Lecture 14: Magnetic Forces on Currents.

Outline:

- Hall Effect.
- Magnetic Force on a Wire Segment.
- Torque on a Current-Carrying Loop.

Lecture 13: Magnetic Forces on Moving Charges
- we considered individual charges;
- and ignored electrostatic interactions among the charges in the electron beam.
Under what circumstances is the total magnetic flux through a closed surface positive?

A. if the surface encloses the north pole of a magnet, but not the south pole

B. if the surface encloses the south pole of a magnet, but not the north pole

C. if the surface encloses both the north and south poles of a magnet

D. none of the above
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When a charged particle moves through a magnetic field, the direction of the magnetic force on the particle at a certain point is

A. in the direction of the magnetic field at that point.
B. opposite to the direction of the magnetic field at that point.
C. perpendicular to the magnetic field at that point.
D. none of the above
E. The answer depends on the sign of the particle’s electric charge.
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E. The answer depends on the sign of the particle’s electric charge.
A charged particle moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the particle to move through this region at a constant velocity,

A. the electric and magnetic fields must point in the same direction.

B. the electric and magnetic fields must point in opposite directions.

C. the electric and magnetic fields must point in perpendicular directions.

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Hall Effect

Current-carrying conductors in external magnetic field: two systems of charges - mobile current carriers and immobile ion lattice - and only the current carriers are affected by $B$.

In the steady state, the magnetic force on moving charges is compensated by the electrostatic force due to *uncompensated* surface charge:

\[
\vec{F}_E = -\vec{F}_B \\
qE = qv_d B \\
E = v_d B
\]

\[
I = qnv_d W t
\]

\[
V_H = EW = v_d BW = \frac{IBW}{qnWt}
\]

- Hall voltage

The sign of $V_H$ depends on the sign of the charge of current carriers $q$, the magnitude $\propto 1/n$. 

Edwin Hall (1855-1938)

(recall “velocity selector”, see L.13)
**Example**: typical two-dimensional \((n \cdot t)\) charge density in Si field-effect transistor (in the “on” state) is \(10^{16} \text{ } 1/\text{m}^2\). Let’s run a 0.1A current and place the transistor in a 0.1T magnetic field:

\[
V_H = \frac{IB}{qnt} = \frac{0.1A \cdot 0.1T}{1.6 \cdot 10^{-19}C \cdot 1 \cdot 10^{16}m^{-2}} \approx 6V
\]
The electrons drift **along** the wire because the net (el.+mag.) force on them in the direction normal to the wire is 0. However, positively charge ions are at rest, they feel only an **uncompensated** electric force:

\[ f = qE_H = \frac{IB}{nWt} \text{ - force per one ion} \]

\[ F = nWtf = IB \text{ - force per unit length of the conductor} \]

The force on a wire segment of length \( l \):

\[ \vec{F} = l(\vec{I} \times \vec{B}) \]

Less accurate treatment in the textbook: if we ignore the Hall voltage, and the electric force on the positive ions, and assume that the momentum in the z direction is somehow transferred from the electrons to the lattice, we still get the same answer for the net force.
Example: a straight horizontal length of wire has a mass of \( m/l = 10 \text{ g/m} \); it carries a current of \( 1 \text{ A} \). What are the magnitude and direction of the minimum magnetic field needed to suspend the wire in the Earth’s gravitational field?

\[
F_B = mg \quad lIB = \left(\frac{m}{l}\right) lg \quad B = \frac{(m/l)g}{I} = \frac{0.01 \text{ kg/m} \cdot 10 \text{ m/s}^2}{1 \text{ A}} = 0.1 \text{T}
\]
Ampere’s Motor

Which orientation?

