The first midterm!!!

First 227 Midterm common hour exam will be held Thursday, October 6, 9:50 to 11:10 PM at night in four locations on the Busch campus. You should go to the room corresponding to the first 3 letters of your last name. If you go to the wrong location, you will not find your exam.

<table>
<thead>
<tr>
<th>ARC 103</th>
<th>Aaa-Jzz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hill 114</td>
<td>Kaa-Nzz</td>
</tr>
<tr>
<td>PHY LH</td>
<td>Oaa-Shz</td>
</tr>
<tr>
<td>SEC 111</td>
<td>Sia-Zzz</td>
</tr>
</tbody>
</table>
The exam will consist of 15 multiple-choice questions that will include answers based on concepts (such as i-clickers), formulae, and simple numerical calculations, covering Chapters 21-24 in the textbook.

All exams are closed book, no calculators or other electronic devices allowed. For the midterm exam, you may bring with you a single "formula sheet", one and only one 8.5 x 11 inch sheet of paper (OK to use both sides) on which you may write any formulae or diagrams or notes or problem solutions that might be helpful to you during the exam. Information on the sheets must be handwritten, no attachments.

The numerical values of relevant constants will be provided to you. You should bring #2 pencils. Bathroom breaks will be permitted one at a time and none in the last 30 minutes of the exam.
Unregistered iClickers?

If you have problems with your iClicker registration:
Email Professor Gershenson (gersh@physics.rutgers.edu) with your name and iClicker number for help!
Electric Field and Potential of a Parallel-Plate Capacitor

Potential is a continuous function of \( r \).
(exception: point charges).
**Equipotential Lines**

*Equipotential lines (and surfaces) are* lines (surfaces) on which the potential (voltage) is constant

They are plotted for fixed differences in voltage.

At each point, a field line is **perpendicular** to an equipotential line:

\[ \int_{\text{along equipot. line (surface)}} \vec{E}(r) \cdot d\vec{l} = 0 \]
Which point corresponds to the greatest magnitude of the electric field?

\[ E(x, y) = -\frac{\partial V(x, y)}{\partial x} \hat{x} - \frac{\partial V(x, y)}{\partial y} \hat{y} \]
Which point corresponds to the greatest magnitude of the electric field?

A. A
B. B
C. C
D. D

\[ \vec{E}(x, y) = -\frac{\partial V(x, y)}{\partial x} \hat{x} - \frac{\partial V(x, y)}{\partial y} \hat{y} \]
The figure shows cross sections through two equipotential surfaces. In both diagrams the potential difference between adjacent equipotentials is the same. Which of these two could represent the field of a point charge?

A.  a
B.  b
C.  Neither one
The figure shows cross sections through two equipotential surfaces. In both diagrams the potential difference between adjacent equipotentials is the same. Which of these two could represent the field of a point charge?

A. a

B. b

C. Neither one
The dashed lines in the diagram represent cross sections of equipotential surfaces drawn in 1 V increments. Which of the following statements is FALSE:

A  The magnitude of the electric field at point C is greater than the magnitude of the electric field at point A.
B  The potential difference between A and D is 1 V.
C  No work $W_{AB}$ is done by the electric force to move a 1 C charge from A to B.
D  No work $W_{AD}$ is done by the electric force to move a 1 C charge from A to D.
The dashed lines in the diagram represent cross sections of equipotential surfaces drawn in 1 V increments. Which of the following statements is FALSE:

A  The magnitude of the electric field at point C is greater than the magnitude of the electric field at point A.
B  The potential difference between A and D is 1 V.
C  No work $W_{AB}$ is done by the electric force to move a 1 C charge from A to B.
D  No work $W_{AD}$ is done by the electric force to move a 1 C charge from A to D.
Example: Dipole

$V(x, y)$

$V_0 = V_0 - 1V$

$V_0 = V_0 + 2V$

$V = +2V$

$V = +1V$

$V = 0V$

$V = -1V$
Intersecting Equipotential Lines

If an equipotential line crosses itself, then $E=0$ at this point.

Recall: a field line never crosses itself!
Compare these two plots. What’s wrong with the right one?

The electric field intensity is proportional to the density of equipotential lines.

At the center \( \vec{E}(x, y, z) = -\frac{\partial V(x, y, z)}{\partial x} \hat{x} \).

