First 227 Midterm common hour exam will be held Thursday, October 6, 9:50 to 11:10 PM at night in four locations on the Busch campus. You should go to the room corresponding to the first 3 letters of your last name. If you go to the wrong location, you will not find your exam.

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<tbody>
<tr>
<td><strong>ARC 103</strong></td>
<td><strong>Aaa-Jzz</strong></td>
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<td><strong>Hill 114</strong></td>
<td><strong>Kaa-Nzz</strong></td>
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<td><strong>PHY LH</strong></td>
<td><strong>Oaa-Shz</strong></td>
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<td><strong>SEC 111</strong></td>
<td><strong>Sia-Zzz</strong></td>
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The exam will consist of 15 multiple-choice questions that will include answers based on concepts (such as i-clickers), formulae, and simple numerical calculations, covering Chapters 21-24 in the textbook.

All exams are closed book, **no calculators or other electronic devices allowed**. For the midterm exam, you may bring with you a single "formula sheet", **one and only one** 8.5 x 11 inch sheet of paper (OK to use both sides) on which you may write any formulae or diagrams or notes or problem solutions that might be helpful to you during the exam. **Information on the sheets must be handwritten**, no attachments.

The numerical values of relevant constants will be provided to you. You should bring #2 pencils. Bathroom breaks will be permitted one at a time and none in the last 30 minutes of the exam.
Next time: Review

Email me questions by Wednesday noon!!
Capacitance of a coax cable (cylindrical capacitor)

\[ \Delta V = \int_{+\text{electrode}}^{-\text{electrode}} \vec{E} \cdot d\vec{l} = \int_{R_1}^{R_2} \frac{\lambda}{2\pi \varepsilon_0 r} dr = \frac{\lambda}{2\pi \varepsilon_0} \ln \left( \frac{R_2}{R_1} \right) \]

\[ C(\text{per length } l) = \frac{Q}{V} = \frac{\lambda l}{V} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{R_2}{R_1} \right)} l \]
How much work it takes to charge a capacitor?

\[ W = \delta q \cdot E(q) \cdot d = \delta q \cdot \frac{q}{\varepsilon_0 A} \cdot d > 0 \]

The potential energy stored in the capacitor:

\[ U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2} \]

50 MJ world's largest capacitor bank (Dresden High Magnetic Field Lab)

The capacitors are discharged through a coil which produces the magnetic field \( \sim 100 \text{ T} \) for a few milliseconds.
How much energy is stored in a stun gun capacitor if its capacitance is $2 \times 10^{-6}$ F (2 μF) and it is charged up to 400 V?

$$U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}$$

A. $4 \times 10^{-4}$ J
B. 0.16 J
C. 4 J
D. 160 J
How much energy is stored in a stun gun capacitor if its capacitance is $2 \times 10^{-6}$ F (2 μF) and it is charged up to 400 V?

$$U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}$$

A. $4 \times 10^{-4}$ J

B. 0.16 J

C. 4 J

D. 160 J
Energy Stored in Electric Field

In electrostatics, both approaches are equivalent. In electrodynamics, the alternating e.-m. field can exist with no obvious relevance to charges, so it’s preferable to think that the energy is stored in the electric field.

**Energy density** (U/volume):

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 \]

- this result is general, applies also to time-dependent electric fields.

**Total energy of the electric field**:

\[ U_E = \int_{all \ space} \frac{1}{2} \varepsilon_0 E^2(r) d\tau \]
You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

\[ C = \frac{\varepsilon_0 A}{d} \]

\[ U_E = \int_{all \ space} \frac{1}{2} \varepsilon_0 E^2(r) d\tau \]

A. becomes 4 times greater.

B. becomes twice as great.

C. remains the same.

D. becomes 1/2 as great.

E. becomes 1/4 as great.
You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

A. becomes 4 times greater.
B. becomes twice as great.
C. remains the same.
D. becomes 1/2 as great.
E. becomes 1/4 as great.

$$C = \frac{\varepsilon_0 A}{d}, \quad d \to \frac{d}{2}$$

$$U_E = \int \frac{1}{2} \varepsilon_0 E^2(r) d\tau$$

$E=\text{const}$, volume – cut in half, $U_E$ is cut in half as well
Capacitors and Dielectrics

Static conditions (no currents)
Two Types of Problems

Capacitor is connected to a voltage source.

