First 227 Midterm common hour exam will be held Thursday, October 6, 9:50 to 11:10 PM at night in four locations on the Busch campus. You should go to the room corresponding to the first 3 letters of your last name. If you go to the wrong location, you will not find your exam.

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The exam will consist of 15 multiple-choice questions that will include answers based on concepts (such as i-clickers), formulae, and simple numerical calculations, covering Chapters 21-24 in the textbook.

All exams are closed book, **no calculators or other electronic devices allowed.** For the midterm exam, you may bring with you a single "formula sheet", **one and only one** 8.5 x 11 inch sheet of paper (OK to use both sides) on which you may write any formulae or diagrams or notes or problem solutions that might be helpful to you during the exam. Information on the sheets must be **handwritten**, no attachments.

The numerical values of relevant constants will be provided to you. You should bring #2 pencils. Bathroom breaks will be permitted one at a time and none in the last 30 minutes of the exam.
A point charge $+Q$ is placed inside a neutral, hollow, spherical conductor. As the charge is moved around inside, the electric field outside

A. is zero and does not change
B. is non-zero but does not change
C. is zero when centered but changes
D. is non-zero and changes
A point charge $+Q$ is placed inside a neutral, hollow, spherical conductor. As the charge is moved around inside, the electric field outside

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Having problems with superposition??

A nice video:
https://www.youtube.com/watch?v=S1TXN1M9t18
What is the total Coulomb force on the -1C charge at the origin from the other two 1 C charges?

\[ \vec{F}_{1\ on\ 2} = \frac{kq_1q_2}{r^2} \hat{r}_{12} \]

A. \( k(\hat{x} + \hat{y}) \)
B. \( k(\hat{x} - \hat{y}) \)
C. \( k(-\hat{x} - \hat{y}) \)
D. \( k\sqrt{2}(\hat{x} + \hat{y}) \)
E. \( k[-\sqrt{2}(\hat{x} + \hat{y})] \)
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E. \( k[-\sqrt{2}(\hat{x} + \hat{y})] \)
This lecture is about forces and **potential energy** $U(r)$, unit = joule, J

Next lecture is about electrical **potential** $V(r)$, unit: volt, V

Don’t mix the two up.

\[ V(r) = \frac{U(r)}{q} \]

Coulomb’s Law and Superposition Principle – sufficient to solve any electrostatic problem. Gauss’ Law simplifies calculation of electric fields for symmetric charge distributions. Why do we need anything else?

**Example**

Two charges $q=+1C$ are 1 m apart at rest. We release the charges. What would be **the net kinetic energy (in SI units) of the system** when the charges are very far from one another?

\[ +1C \quad +1C \]

\[ 0 \quad 1 \text{m} \]
Consider two concentric spherical metallic shells, one with radius $R$ and one with radius $2R$. Both have the same charge $Q$. At a point just inside the outer shell, the magnitude of the electric field is

A. $\frac{2kQ}{R^2}$

B. $\frac{kQ}{R^2}$

C. $\frac{kQ}{2R^2}$

D. 0

E. $\frac{kQ}{4R^2}$
Consider two concentric spherical metallic shells, one with radius $R$ and one with radius $2R$. Both have the same charge $Q$. At a point just inside the outer shell, the magnitude of the electric field is

A. $2kQ/R^2$
B. $kQ/R^2$
C. $kQ/2R^2$
D. 0
E. $kQ/4R^2$
An **insulating** spherical shell of inner radius \(a\) and outer radius \(b\) is uniformly charged with a positive charge density. The radial component of the electric field, \(E_r(r)\), is best depicted by which figure?
An **insulating** spherical shell of inner radius \( a \) and outer radius \( b \) is uniformly charged with a positive charge density. The radial component of the electric field, \( E_r(r) \), is best depicted by which figure?
Coulomb’s Law and Superposition Principle – sufficient to solve any electrostatic problem. Gauss’ Law simplifies calculation of electric fields for symmetric charge distributions. Why do we need anything else?

- Electrostatic forces are conservative, and energy conservation is a very powerful tool for solving a wide range of problems.

