Lecture 13: Magnetic Forces on Moving Charges
- we considered individual charges;
- and ignored electrostatic interactions among the charges in the electron beam.

Outline:

- Hall Effect.
- Magnetic Force on a Wire Segment.
- Torque on a Current-Carrying Loop.
Under what circumstances is the total magnetic flux through a closed surface *positive*?

A. if the surface encloses the north pole of a magnet, but not the south pole

B. if the surface encloses the south pole of a magnet, but not the north pole

C. if the surface encloses both the north and south poles of a magnet

D. none of the above
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A particle with a positive charge moves in the $xz$-plane as shown. The magnetic field is in the positive $z$-direction. The magnetic force on the particle is in

A. the positive $x$-direction.
B. the negative $x$-direction.
C. the positive $y$-direction.
D. the negative $y$-direction.
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\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
Visualization: direction of force on point charge, orientation of electric dipoles.

“Force-on charges” field: direction-wise coincides with $\vec{E}$ field

Acceleration: tangential to $\vec{E}$ field

Trajectory: no relation to $\vec{E}$ field

Visualization: orientation of magnetic dipoles (compass needles)

“Force-on charges” field: no such thing

Acceleration: perpendicular to $\vec{B}$ field

Trajectory: no relation to $\vec{B}$ field
A charged particle moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the particle to move through this region at a constant velocity,

A. the electric and magnetic fields must point in the same direction.

B. the electric and magnetic fields must point in opposite directions.

C. the electric and magnetic fields must point in perpendicular directions.

D. The answer depends on the sign of the particle’s electric charge.
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Hall Effect

Current-carrying conductors in external magnetic field: two systems of charges - mobile current carriers and immobile ion lattice - and only the current carriers are affected by $B$.

In the steady state, the magnetic force on moving charges is compensated by the electrostatic force due to *uncompensated* surface charge:

$$\vec{F}_E = -\vec{F}_B$$

$$qE = qv_d B$$

$$E = v_d B$$

$$I = qnv_d W t$$

- $n$ – the density of mobile carriers
- $W$ – the width of the conductor
- $t$ – the thickness of the conductor

$$V_H = EW = v_d BW = \frac{IBW}{qnWt}$$

- Hall voltage

The sign of $V_H$ depends on the sign of the charge of current carriers $q$, the magnitude $\propto 1/n$.  

(Recall “velocity selector”, see L.13)
**Example**: typical two-dimensional \((n \cdot t)\) charge density in Si field-effect transistor (in the “on” state) is \(10^{16} \text{ 1/m}^2\). Let’s run a 0.1A current and place the transistor in a 0.1T magnetic field:

\[
V_H = \frac{IB}{qnt} = \frac{0.1A \cdot 0.1T}{1.6 \cdot 10^{-19}C \cdot 1 \cdot 10^{16} m^{-2}} \approx 6V
\]
The electrons drift *along* the wire because the net (el.+mag.) force on them in the direction normal to the wire is 0. However, positively charge ions are at rest, they feel only an *uncompensated* electric force:

\[ f = qE_H = \frac{IB}{qnWt} \]

\[ F = nWtf = IB \]

The force on a wire segment of length \( d\hat{l} \):

\[ \vec{F} = I(d\hat{l} \times \vec{B}) \]

Less accurate treatment in the textbook: if we ignore the Hall voltage, and the electric force on the positive ions, and assume that the momentum in the \( z \) direction is somehow transferred from the electrons to the lattice, we still get the same answer for the net force.
\( \vec{j}(r) \) - the current density, which is a \textit{vector} field

By definition, the total current through a surface \( S \) is the flux of the current density through this surface:

\[
I \equiv \int_S \vec{j}(r) \cdot d\vec{a}
\]

\( d\vec{a} \) - the element of surface, as usual

Thus, the total current is a \textit{scalar}. In the equation for the force on the segment of current-carrying wire of length \( dl \) in the external \( B \) field, \( \vec{F} = I \vec{d\ell} \times \vec{B} \), the vector \( d\vec{\ell} \) is tangential to the wire and its direction coincides with the current direction:
Visualization: direction of force on point charge, orientation of electric dipoles.

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Example: a straight horizontal length of wire has a mass of $m/l=10 \text{ g/m}$; it carries a current of $1 \text{ A}$. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire in the Earth’s gravitational field?

$$F_B = mg \quad lIB = \left(\frac{m}{l}\right)lg \quad B = \frac{(m/l)g}{I} = \frac{0.01 kg/m \cdot 10 m/s^2}{1\text{ A}} = 0.1 T$$
Dipoles in Uniform Fields

Net force = 0, torque ≠ 0

\[
\tau = \vec{r} \times \vec{F}
\]

Electric

\[
\vec{F} = \vec{p} \times \vec{E}
\]

\[
U(\phi) = -pE\cos(\phi)
\]

Magnetic

\[
F = ILB
\]
Torque on a Current Loop

Consider $\vec{A} \perp \vec{B}$ (\(\vec{B}\) in the loop’s plane): $F_1 = F_2 = aIB$  $F_3 = F_4 = 0$

$$\vec{\tau} = \frac{b}{2} \hat{z} \times \vec{F}_1 + \frac{b}{2} (-\hat{z}) \times \vec{F}_2 = \frac{b}{2} aIB(-\hat{y}) + \frac{b}{2} aIB(-\hat{y}) = abIB(-\hat{y})$$

**Magnetic dipole moment:**

$$\mu = abl = AI \quad (A – the\ loop’s\ area)$$

Direction of $\vec{\mu}$: the right-hand rule.

In general:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \tau = \mu B \sin \phi$$

(compare with $\vec{\tau} = \vec{p} \times \vec{E}$)

If there are $N$ turns, the total magnetic dipole moment is $\mu = NAI$

$$\tau = 0 \text{ if } \phi = 0^0, 180^0$$
Potential Energy of a Current Loop in Magnetic Field

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \mu B \sin(\phi) \]

\[ U(\phi) = -\mu B \cos(\phi) \]

unstable equilibrium

\[ \tau = -\text{grad } U(\phi) \]

\[ = -\frac{dU(\phi)}{d\phi} \hat{\phi} \]

stable equilibrium

\( \phi \)

\( \pi \)

\( \pi/2 \)

\( 0 \)
A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the net magnetic torque on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.

B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.

C. tends to make the loop rotate around its axis.

D. is zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

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Magnetic Dipole vs. Electric Dipole

**Similarity:**

- a magnet’s magnetic field is very similar to a dipole’s electric field *at points far from the dipoles*;
- both repel/attract each other;
- both align along the field lines.

**Difference:**

- unlike electric dipoles, magnetic poles cannot be separated;
- magnets have no effect on stationary charges.
The Earth's North Magnetic Pole is actually a magnetic *south* pole and the Earth's South Magnetic Pole is a magnetic *north* pole.
Next time: Lecture 15: Magnetic Fields of Currents.
§§ 28.1- 28.5
Appendix I: Ampere’s Motor

Which orientation?

Conducting bar

Conducting rails

Switch

Vector $\vec{F}$

Vector $\vec{B}$
Appendix II: Magnetic Dipole in Non-Uniform Magnetic Field

http://www.ru.nl/hfml/research/levitation/diamagnetic/

F = \mu \cdot \frac{dB}{dz} \propto B \cdot \frac{dB}{dz}

In contrast, the induced electric dipoles are oriented along the electric field, they are always **attracted** to the region of a stronger field.