The voltage difference is fixed, the charge may vary if the capacitance varies.

\[ V = \text{const} \]
\[ Q = CV \]

Capacitor is charged and disconnected from the voltage source.

The charge is fixed, the voltage difference may vary if the capacitance varies.

\[ Q = \text{const} \]
\[ V = \frac{Q}{C} \]
A capacitor is charged and **disconnected from the voltage source** (charges $+Q$ and $-Q$ on the two plates are kept constant). You reposition the two plates of a capacitor so that the capacitance doubles.

What happens to the potential difference $V_{ab}$ between the two plates?

A. $V_{ab}$ becomes 4 times as great

B. $V_{ab}$ becomes twice as great

C. $V_{ab}$ remains the same

D. $V_{ab}$ becomes 1/2 as great

E. $V_{ab}$ becomes 1/4 as great
A capacitor is charged and disconnected from the voltage source (charges +Q and –Q on the two plates are kept constant). You reposition the two plates of a capacitor so that the capacitance doubles.

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\[ C = \frac{\varepsilon_0 A}{d} \]

\[ V = \frac{Q}{C} \]
Iclicker Question

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the capacitor is **connected to the voltage source in this process**, what happens to the charges on the plates?

A. $Q$ becomes 4 times as great

B. $Q$ becomes twice as great

C. $Q$ remains the same

D. $Q$ becomes $1/2$ as great

E. $Q$ becomes $1/4$ as great
You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the capacitor is connected to the voltage source in this process, what happens to the charges on the plates?

A. $Q$ becomes 4 times as great

B. $Q$ becomes twice as great

C. $Q$ remains the same

D. $Q$ becomes 1/2 as great

E. $Q$ becomes 1/4 as great

\[ C = \frac{\epsilon_0 A}{d} \]

\[ V = \frac{Q}{C} \]
Parallel Connection of Capacitors

What quantity is the same for both capacitors?

\[ V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \]

- the charges on capacitors are redistributed to keep \( V_1 = V_2 = V \)

\[
\begin{align*}
Q_1 &= C_1 V \\
Q_2 &= C_2 V \\
Q_\Sigma &= Q_1 + Q_2 = (C_1 + C_2)V \\
C_\Sigma &= C_1 + C_2
\end{align*}
\]
A 12–μF capacitor and a 6–μF capacitor are connected together as shown. If the charge on the 12–μF capacitor is 24 microcoulombs (24 μC), what is the charge on the 6–μF capacitor?

A. 48 μC  
B. 36 μC  
C. 24 μC  
D. 12 μC  
E. 6 μC
A 12–μF capacitor and a 6–μF capacitor are connected together as shown. If the charge on the 12–μF capacitor is 24 microcoulombs (24 μC), what is the charge on the 6–μF capacitor?

\[ Q = CV \]

Voltage is the same for both capacitors.

A. 48 μC
B. 36 μC
C. 24 μC
D. 12 μC
E. 6 μC
Series Connection of Capacitors

What quantity is the same for both capacitors?

- the “central” conductor is uncharged, it can only be polarized.

$Q_1 = Q_2 = Q$

$V_1 = V - V^* = \frac{Q}{C_1}$

$V_2 = V^* - 0 = \frac{Q}{C_2}$

$V_1 + V_2 = V$

$V = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_\Sigma}$

$C_\Sigma = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

$C_\Sigma = \frac{C_1 C_2}{C_1 + C_2}$
Three capacitors, $C_1 = C_2 = C_3 = C$ are connected as shown in the Figure. Find the equivalent capacitance of the circuit.