Touching equipotential lines (corresponding to different potentials) imply an infinitely strong field.
Charges are fixed on a grid having the same spacing (see the Figure). Each charge has the same magnitude \(-Q\). What are the electrostatic potential and the electric field at the location marked with an “x”? Assume that the reference point for \(V\) is at infinity.

\[
A. \quad V = 0, \quad E = 0
\]

\[
B. \quad V = 0, \quad E = -k \frac{4Q}{a^2}
\]

\[
C. \quad V = k \frac{4Q}{a}, \quad E = 0
\]

\[
D. \quad V = -k \frac{4Q}{a}, \quad E = 0
\]

\[
E. \quad V = -k \frac{4Q}{a}, \quad E = -k \frac{4Q}{a^2}
\]
Charges are fixed on a grid having the same spacing (see the Figure). Each charge has the same magnitude $-Q$. What are the electrostatic potential and the electric field at the location marked with an “x”? Assume that the reference point for $V$ is at infinity.

A. $V = 0, \ E = 0$

B. $V = 0, \ E = -k \frac{4Q}{a^2}$

C. $V = k \frac{4Q}{a}, \ E = 0$

D. $V = -k \frac{4Q}{a}, \ E = 0$

E. $V = -k \frac{4Q}{a}, \ E = -k \frac{4Q}{a^2}$
The equipotential curves in a certain region of an equipotential diagram for the \( xy \) plane are parallel to the \( y \) axis and are equally spaced, with the potential increasing in the \(-x\) direction. An electric field vector in this region would point in which direction?

A. +x direction  
B. -x direction  
C. +y direction  
D. -y direction  
E. Other
The equipotential curves in a certain region of an equipotential diagram for the $xy$ plane are parallel to the $y$ axis and are equally spaced, with the potential increasing in the $-x$ direction. An electric field vector in this region would point in which direction?

A. $+x$ direction
B. $-x$ direction
C. $+y$ direction
D. $-y$ direction
E. Other

$$\vec{E}(x) = -\frac{dV(x)}{dx} \hat{x}$$
At metallic surfaces the electric field lines are *perpendicular* to the surface.

What can we conclude about the electric potential distribution over the surface?

**Conducting Surface = Equipotential Surface**

The surface of a conductor is always an *equipotential surface* (in electrostatics!): when we move a test charge along the surface, no work is done ($\vec{F} \perp d\vec{l}$).
Conducting Surface = Equipotential Surface

Since $E=0$ inside, *any* point in the volume of a conductor is at the same potential as its surface.
**Capacitors**

**Capacitor**: a system of two **conducting** surfaces (electrodes), the net charge is zero.

**Important**: because the surfaces are conducting, they are **equipotential**, and

\[ \Delta V = \int_{+\text{electrode}}^{-\text{electrode}} \vec{E} \cdot d\vec{l} \]

- is the same for any path between the electrodes.

Parallel-plate capacitor:
Capacitance

\[ C \equiv \frac{Q}{\Delta V} = \frac{Q}{\int_{-\text{electrode}}^{+\text{electrode}} E \cdot d\vec{l}} \]

Capacitance: Again, this definition makes sense for conducting surfaces only.

Units: \( C/V = \text{Farad (F)} \)

\[ \Delta V = \frac{Q}{C} \]

- \( Q \) – amount of water
- \( C \) – volume (capacity) of the bucket
- \( \Delta V \) – level (height) of water in the bucket
The two conductors \( a \) and \( b \) form a capacitor. You increase the charge on \( a \) to \(+2Q\) and increase the charge on \( b \) to \(-2Q\), while keeping the conductors in the same positions.

As a result of this change, the capacitance \( C \) of the two conductors

A. becomes 4 times great. 
B. becomes twice as great. 
C. remains the same. 
D. becomes 1/2 as great. 
E. becomes 1/4 as great.

\[ C \equiv \frac{Q}{\Delta V} = \frac{Q}{\int_{+electrode}^{electrode} \vec{E} \cdot d\vec{l}} \]
The two conductors $a$ and $b$ form a capacitor. You increase the charge on $a$ to $+2Q$ and increase the charge on $b$ to $-2Q$, while keeping the conductors in the same positions.

