The second common hour in Physics 227 will be held Thursday, November 12 from 9:50 to 11:10 PM at night in 4 locations on two campuses, Livingston and Busch. You should go to the exam location assigned to the first letters of your last name. Note that the exam locations have changed since the first exam. Make sure you go to the correct exam location where you will find your exam.

<table>
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If you have a conflict for this exam you must send your request for a conflict and your entire schedule for the week of November 8 to Professor Cizewski Cizewski@physics.rutgers.edu no later than 5:00 PM on Wednesday, November 4. If you do not request a conflict exam before the deadline you will have to take the exam as scheduled on November 12.
The magnetic field at a distance $r$ from a straight wire with current:

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

The circulation of the field $B \neq 0$ if the enclosed-by-the-loop current $\neq 0$.

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_0 I$$

For a loop that doesn’t enclose any current, the circulation is 0.
**Ampere’s Law**

**Magnetostatics**: both $B$ and $I$ are time-independent.

Circulation of the magnetic field around a loop

\[ \oint \vec{B}(r) \cdot d\vec{l} = \mu_0 I_{encl} \]

(“Discrete”) enclosed currents:

\[ I_{encl} = \sum_i I_i \]

The line integral could go around the loop in either direction (clockwise or counterclockwise); the sign of currents is determined by the right-hand rule: the curled fingers are aligned along $d\vec{l}$, the thumb points in the direction of “positive” currents.

\[ \sum_i I_i = I_1 + I_3 - I_2 \]
The figure shows, in cross section, three conductors that carry currents perpendicular to the plane of the figure.

If the currents $I_1$, $I_2$, and $I_3$ all have the same magnitude, for which path(s) is the line integral of the magnetic field equal to zero?

A. path $a$ only  
B. paths $a$ and $c$
C. paths $b$ and $d$  
D. paths $a$, $b$, $c$, and $d$
E. The answer depends on whether the integral goes clockwise or counterclockwise around the path.
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Continuous Current Distribution

\[ \vec{j}(\vec{r}) \] - local current density

\[ I_{encl} = \int_{surface} \vec{j}(\vec{r}) \cdot d\vec{A} \]

The flux of the current vector field

\[ \oint \vec{B}(r) \cdot d\vec{l} = \mu_0 \int_{surface} \vec{j}(\vec{r}) \cdot d\vec{A} \]

There are infinitely many possible surfaces that have the loop as their border. Which of those surfaces is to be chosen? It does not matter; for all these surfaces \( \int_{surface} \vec{j}(\vec{r}) \cdot d\vec{A} \) would be the same (due to the continuity equation for charge).
Similar to Gauss’ Law, Ampere’s Law is very useful whenever it is possible to reduce a 3D (vector) problem to a 1D (scalar) problem. The key is the proper symmetry of a problem.

\[ \int_{\text{surface}} \vec{E}(r) \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \]

**Symmetry:** if a charge distribution is unchanged by rotations, translations, and reflections, then the \( E \) field is also unchanged by the same transformation.

\[ \int_{\text{loop}} \vec{B}(r) \cdot d\vec{l} = \mu_0 I_{\text{encl}} \]

**Symmetry:** if a current distribution is unchanged by rotations and translations, then the \( B \) field is also unchanged by the same transformation.

**Exception:** reflections.

**«Useful» symmetries:**
- Axial + translational (an infinite cylinder)
- Spherical
- Infinite slab (plane)

- Axial + translational (an infinite cylinder)
- Infinite solenoid
- Infinite slab (plane)
Mirror Reflection Symmetry

Electric field

uniformly charged circle

\[ \vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda dl \left( \vec{r} - \vec{r}' \right)}{|\vec{r} - \vec{r}'|^3} \]

\( \vec{E} \) is **parallel** to the mirror plane at any point on the plane.

Magnetic field

\[ \vec{B}(r) = \frac{\mu_0}{4\pi} \int \frac{I dl \times \left( \vec{r} - \vec{r}' \right)}{|\vec{r} - \vec{r}'|^3} \]

\( \vec{B} \) is **perpendicular** to the mirror plane at any point on the plane.

**The Mirror Rule for Magnetic Fields:**

if we can slice a current distribution with a mirror in such a way that the distribution looks exactly the same after we insert the mirror as before, then \( \vec{B} \) at any point on the mirror’s surface will be perpendicular to that surface.
Axial + Translational Symmetry

An infinite cylinder carrying a current whose density depends (at most) on the distance \( r \) from the axis.