A  3C
B  C
C  C/2
D  C/3
E  0
Three capacitors, \( C_1 = C_2 = C_3 = C \) are connected as shown in the Figure. Find the equivalent capacitance of the circuit.

\[ \Delta V = V \]

A  3C
B  C
C  C/2
D  C/3
E  0
Three capacitors, $C_1 = C_2 = C_3 = C$ are connected as shown in the Figure. Find the equivalent capacitance of the circuit.

\[
\begin{align*}
\frac{C \cdot C}{C + C} &= \frac{C}{2} \\
\frac{C \cdot \frac{C}{2}}{C + \frac{C}{2}} &= \frac{C}{3}
\end{align*}
\]

\[
\frac{C_1 \cdot C_2 \cdot C_3}{C_1 \cdot C_2 + C_1 \cdot C_3 + C_2 \cdot C_3} = \frac{C}{3}
\]

A 3C  
B C  
C $C/2$  
D $C/3$  
E 0
More Complex Circuits

\[ \Delta V = V \]

\[ C_1 = 0.33 + 0.33 + 0.33 = 1 \mu F \]

\[ C_2 = \frac{3 \cdot 3}{3 + 3} = 1.5 \mu F \]

\[ C_2 \ldots C_5 = \frac{1.5 \cdot 3}{1.5 + 3} = 1 \mu F \]

\[ C_4 \parallel (C_2 \ldots C_5) = 1 + 1 = 2 \mu F \]

\[ C_3 \ldots [C_4 \parallel (C_2 \ldots C_5)] = C_4 \parallel (C_2 \ldots C_5) = \frac{2 \cdot 2}{2 + 2} = 1 \mu F \]

\[ C_\Sigma = C_1 \parallel \{C_3 \ldots [C_4 \parallel (C_2 \ldots C_5)]\} = 1 + 1 = 2 \ \mu F \]
Initially, only $C_1$ was connected to the voltage source, $C_2$ was uncharged. We disconnect $C_1$ from the source and connect it to $C_2$. Find $V^*$, $Q_1$ and $Q_2$.

After $C_1$ was disconnected from the voltage source, its charge was $Q_1^{\text{init}} = C_1V$. Connection to $C_2$ results in redistribution of charges, but the net charge ($Q_\Sigma = Q_1^{\text{init}}$) is conserved:

\[
Q_\Sigma = Q_1^{\text{init}} = C_1V \quad \text{ switch in position 1} \quad \quad Q_\Sigma = C_1V^* + C_2V^* \quad \text{ switch in position 2}
\]

\[
V^* = \frac{C_1}{C_1 + C_2} V \quad \quad Q_1 = \frac{C_1}{C_1 + C_2} C_1V \quad \quad Q_2 = \frac{C_1}{C_1 + C_2} C_2V
\]

\[
C_2 \gg C_1 \quad V \gg V^* \quad Q_1 \ll Q_\Sigma \quad Q_2 \approx Q_\Sigma
\]
Uncompensated charges reside on the surfaces, the total field is the sum of the external field and the field due to uncompensated charges.

Has the field outside been changed?

Dielectric constant: \[ K \equiv \frac{E_0}{E_{in}} > 1 \]

Uncompensated charges reside on the surfaces, the total field is the sum of the external field and the field due to uncompensated charges.

Has the field outside been changed?

Dielectric constant: \[ K \equiv \frac{E_0}{E_{in}} > 1 \]

\[ \frac{\sigma_i}{\varepsilon_0} = E_0 - E_{in} = E_0 \left(1 - \frac{1}{K}\right) \]

What would be \( K \) for metal?

