Outline: (Chapter 32 in the book)

- Electromagnetic Waves in Free Space
- Transfer of EM Energy in Space: Poynting Formalism

Today, we will consider electromagnetic waves in vacuum only!

Cell phones $\lambda \sim 0.3 \text{ m}$
Maxwell’s Equations

- **Gauss’s Law**
  \[ \oint_{\text{surf}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

- **No magnetic monopoles**
  \[ \oint_{\text{surf}} \vec{B} \cdot d\vec{A} = 0 \]

- **Faraday’s Law of electromagnetic induction**
  \[ \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_{\text{surf}} \vec{B} \cdot d\vec{A} \]

- **Generalized Ampere’s Law**
  \[ \oint_{\text{loop}} \vec{B} \cdot d\vec{l} = \mu_0 \oint_{\text{surf}} \left( \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{A} \]

Different types of solutions, e.g.

- “Coulomb” electric fields generated by charges at rest \((E \propto 1/r^2)\);

- Spherical electromagnetic waves generated by accelerated charges \((E \propto 1/r)\).

We’ll consider the simplest case of waves: plane EM waves in vacuum.
Maxwell’s Equations in Vacuum

\[ \oint_{\text{surf}} \mathbf{E} \cdot d\mathbf{A} = 0 \]

\[ \oint_{\text{surf}} \mathbf{B} \cdot d\mathbf{A} = 0 \]

\[ \oint_{\text{loop}} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint_{\text{surf}} \mathbf{B} \cdot d\mathbf{A} \]

\[ \oint_{\text{loop}} \mathbf{B} \cdot d\mathbf{l} = \varepsilon_0 \mu_0 \oint_{\text{surf}} \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{A} \]

\[ q = 0, J = 0 \]

Do these equations have non-trivial \((E \neq 0, B \neq 0)\) solutions in vacuum?

Yes, these solutions describe electromagnetic waves that propagate at the speed of light!

Maxwell’s greatest triumph: prediction of the existence of electromagnetic waves that could travel through empty space at the speed of light and identification of light as electromagnetic waves.
Plane EM Waves in Vacuum

Generation of EM waves: by accelerated charges and, thus, by AC currents. We’ll consider the simplest case of a plane monochromatic wave.

\[ \oint_{\text{surf}} \vec{E} \cdot d\vec{A} = 0 \]
\[ \oint_{\text{surf}} \vec{B} \cdot d\vec{A} = 0 \]

- ensure transverse character of EM waves (E and B are perpendicular to dA)

- provide the wave equations for both E and B waves, e.g. plane E and B waves that travel along +x:

\[ \frac{\partial^2 E(x, t)}{\partial x^2} - \epsilon_0 \mu_0 \frac{\partial^2 E(x, t)}{\partial t^2} = 0 \]
\[ \frac{\partial^2 B(x, t)}{\partial x^2} - \epsilon_0 \mu_0 \frac{\partial^2 B(x, t)}{\partial t^2} = 0 \]
First important relationship $E = cB$

\[ \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{\text{surf}} \vec{B} \cdot d\vec{A} \]

\[ \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = -Bac \]

\[ \frac{d}{dt} \int_{\text{surf}} \vec{B} \cdot d\vec{A} = -Ea \]
Second important relationship $c = 1/\sqrt{\varepsilon_0 \mu_0}$

\[ \oint_{\text{loop}} \vec{B} \cdot d\vec{l} = \mu_0 \int_{\text{surf}} \left( \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{A} \]

$E a c = \int_{\text{surf}} \left( \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{A}$

\[ \oint_{\text{loop}} \vec{B} \cdot d\vec{l} = B \ a \]

$B \ a = E a \ c \ \varepsilon_0 \mu_0$

$c = 1/\sqrt{\varepsilon_0 \mu_0}$
In the same way: The wave equation

\[
\frac{\partial^2 E(x, t)}{\partial x^2} - \varepsilon_0 \mu_0 \frac{\partial^2 E(x, t)}{\partial t^2} = 0
\]

\[
\frac{\partial^2 B(x, t)}{\partial x^2} - \varepsilon_0 \mu_0 \frac{\partial^2 B(x, t)}{\partial t^2} = 0
\]
Phase velocity: 

\[ kx - \omega t = \text{const} \]  
\[ kdx - \omega dt = 0 \]  

where \( k \) is the wave vector, \( \omega \) is the angular velocity, \( x \) and \( t \) are position and time, respectively. 

\[ v_p \equiv \frac{dx}{dt} = \frac{\omega}{k} \]

The general wave equation in 1D is:

\[ \frac{\partial^2 f(x,t)}{\partial x^2} - \frac{1}{v_p^2} \frac{\partial^2 f(x,t)}{\partial t^2} = 0 \]

