has its dipole moment oriented at an angle $\theta$ with respect to the $y$ axis. There is a uniform external electric field of magnitude $E$ pointing in the $+y$ direction. The positive and negative ends of the dipole have charges $+q$ and $-q$, respectively, and the two charges are a distance $d$ apart. The dipole is free to rotate about a pivot through its center. What is the magnitude of the torque $\tau$ that the electric field exerts about the center of mass of the dipole?

\[ \begin{align*}
\text{a)} & \quad \tau = qEd \\
\text{b)} & \quad \tau = qEd\sin\theta \\
\text{c)} & \quad \tau = qEd\cos\theta \\
\text{d)} & \quad \tau = qEd/2\sin\theta \\
\text{e)} & \quad \tau = qEd/2\sin\theta 
\end{align*} \]
13. In the figure \( A = (x_1, y_1) \) and \( B = (x_2, y_2) \) are two points near and on the same side of a charged sheet with positive surface charge density \( +\sigma \). The electric field \( \mathbf{E} \) due to such a charged sheet has magnitude \( E = \sigma / 2\varepsilon_0 \) everywhere and the field points away from the sheet. What is the potential difference \( V_{AB} = V_A - V_B \) between points \( A \) and \( B \)?

a) \( V_{AB} = E(y_2 - y_1)(x_2 - x_1) \)
b) \( V_{AB} = E(y_1 - y_2) \)
c) Cannot determine \( V_{AB} \) unless
   know the detailed path.
d) \( V_{AB} = E(y_2 - y_1) \)
e) \( V_{AB} = E(x_2 - x_1) \)
A particle with mass $m$ and charge $q$ is emitted from the origin $(x, y) = (0, 0)$ with velocity $v_0$ in the $+x$ direction. A large flat screen is located at $x = L$. There is a target on the screen at position $y_h$ where $y_h > 0$. Between the origin and the screen, the charge travels through a constant electric field pointing in the $+y$ direction. See the figure. What should the magnitude $E$ of the electric field be if the charge is to hit the target on the screen? You can ignore gravity in this problem.

a) $E = 2y_h(m/q)(v_0/L)^2$

b) $E = y_h(1/2q)(v_0/L)^2$

c) $E = 2y_h(q/m)(v_0/L)^2$

d) $E = 2y_h(1/q)(v_0/L)^2$

e) $E = y_h(q/2m)(L/v_0)^2$
9. Find the equivalent capacitance of the combination.

a) $\frac{1}{4} \ \mu F$
b) $4 \ \mu F$
c) $\frac{4}{3} \ \mu F$
d) $\frac{3}{4} \ \mu F$
e) $3 \ \mu F$
6. Two small identical metal balls hold charges of $-10\mu\text{C}$ and $+6\mu\text{C}$, respectively. When they are placed a certain distance apart, the magnitude of the force between them is $F$. They are then allowed to touch and brought back to their original position. The force $F$ between them is now

a) $F/15$ and attractive.
b) $F/15$ and repulsive.
c) $4F/15$ and attractive.
d) $4F/15$ and repulsive.
e) $4F/15$ and repulsive.

$$F = k \frac{q_1 q_2}{r^2}$$

Charge conservation:

$$q_{net} = q_1 + q_2 = q_1' + q_2'$$

$$q_1' = q_2' = \frac{q_{net}}{2} = -2\mu\text{C}$$

$$\frac{F'}{F} = \frac{q_1' q_2'}{q_1 q_2} = \frac{(-2)(-2)}{(-10)(+6)} = \frac{1}{15}$$
15. Two charges, \( q_1 = +7 \, \mu C \) and \( q_2 = -4 \, \mu C \) are separated by a distance \( D = 0.1 \, m \). How much work must be done to bring from infinity a negative charge \( q_3 = -5 \, \mu C \) to form the third vertex of an equilateral triangle, a distance \( D = 0.1 \, m \) from each of the others. Answer in terms of \( k = 1/4\pi\epsilon_0 \).

   a) \(-28 \times 10^{-12} \, k \, J\)
   b) \(-150 \times 10^{-12} \, k \, J\)
   c) \(-280 \times 10^{-12} \, k \, J\)
   d) \(+280 \times 10^{-12} \, k \, J\)
   e) \(-15 \times 10^{-12} \, k \, J\)

   \[ W = U(r) - U(\infty) = q_0 V(r) = k q_0 \left( \frac{q_1}{r_{01}} + \frac{q_2}{r_{02}} \right) \]

   \[ W = k \frac{q_0}{D} (q_1 + q_2) \]

   \[ W = k \frac{(-5)}{0.1} (+3) \times 10^{-12} \, J = -150 \times 10^{-12} \, k \, J \]
Total energy of interactions between several charges

The total work required to assemble this charge distribution:

\[ U_\Sigma = \frac{q_0 q_1}{4\pi \varepsilon_0} \left( \frac{1}{r_{01}} \right) + \frac{q_0 q_2}{4\pi \varepsilon_0} \left( \frac{1}{r_{02}} \right) + \frac{q_1 q_2}{4\pi \varepsilon_0} \left( \frac{1}{r_{12}} \right) + \ldots = \frac{1}{4\pi \varepsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}} \]

- all possible pairs, but each pair we count just once.

Example: four equal charges in the corners of a square.

\[ U_e^{\text{total}} = \sum_{\text{all pairs}} \frac{q_i q_j}{4\pi \varepsilon_0 r_{ij}} = 4 \frac{q^2}{4\pi \varepsilon_0 a} + 2 \frac{q^2}{4\pi \varepsilon_0 a\sqrt{2}} \]

Each pair counts only once

The energy is positive (the charges repel each other), the system can do some work if we let the charges go.

On the other hand, the potential energy of one of these charges in the field due to the other three charges:

\[ U_e = 2 \frac{q^2}{4\pi \varepsilon_0 a} + \frac{q^2}{4\pi \varepsilon_0 a\sqrt{2}} \]