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant</th>
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<tbody>
<tr>
<td>Vacuum</td>
<td>1.0000</td>
</tr>
<tr>
<td>Air (1 atm)</td>
<td>1.0006</td>
</tr>
<tr>
<td>Paraffin</td>
<td>2.2</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.6</td>
</tr>
<tr>
<td>Vinyl (plastic)</td>
<td>2–4</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
</tr>
<tr>
<td>Quartz</td>
<td>4.3</td>
</tr>
<tr>
<td>Oil</td>
<td>4</td>
</tr>
<tr>
<td>Glass, Pyrex</td>
<td>5</td>
</tr>
<tr>
<td>Rubber, neoprene</td>
<td>6.7</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6–8</td>
</tr>
<tr>
<td>Mica</td>
<td>7</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>80</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>300</td>
</tr>
</tbody>
</table>
Dielectric-filled Capacitors

The voltage source \((V=\text{const})\) is connected to (1) a “vacuum” capacitor and (2) the same capacitor filled with dielectric.

Charge on metallic electrodes (that’s what we measure/control)

\[
\begin{align*}
(1) & \quad E_1 = \frac{V}{d} = E \\
& \quad \sigma_1 = \sigma_m = \varepsilon_0 E = \varepsilon_0 \frac{V}{d} \\
& \quad C_1 = \frac{Q}{V} = \frac{\sigma_m A}{V} = \varepsilon_0 \frac{A}{d} \\
(2) & \quad E_2 = \frac{V}{d} = E \\
& \quad \sigma_2 = \sigma_m - \sigma_i = \varepsilon_0 E \\
& \quad \sigma_i = \varepsilon_0 E \text{in}(K-1) = \varepsilon_0 E(K-1) \\
& \quad C_2 = \frac{Q}{V} = \frac{\sigma_m A}{V} = \varepsilon_0 \frac{A}{d} = K \frac{A}{d} = KC_1 \\
\end{align*}
\]

\(K\) times greater than that for a vacuum capacitor

\[
C = K \varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d} \\
\varepsilon \equiv K \varepsilon_0 \quad - \text{permittivity of a dielectric}
\]
We want to calculate the total reversible work done on the free charges in the charging circuit (these are the charges that we control); this work can be retrieved from the capacitor.

\[ \delta W = \delta q_m \cdot V(q_m, q_i) = \delta q_m \cdot \frac{\sigma_m - \sigma_i}{\varepsilon_0} \cdot d \]

\[ \sigma_i = \sigma_m \left(1 - \frac{1}{K}\right) \quad \sigma_m - \sigma_i = \frac{\sigma_m}{K} = \frac{q_m}{KA} \]

We took into account the reaction of bound (polarization) charges: \( V \) depends on \( \sigma_i \).

\[
W = \int_{q_m=0}^{q_m=Q} \delta q_m \cdot \frac{q_m}{\varepsilon_0 KA} \cdot d = \frac{d}{\varepsilon_0 KA} \cdot \frac{Q^2}{2} = \frac{1}{2} C_{filled} V^2
\]

\[ C_{filled} = \frac{\varepsilon_0 KA}{d} \]

Thus, if we take two identical capacitors - one empty, another one filled with a dielectric – and **charge them from the same voltage source**, the energy stored in the filled one will be greater (extra reversible work on polarization of the dielectric).
**Energy Stored in a Dielectric-filled Capacitor (cont’d)**

*Different experiment:* we charge an “empty” capacitor to charge $Q$, *disconnect it from the voltage source*, and insert dielectric.

$$U_{filled}/U_{empty} = ?$$

Now $Q$ is fixed (rather than $V$).

$$U_{empty} = \frac{d}{\varepsilon_0A} \cdot \frac{Q^2}{2} = \frac{Q^2}{2C_{empty}}$$

$$U_{filled} = \frac{d}{\varepsilon_0KA} \cdot \frac{Q^2}{2} = \frac{Q^2}{2C_{filled}}$$

$$\frac{U_{filled}}{U_{empty}} = \frac{1}{K}$$

Now the final energy is **smaller** than the initial one. The energy was wasted: we have to do some “negative” work in order to counteract the force on dielectric (which is pulled in the region of stronger field).