- The concept of the potential energy of the electrostatic field is the first step towards the appreciation of the energy of electromagnetic field.
Recall: Potential Energy in Gravitational Field

Work done by us while lifting the body from \( a \) to \( b \) at constant speed:

\[
W_{a \to b}^{"us"} = \int_{a}^{b} \mathbf{F}_{us} \cdot d\mathbf{r}
\]

Work done by the grav. force \( F_g \):

\[
W_{a \to b}^{"field"} = \int_{a}^{b} \mathbf{F}_g \cdot d\mathbf{r}
\]

Change in potential energy is equal to the work done by external forces ("us") to move an object in gravitational field (assuming that kinetic energy remains constant).

Potential energy: always the energy of interaction between masses, not a characteristic of a single mass (the potential energy of the system “Earth + mass \( m \)”).
Gravitational Potential Energy of the system “Earth + astronaut”

Potential energy with respect to “\( \infty \)”: 

\[
U(r) - U(\infty) = \int_{\infty}^{r} \vec{F}_{us} \cdot d\vec{r} = \int_{\infty}^{r} \left( G \frac{m_Em}{r^2} \hat{r} \right) \cdot (dr\hat{r}) = \int_{\infty}^{r} G \frac{m_Em}{r^2} dr = -G m_E m \frac{1}{r}
\]

Energy conservation:

\[
U(r) + K(r) = \text{const}
\]
**Reference Point(s)**

### Reference point

Reference point: matter of convenience, only \( \Delta U \) matters (because only forces can be measured, and the forces depend on \( \Delta U \), not \( U \)).

\[
\vec{F}(r) = -\frac{\partial U(r)}{\partial r} \hat{r}
\]

**“sea level” as a reference:**

\[
U(r) - U(r_E) = -Gm_E m \left( \frac{1}{r} - \frac{1}{r_E} \right)
\]

\[
\cong Gm_E m \frac{r - r_E}{r_E^2} = m \frac{G m_E}{r_E^2} \Delta h = mg \Delta h
\]

\( r - r_E \ll r_E \)

\( \Delta U(h) = mg \Delta h \)
Conservative Vector Fields

**Gravitational and Electrostatic Fields are conservative.**

The reason: both fields are *central* (a central force depends only on the distance between interacting objects and is directed along the line joining them).

\[ \oint F_E \cdot d\mathbf{l} = 0 \]

- for any loop

\( \Delta U \) depends only on the *initial* and *final* points of the trajectory:

No closed \( E \) field lines in electrostatics:
Consider two point charges, $+q_1$ and $+q_2$. Place $+q_1$ at the origin.

Let’s bring $q_2$ from $\infty$ to a point $P$ at a distance $r$ from the origin. The work done by us:

$$W_{\infty \rightarrow r}^{"us"} = \int_{\infty}^{r} \vec{F}_{us} \cdot d\vec{l} = \int_{\infty}^{r} \left( -\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \right) \cdot d\vec{r} = \frac{q_1 q_2}{4\pi\varepsilon_0} \left( \frac{1}{r} - \frac{1}{\infty} \right) = \frac{q_1 q_2}{4\pi\varepsilon_0} \left( \frac{1}{r} \right)$$

along the green trajectory

We worked against the electrostatic forces and increased the potential energy of this charge distribution.
Electrostatic Potential Energy

- the potential energy of the system “charge $q$ + external electrostatic field $\vec{E}(r)$”

$$U(r) = \int_{\text{some ref. point}}^{r} \vec{F}_{us} \cdot d\vec{l} = - \int_{\text{some ref. point}}^{r} q\vec{E}(r) \cdot d\vec{l}$$

$$U(r) = - \int_{\infty}^{(\cdot)P} q_2\vec{E}_1(r) \cdot d\vec{l} = - \int_{\infty}^{(\cdot)Q} q_1\vec{E}_2(r) \cdot d\vec{l} = \frac{q_1q_2}{4\pi\varepsilon_0} \left( \frac{1}{r} \right)$$

$$E_{tot} = K(r) + U(r)$$

$++$ or $--$

$+-$ or $-+$
Two charges $q=+1\text{C}$ are 1 m apart at rest. We release the charges. What would be the net kinetic energy of the system when the charges are very far from one another?