As a result of this change, the capacitance $C$ of the two conductors

A. becomes 4 times great.

**C. remains the same.**

B. becomes twice as great.

D. becomes 1/2 as great.

E. becomes 1/4 as great.

$C$ depends only on the geometry (shape of the electrodes, separation between them) and the medium between the electrodes.
Capacitance of a parallel-plate capacitor

Recipe #1 for computing $C$: for a given $Q$, calculate $\Delta V$, and use the definition of capacitance.

$$C \equiv \frac{Q}{\Delta V} = \frac{Q}{\int_{+\text{electrode}}^{-\text{electrode}} \vec{E} \cdot d\vec{l}}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \Delta V = Ed = \frac{qd}{\varepsilon_0 A}$$

$$C = \frac{\varepsilon_0 A}{d} \quad \varepsilon_0 \approx 9 \cdot 10^{-12} \frac{F}{m}$$

Typical numbers:

$A \sim 1 \, \text{m}^2$

$d \sim 10 \, \mu\text{m} = 10^{-5} \text{m}$

$$C \approx 9 \cdot 10^{-12} \frac{F}{m} \frac{1\text{m}^2}{1 \cdot 10^{-5} \text{m}} \approx 1 \cdot 10^{-6} \text{F}$$
Iclicker Question

Each diagram shows two very large parallel charged sheets with uniform charge densities and separations as shown. Which diagram corresponds to the greatest absolute value of the electric potential difference between the sheets?
Each diagram shows two very large parallel charged sheets with uniform charge densities and separations as shown. Which diagram corresponds to the greatest absolute value of the electric potential difference between the sheets?
Capacitance of a coax cable (cylindrical capacitor)

\[ \Delta V = \int_{+\text{electrode}}^{-\text{electrode}} \vec{E} \cdot d\vec{l} = \int_{R_1}^{R_2} \frac{\lambda}{2\pi\epsilon_0 r} \, dr = \frac{\lambda}{2\pi\epsilon_0} \ln \left( \frac{R_2}{R_1} \right) \]

\[ C(\text{per length } l) = \frac{Q}{V} = \frac{\lambda l}{V} = \frac{2\pi\epsilon_0}{\ln \left( \frac{R_2}{R_1} \right)} \, l \]
Energy Stored in a Capacitor

How much work it takes to charge a capacitor?

\[ \delta W = \delta q \cdot E(q) \cdot d = \delta q \cdot \frac{q}{\varepsilon_0 A} \cdot d > 0 \]

The potential energy stored in the capacitor:

\[ W = \int_{q=0}^{q=Q} \delta q \cdot \frac{q}{\varepsilon_0 A} \cdot d = \frac{d}{\varepsilon_0 A} \cdot \frac{Q^2}{2} = \frac{Q^2}{2C} \]

\[ U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2} \]

50 MJ world's largest capacitor bank (Dresden High Magnetic Field Lab)

The capacitors are discharged through a coil which produces the magnetic field ~ 100 T for a few milliseconds.
How much energy is stored in a stun gun capacitor if its capacitance is $2 \times 10^{-6}$ F (2 μF) and it is charged up to 400 V?

\[ U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2} \]

A. $4 \times 10^{-4}$ J

B. 0.16 J

C. 4 J

D. 160 J
Iclicker Question

How much energy is stored in a stun gun capacitor if its capacitance is $2 \times 10^{-6}$ F (2 μF) and it is charged up to 400 V?

$$U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}$$

A. $4 \times 10^{-4}$ J

B. 0.16 J

C. 4 J

D. 160 J
Iclicker Question

Given:

\[ C = 26,000 \ \mu F \quad V = 27 \ \text{Volts} \]

The energy stored in the capacitor is transferred to the motor and the motor lifts mass = 50 g

How high will the mass rise?

A. 0.4 m
B. 1.0 m
C. 2.0 m
D. 4.0 m
E. 20.0 m

\[ U_{cap} = \frac{1}{2} CV^2 \quad U_{grav} = mgh \]
Iclicker Question

Given:

C = 26,000 \mu F \quad V = 27 \text{ Volts}

The energy stored in the capacitor is transferred to the motor and the motor lifts mass = 50 g

How high will the mass rise?

A. 0.4 m
B. 1.0 m
C. 2.0 m
D. 4.0 m
E. 20.0 m

\[
U_{\text{cap}} = \frac{1}{2} CV^2 = U_{\text{grav}} = mgh
\]

\[
h = \frac{1}{2} \frac{CV^2}{mg} = \frac{1}{2} \frac{(26,000 \cdot 10^{-6} \text{ F}) \cdot (27 \text{ V})^2}{0.05 \text{ kg} \cdot 10 \text{ m/s}^2} = 19m!!
\]

What have we forgotten?