A **non-uniform** electric field exerts a force on dipoles ($F = qE_1 - qE_2 = qd\frac{dE}{dx}$):
Dielectric-Filled Capacitors

\[ C_\Sigma = C_1 + C_2 = \frac{\varepsilon_0}{d} (A_1 + k_2 A_2) \]

\[ C_\Sigma = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0 k_2 A} \right)^{-1} \]
Parallel and serial connection of capacitors.
Dielectrics in electric field.
Capacitance of dielectric-filled capacitors
Energy stored in dielectric-filled capacitors.

Next time: Lecture 9. Review.
Please prepare/email your questions!
Student: “After we introduced the concept of dielectric constant, why bother to distinguish among conductors, vacuum, and dielectrics? Can’t we just say “a spherical region of $K = \infty$” instead of “a metal sphere”?

Distinctions:

(a) Physical mechanisms responsible for polarization (emerging induced charges) of metals and dielectrics are quite different (redistribution of mobile electrons vs. polarization of bound charges). This becomes especially important when the electric field is time-dependent: the free and bound electrons respond differently to high-frequency electric fields.

(b) Boundary conditions are different for metals and dielectrics, this becomes clear when we’ll consider steady currents (supported by metals, not dielectrics).
Two uniformly charged spherical \( (\sigma = \sigma_0) \) shells are placed in a dielectric. The dielectric is polarized: the polarization charge partially “screens” the charges on the spheres.

\[
\sigma_i = \varepsilon_0 E_0 \left(1 - \frac{1}{K}\right) = \sigma_0 \left(1 - \frac{1}{K}\right) \quad \text{ - absolute value, the sign is opposite to} \ \sigma_0
\]

Net charge (initial charge + polarization “cloud”): \( Q_{\Sigma} = Q - Q \left(1 - \frac{1}{K}\right) = \frac{Q}{K} \)

The electric field due to the \textit{screened} +Q acting upon −Q: \( E = k \frac{Q}{Kr^2} \)

The force between +Q and −Q charges surrounded by dielectric: \( F = QE = \frac{F_0}{K} \)

\( (F_0 \text{ is the force between these charges in vacuum}) \)

Polar (large \( K \)) solvents at work: strongly polar compounds (e.g., sugars) or ionic compounds (e.g., table salt) dissolve only in very polar solvents like water.
A parallel plate capacitor is charged to a total charge \( Q \) and the battery removed. A slab of material with dielectric constant \( k \) is inserted between the plates. The charge stored in the capacitor

1. Increases
2. Decreases
3. Stays the Same
Answer: 3. Charge stays the same

Since the capacitor is disconnected from a battery there is no way for the amount of charge on it to change.
A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $k$ is inserted between the plates. The energy stored in the capacitor

1. Increases
2. Decreases
3. Stays the Same
Concept Question Answer: Dielectric

Answer: 2. Energy stored decreases

The dielectric reduces the electric field and hence reduces the amount of energy stored in the field.

The easiest way to think about this is that the capacitance is increased while the charge remains the same so

\[ U = \frac{Q^2}{2C} \]

Also from energy density:

\[ u_{E,0} = \frac{1}{2} \varepsilon_0 E^2 \Rightarrow \frac{1}{2} \left( \kappa \varepsilon_0 \right) \left( \frac{E}{\kappa} \right)^2 < u_{E,0} \]
Concept Question: Dielectric

A parallel plate capacitor is charged to a total charge $Q$ and the battery removed. A slab of material with dielectric constant $k$ in inserted between the plates. The force on the dielectric

1. pulls in the dielectric
2. pushes out the dielectric
3. is zero
Concept Question Answer: Dielectric

Answer: 1. The dielectric is pulled in

We just saw that the energy is reduced by the introduction of a dielectric. Since systems want to reduce their energy, the dielectric will be sucked into the capacitor.
What’s wrong with this picture?

\[ k_1 = 1 \quad \text{and} \quad k_2 \]