Symmetries of the current distribution:
- rotations around the axis;
- translations along the axis;
- reflections across any plane containing the axis.

\[ B(r) \text{ is tangent to a circle centered at the axis and depends only on the distance from the axis.} \]

Amperian loop: a circle centered at the axis
\[ \oint \vec{B}(r) \cdot d\vec{l} = \mu_0 I_{encl} \]

**Example**: A circular wire of radius \( R \) with a **uniform** current density \( j = I/(\pi R^2) \):

\[
\begin{align*}
  r < R: & \quad B(r)2\pi r = \mu_0 I \frac{r^2}{R^2} \\
  r > R: & \quad B(r)2\pi r = \mu_0 I
\end{align*}
\]

\[ B(r) = \frac{\mu_0 I r}{2\pi R^2} \]
More Complex Cases

Any charge distribution that can be considered as superposition of symmetrical charge distributions can be treated on the basis of Ampere’s Law.

\[
B(r) = \frac{\mu_0 I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}
\]

Coaxial cable

\[
b < r < c \quad B(r)2\pi r = \mu_0 \left( I - I \frac{\pi(r^2-b^2)}{\pi(c^2-b^2)} \right) = \mu_0 I \frac{c^2-r^2}{c^2-b^2}
\]
**Field of an Infinite Solenoid**

**Approximation**: the solenoid’s radius is much smaller than its length, and we evaluate the field far from the solenoid’s ends.

The solenoid carries current $I$, the number of turns per unit length is $n$.

Symmetries of the current distribution:
- rotations around the axis;
- translations along the axis;
- reflections across any plane perpendicular to the axis.

$\mathbf{B}$ at any point is directed **along** the axis; it can depend only on the distance from the axis.

**Loop 1**: \[ \oint \mathbf{B}(r) \cdot d\mathbf{l} = B_{\text{top}}L - B_{\text{bot}}L = 0 \] - field inside is uniform

**Loop 2**: \[ \oint \mathbf{B}(r) \cdot d\mathbf{l} = B_{\text{top}}L \pm B_{\text{bot}}L = \mu_0 nIL \] - field outside is also uniform

Ampere’s Law allows us to calculate only the **combination** $B_{\text{top}} \pm B_{\text{bot}} = \mu_0 nI$.

Experimental fact: $B_{\text{outside}} = 0$. 

\[ B_{\text{inside}} = \mu_0 nI \]
Field of a Finite Solenoid

\[ B = \frac{1}{2} \mu_0 I n \]

- only for a “semi-infinite” solenoid
Field of a Toroidal Solenoid

Symmetries of the current distribution:
- rotations around the center in the plane of the toroid;
- reflections across any plane perpendicular to the plane of the toroid and going through its center.

the radial component of $B$ is zero; $B$ can depend only on the distance from the axis.

Amperian loops: circles centered at the toroid’s axis

Loop 1 \[ B(r)2\pi r = 0 \quad B(r) = 0 \quad r < r_1 \]

Loop 2 \[ B(r)2\pi r = \mu_0 IN \quad B(r) = \frac{\mu_0 IN}{2\pi r} \quad r_1 < r < r_2 \]

Loop 3 \[ B(r)2\pi r = 0 \quad B(r) = 0 \quad r > r_2 \]
Symmetries of the current distribution:
- translations along the plane;
- reflections across any $xz$ plane
- reflections across the $yz$ plane centered at the slab.

$B$ is directed along $y$ everywhere, it is zero along the $yz$ plane centered at the slab.

Amperian loop: rectangle in the $xy$ plane of length $l$ along $y$, one side is centered at the slab ($B=0$ at the center due to symmetry)

$$Bl = \mu_0 jxl \quad \vec{B} = -\mu_0 jx\hat{j}$$

For an infinitely thin slab with the linear current density $K = 2aj$ (current per unit length)
Pressure on Solenoid Walls

\[ B_{in} = \mu_0 nI = \mu_0 K \]

\( K = nI \) – linear current density (per unit length)

**Total force per unit area** 1m² (pressure) on a current-carrying sheet:

\[ P = nIB_{ext} = KB_{ext} = K \frac{\mu_0 K}{2} = \frac{1}{2\mu_0} B_{in}^2 \]

For a 10T solenoid:

\[ P = \frac{10^7}{8\pi} 10^2 Pa \approx 4 \cdot 10^7 Pa \approx 400\text{bar} \]

**Ultra-strong mag. fields: problem of mechanical stability of solenoids**
Axial + Translational Symmetry

Infinite Solenoid

\( B_{\text{inside}} = \mu_0 n I \)

\( B_{\text{outside}} = 0 \)

Toroidal Solenoid

\( r < R: B(r) = \frac{\mu_0 I r}{2\pi R^2} \)

\( r > R: B(r) = \frac{\mu_0 I}{2\pi r} \)

Infinite Slab

Infinitely thin slab with linear current density \( K \)

\( B = \mu_0 \frac{K}{2} \)
Appendix I. Magnetic Field as a Pseudovector

The magnetic field, being a cross product of two polar (or true) vectors, $\propto \vec{r} \times \vec{I}$, is a pseudovector (or an axial vector).

A pseudovector transforms like a true vector under a proper rotation, but gains an additional sign flip under an improper rotation such as a reflection (including inversion). Geometrically the pseudovector is the opposite of its mirror image (in contrast to a polar vector, which on reflection matches its mirror image). Examples of pseudovectors: angular velocity, torque, and angular momentum.

Another example: magnetic field of a current-carrying wire loop. If the position and current of the wire are reflected across the dashed line, the magnetic field it generates would be reflected and reversed.