Inefficient motor, friction, etc. etc.
Energy Stored in Electric Field

Energy stored in charges ? energy stored in the field

In electrostatics, both approaches are equivalent. In electrodynamics, the alternating e.-m. field can exist with no obvious relevance to charges, so it’s preferable to think that the energy is stored in the electric field.

**Energy density** (U/volume):

\[
\begin{align*}
    u_E &= \frac{CV^2}{2} \cdot \frac{1}{Ad} = \frac{\varepsilon_0 A (Ed)^2}{d} \frac{1}{Ad} = \frac{1}{2} \varepsilon_0 E^2 \\
    u_E &= \frac{1}{2} \varepsilon_0 E^2
\end{align*}
\]

- this result is general, applies also to time-dependent electric fields.

**Total energy of the electric field**:

\[
U_E = \int_{all \ space} \frac{1}{2} \varepsilon_0 E^2(r) d\tau
\]
You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

A. becomes 4 times greater.
B. becomes twice as great.
C. remains the same.
D. becomes 1/2 as great.
E. becomes 1/4 as great.

$$C = \frac{\varepsilon_0 A}{d}$$

$$U_E = \int_{all \ space} \frac{1}{2} \varepsilon_0 E^2(r) d\tau$$
Iclicker Question

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

A. becomes 4 times greater.
B. becomes twice as great.
C. remains the same.
D. becomes $1/2$ as great.
E. becomes $1/4$ as great.

$E=\text{const, volume – cut in half, } U_E \text{ is cut in half as well}$
**Field Energy vs. Energy of Interactions btw Charges**

**Example**: the electric field energy of a charged metal sphere.

\[
U_E = \int \frac{1}{2} \varepsilon_0 E^2(r) d\tau = \int_{\text{all space}} \frac{1}{2} \varepsilon_0 \left( \frac{Q}{4\pi \varepsilon_0 r^2} \right)^2 4\pi r^2 dr = k \frac{Q^2}{2R}
\]

- the field energy is *always positive* \((\propto E^2)\).

Potential energy of **interaction** between the charged electrodes

- sign?

\(U_E > 0\)

The potential energy of **interaction** \(U_{\text{int}}\) between charges can be either *positive* or *negative*.

\(U_E\) takes into account the work on assembling the individual charges, whereas \(U_{\text{int}}\) doesn’t include the self-energies of these charges (doesn’t include the work on assembling the individual charges).
Example

Two dielectric spheres with uniform surface charge density.

\[ U_{tot} = U_1 + U_2 + U_{int} \]

1. Charged spheres are far away \((U_{int} = 0)\).

\[ U_{tot}(\infty) = \int \frac{1}{2} \varepsilon_0 E^2(r) d\tau = k \frac{Q^2}{2R} + k \frac{Q^2}{2R} = k \frac{Q^2}{R} \]

2. Charged spheres are nearby.

\[ U_{int} = -k \frac{Q^2}{r} \]

\[ U_{tot}(\infty) > U_{tot}(r) > 0 \]
Conductors as equipotential surfaces. Grounding.

Capacitors.

Energy stored by a capacitor.

Electric field energy.

Next time: Lecture 8. Capacitors and Dielectrics
§§ 24.2, 24.4, 24.5
Appendix I. Self-Capacitance

One can consider the \textit{capacitance of an isolated conductor} (assuming that the second electrode is infinitely far away), \textit{self-capacitance}. For example, for a metal sphere:

\[
\Delta V = \int_{+\text{electrode}}^{\infty} \vec{E} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon_0 R} \quad C[\text{F}] = \frac{Q}{\Delta V} = 4\pi\epsilon_0 R \approx 10^{-10} R[\text{m}]
\]

Capacitance of the Earth \((R \sim 6,400 \text{ km})\): \(C_{\text{Earth}} \approx 10^{-3} \text{F}\)

Capacitance of the top electrode of a van de Graaff generator \((R \sim 0.2 \text{ m})\): 20 pF.
Appendix II. Faraday Cage vs. Grounding

**Faraday cage:**
- total charge = 0,
- field inside = 0,
- equipotential surface at $V=???$

**Earth:**
- a huge conductor at a (reasonably) constant potential
- (often considered as a reference $V = 0$).

What is the total charge of the grounding shell?
Appendix III. Method of Mirror Images

The equation for the potential has a unique solution if the boundary conditions are fixed. These conditions are the same for the left half-spaces of the figures. Thus, the electric field (to the right of the metal surface) is also the same!

Force of attraction between a point charge and a grounded conducting surface:

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{(2d)^2} \]