\[ U(r) = \frac{kq_1q_2}{r} \]

A. $k_1 \text{J}$  B. $-k_1 \text{J}$  C. $k_2 \text{J}$  D. $-k_2 \text{J}$  E. 0
Two charges \( q=+1\text{C} \) are 1 m apart at rest. We release the charges. What would be the net kinetic energy (in SI units) of the system when the charges are very far from one another?

\[
\begin{align*}
+1\text{C} & \quad \quad +1\text{C} \\
\quad \quad 1\text{m}
\end{align*}
\]

A. \( k_1 \)  
B. \(-k_1 \)  
C. \( k_2 \)  
D. \(-k_2 \)  
E. 0
Example

The figure shows a region of space with a constant electric field given by $\vec{E} = 2\hat{x} + 3\hat{y}$, in units of N/C. The dashed lines indicate the electric field. A charge $Q = -3 \text{ C}$ is moved from a start position $(x = 0 \text{ m}, y = 0 \text{ m})$ to an end position $(x = 1 \text{ m}, y = -2 \text{ m})$. What is the change in potential energy of the charge?

\[
\Delta U(r) = - \int_{\text{start}}^{\text{end}} Q\vec{E}(r) \cdot d\vec{l}
\]

\[
= -Q \left[ \int_{(0,0)}^{(1,0)} \vec{E}(x, y) \cdot dx \hat{x} + \int_{(0,0)}^{(1,0)} \vec{E}(x, y) \cdot dy \hat{y} \right]
\]

\[
= -Q \left[ \int_{(0,0)}^{(1,0)} (2\hat{x} + 3\hat{y}) \cdot dx \hat{x} + \int_{(0,0)}^{(1,0)} (2\hat{x} + 3\hat{y}) \cdot dy \hat{y} \right]
\]

\[
= +3 \left[ 2 - 6 \right] J = -12 \text{ J}
\]
From $U(r)$ to $F(r)$

$U(r) = - \int_{\text{ref. point}}^{r} \vec{F}(r) \cdot d\vec{l}$

One-dimensional case:

$\vec{F}(x) = - \frac{dU(x)}{dx} \hat{x}$

+q $\longleftrightarrow$ r $\longleftrightarrow$ −q

(·)P (·)Q

$U(r) = \frac{q_1 q_2}{4\pi \varepsilon_0} \left( \frac{1}{r} \right) = - \frac{q^2}{4\pi \varepsilon_0 r}$

$\vec{F}(r) = - \frac{dU(r)}{dr} \hat{r}$

$dU(r)/dr$ – positive, $\vec{F}(r)$ is directed along $-\hat{r}$.

(as it should be, because of attraction)

Q: is it possible to find $E(r)$ if we know $U(r)$ only at this particular point?
Iclicker Question

$U(x)$ is shown on the plot:

Which plot shows the correct $F(x)$?

\[ \vec{F}(x) = -\frac{dU(x)}{dx} \hat{x} \]

A.  

B.  

C.  

D. None of them.
$U(x)$ is shown on the plot:

Which plot shows the correct $F(x)$?

A. 

B. 

C. 

D. None of them.
The reference point can be *any* point. By shifting the reference point, we add the same constant values to the potential energies at all other points:

\[ U(\vec{r}) = -\int_{\vec{r}_{ref}}^{\vec{r}} q\bar{E} \cdot d\vec{l} \]

This ambiguity shouldn’t bother us, because only the *potential difference* is meaningful

\[ (\vec{F}_E(r) = -\frac{dU(r)}{dr} \hat{r}) \]

and the difference doesn’t depend on the reference point.

**Localized (“finite”) charge distribution.**

The natural choice is to set \( U=0 \) at an infinitely distant point. Again, it’s just a matter of convenience. For many problems, it’s convenient to set the potential energy of charges on the “ground” to zero.
“Infinite” Charge Distributions

- *charge distribution that extends to infinity* (e.g., a charged infinite plane).