Solutions:

\[ f(x,t) = f(x - v_p t) \]

Using dimensionless units:

\[ x - v_p t \]

- phase

- wavelength

Units of \( \alpha \): \( \frac{s^2}{m^2} \Rightarrow 1/v^2 \)

The wave vector:

\[ k = \frac{2\pi}{\lambda} \]

The angular velocity:

\[ \omega = 2\pi \frac{v_p}{\lambda} = 2\pi f \]
Basic Properties of Plane E.-M. Waves in Vacuum

Plane-front wave traveling in +x direction:
\[
\vec{E}(x,t) = E_0 \cos(kx - \omega t) \hat{\jmath} \\
\vec{B}(x,t) = B_0 \cos(kx - \omega t) \hat{k}
\]

\[\vec{k}\vec{r} - \omega t \equiv \text{the phase}\]
\[\vec{k}\vec{r} = \text{const} - \text{the phase front}\]
(the surface of constant phase).

1. In vacuum, the EM waves travel at the speed of light \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \).

\[
\frac{\partial^2 E(x,t)}{\partial x^2} - \varepsilon_0 \mu_0 \frac{\partial^2 E(x,t)}{\partial t^2} = 0 \quad \leftrightarrow \quad \frac{\partial^2 f(x,t)}{\partial x^2} - \frac{1}{v_p^2} \frac{\partial^2 f(x,t)}{\partial t^2} = 0
\]

\[v_p = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c \quad c \approx 3 \cdot 10^8 \text{ m/s}\]

2. The e.-m. wave in free space is a **transverse** wave.

\[\vec{E} \times \vec{B} = \frac{E^2}{\omega} \hat{k}\]

\(\vec{E}\) and \(\vec{B}\) in the traveling e.-m. wave are perpendicular to the direction of propagation (\(\vec{k}\)).
In a sinusoidal electromagnetic wave in vacuum, the electric field \( \vec{E} = \hat{i}E_0 \cos(ky - \omega t) \)

The wave travels along

A. \( X \) axis.
B. \( Y \) axis.
C. \( Z \) axis.
D. not enough information given to decide
In a sinusoidal electromagnetic wave in vacuum, the electric field \( \vec{E} = \hat{i}E_0 \cos(ky - \omega t) \).

The wave travels along

A. X axis.
B. Y axis.  \(\boxed{\text{B. Y axis.}}\)
C. Z axis.
D. not enough information given to decide
In a sinusoidal electromagnetic wave in vacuum, the electric field $\vec{E} = \hat{k}E_0 \cos(ky + \omega t)$.

The wave travels in

A. $+Z$ direction
B. $-Z$ direction
C. $+Y$ direction
D. $-Y$ direction
In a sinusoidal electromagnetic wave in vacuum, the electric field \( \vec{E} = \hat{k}E_0 \cos(ky + \omega t) \).

The wave travels in

A. +Z direction  \( ky + \omega t = \text{const} \)

B. -Z direction  \( kdy + \omega dt = 0 \)

C. +Y direction  \( \frac{dy}{dt} = -\frac{\omega}{k} < 0 \)

D. -Y direction
In a sinusoidal electromagnetic wave in a vacuum, the electric field \( \vec{E} = \hat{i}E_0 \cos(ky + \omega t) \).

The magnetic field of this wave

A. has only an x-component.  
B. has only a y-component.  
C. has only a z-component.  
D. not enough information given to decide 

\[ \vec{E} \times \vec{B} = \frac{E^2}{\omega} \hat{k} \]
In a sinusoidal electromagnetic wave in a vacuum, the electric field \( \vec{E} = \hat{i}E_0 \cos(ky + \omega t) \).

The magnetic field of this wave

A. has only an x-component.
B. has only a y-component.
C. has only a z-component.
D. not enough information given to decide

\[ \vec{E} \times \vec{B} = \frac{E^2}{\omega} \hat{k} \]
Imagine that the electric part of an electromagnetic wave moving in the +z direction is given by $E_x = A \sin(kz - \omega t), E_y = E_z = 0$. The y component of the magnetic field part is

A. $B_y \propto \cos(ky - \omega t)$

B. $B_y \propto \cos(kz - \omega t)$

C. $B_y \propto \sin(kz - \omega t)$

D. $B_y = 0$

E. Given by some other expression.
Imagine that the electric part of an electromagnetic wave moving in the +z direction is given by $E_x = A \sin(kz - \omega t), E_y = E_z = 0$. The y component of the magnetic field part is

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D. $B_y = 0$

E. Given by some other expression.