Switch

Conducting bar

Conducting rails

\[ \vec{F} \quad \vec{B} \]
Torque on a Current Loop

Consider \( \vec{A} \perp \vec{B} \) (\( \vec{B} \) in the loop’s plane): \( F_1 = F_2 = aIB \) \( F_3 = F_4 = 0 \)

\[
\vec{\tau} = \frac{b}{2} \hat{z} \times \vec{F}_1 + \frac{b}{2} (-\hat{z}) \times \vec{F}_2 = \frac{b}{2} aIB(-\hat{y}) + \frac{b}{2} aIB(-\hat{y}) = abIB(-\hat{y})
\]

**Magnetic dipole moment:**

\[
\mu = abI = AI \quad (A \text{ – the loop’s area})
\]

Direction of \( \vec{\mu} \): the right-hand rule.

If there are \( N \) turns, the total magnetic dipole moment is \( \mu = NAI \)

In general:

\[
\vec{\tau} = \vec{\mu} \times \vec{B} \quad \tau = \mu B \text{sin} \phi
\]

(compare with \( \vec{\tau} = \vec{p} \times \vec{E} \))
Potential Energy of a Current Loop in Magnetic Field

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \mu B \sin(\phi) \]

\[ U(\phi) = -\mu B \cos(\phi) \]

unstable equilibrium

\[ \vec{\tau} = -\text{grad} U(\phi) \]

\[ = -\frac{dU(\phi)}{d\phi} \hat{\phi} \]
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic torque on the loop

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic torque* on the loop

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
**Direct Current Motor**

*C. tends to make the loop rotate around its axis.*

How can we realize this situation? – By changing the direction of current once per period of rotation.

- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.
Magnetic Dipole vs. Electric Dipole

**Similarity:**
- a magnet’s magnetic field is very similar to a dipole’s electric field at points far from the dipoles;
- both repel/attract each other;
- both align along the field lines.

**Difference:**
- unlike electric dipoles, magnetic poles cannot be separated;
- magnets have no effect on stationary charges.
Imagine that you place a magnet in a **uniform** magnetic field. Will the magnetic field exert a net force on the magnet? If so, what is the direction of the force (**Hint: use the electric dipole analogy**).  

A. yes, in the direction of the magnetic field  
B. yes, opposite to the direction of the magnetic field  
C. yes, but the direction depends on the magnet’s orientation  
D. no, the net force on the magnet is zero.
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D. no, the net force on the magnet is zero.
Magnetic Dipole in a Non-Uniform Magnetic Field

http://www.ru.nl/hfml/research/levitation/diamagnetic/

\[ F = \mu \cdot \frac{dB}{dz} \propto B \cdot \frac{dB}{dz} \]

Paramagnetic response: the induced \( \vec{\mu} \) is parallel to \( \vec{B} \)

Diamagnetic response: the induced \( \vec{\mu} \) is anti-parallel to \( \vec{B} \)

Levitation of a diamagnetic object

In contrast, the induced electric dipoles are oriented along the electric field, they are always **attracted** to the region of a stronger field.

Andre Geim
Nobel 2010
IgNobel 2000

Top view
Next time: Lecture 15: Magnetic Fields of Currents.
§§ 28.1- 28.5
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic force on the loop is

A. perpendicular to the plane of the loop, in a direction given by a right-hand rule.

B. perpendicular to the plane of the loop, in a direction given by a left-hand rule.

C. in the same plane as the loop.

D. zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the \textit{net magnetic torque} on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.

B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.

C. tends to make the loop rotate around its axis.

D. is zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.
A particle with a positive charge moves in the $xz$-plane as shown. The magnetic field is in the positive $z$-direction. The magnetic force on the particle is in

A. the positive $x$-direction.
B. the negative $x$-direction.
C. the positive $y$-direction.
D. the negative $y$-direction.
E. none of these
A positively charged particle moves in the positive $z$-direction. The magnetic force on the particle is in the positive $y$-direction. What can you conclude about the $x$-component of the magnetic field at the particle’s position?

A. $B_x > 0$

B. $B_x = 0$

C. $B_x < 0$

D. not enough information given to decide