The choice of an infinitely distant reference point is inconvenient – all electrostatic energies would be infinitely large. The remedy is to choose less remote reference point.

**Example:** uniformly charged plane.

Convenient reference point:
any point within the plane *(x=0)*. All points within the plane have the same *U* (we can move the charge over the plane without doing any work).

\[
\Delta U = U(x, y) - U(0,0) = - \int_{0,0}^{x,y} q \vec{E}(x) \cdot d\vec{l} \\
= - \int_{0,0}^{0,y} q \vec{E}(x) \cdot d\vec{y} - \int_{0,y}^{x,y} q \vec{E}(x) \cdot d\vec{x} = -q \frac{\sigma}{2\varepsilon_0} x
\]
Superposition: several interacting charges

If the test charge interacts with several charges, the potential energy of the interaction between the test charge and all other charges is an algebraic sum of \( U_{0i} \):

\[
U_0 = \frac{1}{4\pi \varepsilon_0} \left( \frac{q_0 q_1}{r_{01}} + \frac{q_0 q_2}{r_{02}} + \frac{q_0 q_3}{r_{03}} \right) = \frac{q_0}{4\pi \varepsilon_0} \sum_i \frac{q_i}{r_{0i}}
\]

Example: a positive test charge interacting with a dipole

\[
U_0 = \frac{q_0 q_1}{4\pi \varepsilon_0} \left( \frac{1}{r_{01}} \right) + \frac{q_0 q_2}{4\pi \varepsilon_0} \left( \frac{1}{r_{02}} \right) = \frac{q_0 q}{4\pi \varepsilon_0} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)
\]
Conclusion

Conservative vector fields $\rightarrow$ Potential energy.
Electrostatic potential energy.
Calculation of $U$ for simple charge distributions.

Next time: Lecture 6. Calculation of Electric Potentials
§§ 23.3 - 23.5
Appendix I. Conservative Vector Fields

**Conservative vector fields**: the work by the field on a “charge” depends only on the initial and final points of a trajectory, but not on the shape of the trajectory.

Math involved: \( \text{curl } \vec{a}(r) = 0 \quad \int \vec{a} \cdot d\vec{l} = 0 \) - circulation of a vector field

\[
\text{curl } \vec{E}(r) = \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{x} + \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{y} + \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \hat{z}
\]

- looks scary...

But we can tell at a glance if the curl is zero or not: think of a vector field as a water flow and place a paddle wheel at the point in question. If the wheel rotates, the *curl* is non-zero.

\[
\partial E_y \quad \frac{\partial E_y}{\partial z} = \frac{\partial E_y}{\partial x} = 0
\]

\[
\text{curl } \vec{a}(r) = 0
\]

\[
\partial E_y \quad \frac{\partial E_y}{\partial x} = 1
\]

\[
\text{curl } \vec{a}(r) \neq 0
\]
Appendix II: Total energy of interactions between several charges

The total work required to assemble this charge distribution:

\[ U_\Sigma = \frac{q_0 q_1}{4\pi \varepsilon_0} \left( \frac{1}{r_{01}} \right) + \frac{q_0 q_2}{4\pi \varepsilon_0} \left( \frac{1}{r_{02}} \right) + \frac{q_1 q_2}{4\pi \varepsilon_0} \left( \frac{1}{r_{12}} \right) + \cdots = \frac{1}{4\pi \varepsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}} \]

- all possible pairs, but each pair we count just once.

**Example:** four equal charges in the corners of a square.

\[ U_e^{\text{total}} = \sum_{\text{all pairs}} \frac{q_i q_j}{4\pi \varepsilon_0 r_{ij}} = 4 \frac{q^2}{4\pi \varepsilon_0 a} + 2 \frac{q^2}{4\pi \varepsilon_0 a\sqrt{2}} \]

The energy is positive (the charges repel each other), the system can do some work if we let the charges go.

On the other hand, the potential energy of one of these charges in the field due to the other three charges:

\[ U_e = 2 \frac{q^2}{4\pi \varepsilon_0 a} + \frac{q^2}{4\pi \varepsilon_0 a\sqrt{2}} \]