Outline:

- Intro to Magnetostatics.
- Magnetic Field Flux, Absence of Magnetic Monopoles.
- Force on charges moving in magnetic field.
Our goal: to describe the magnetostatic field the same way we’ve described the electrostatic field.

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \text{always true} \]

\[ \oint \vec{E} \cdot d\vec{l} = 0 \quad \text{true only if the fields are static} \]

\[ \vec{F} = q\vec{E} \]

Do you know a vector field \( \vec{a} \) that has both \( \oint \vec{a} \cdot d\vec{A} = 0 \) and \( \oint \vec{a} \cdot d\vec{l} = 0 \)?
Sources of Magnetic Field

Charges at rest do not generate $B$. What would be our “best bet” for the source of the magnetic field?

“magnetic point charge” (a magnetic monopole): have not been observed yet (though its existence doesn’t contradict anything)

a moving single charge: the field is non-stationary, not good for magnetostatics

steady (time-independent) current: our best bet

Sources of $B$:

- Charges in motion: currents, orbital motion of electrons in atoms.

- Ferromagnetic materials (electron spins – purely quantum phenomenon).

- Time-dependent electric fields (we’ll consider this source later in Electrodynamics).

Units: tesla, T
Characteristic Magnetic Fields

\[ B \approx 10^{-4} T \]

\[ B \approx 1 - 2T \]

\[ B \text{ up to } 8T \]

\[ B \text{ up to } 30T \]

rare-earth magnets

Superconducting solenoids in LHC

Superconducting solenoids
Flux of the Magnetic Field

### Flux of the magnetic field:

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

compare with

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

No “magnetic point charges” (magnetic monopoles): have not been observed yet.

- **Consequence:** for ANY closed surface \[ \oint \vec{B} \cdot d\vec{A} = 0 \]
  - always true, not only in electrostatics but also in electrodynamics

Units: \( T \cdot m^2 = \text{Weber, Wb} \)

Breaking a magnet in two ...

... yields two magnets, not two isolated poles.

Magnetic dipole

Electric dipole
Magnetic field is a \textit{non-conservative} vector field. In general

\[
\oint \mathbf{B} \cdot d\mathbf{l} \neq 0
\]

to be specified later

Magnetic field lines are \textit{closed loops} (no mag. monopoles):
Force on a Charge Moving in Magnetic Field

Lorentz force

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]

\[ F = qvB \sin \phi \]

\[ \vec{F} = 0 \] for charges moving along \( \vec{B} \)

\( F \) is max when \( \vec{v} \perp \vec{B} \)

Magnetic field lines are not lines of force!
When does a magnetic field exert a force on a charged particle?

A. Always.

B. Only when the particle moves exactly perpendicular to the magnetic field lines.

C. When the particle is moving at a non-zero angle with respect to the magnetic field lines.

D. When the particle is moving along the magnetic field lines.

E. When the particle is moving.

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
When does a magnetic field exert a force on a charged particle?

A. Always.

B. Only when the particle moves exactly perpendicular to the magnetic field lines.

C. When the particle is moving at a non-zero angle with respect to the magnetic field lines.

D. When the particle is moving along the magnetic field lines.

E. When the particle is moving.

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
Imagine that you are looking at the face of a CRT. The bright spot indicating where the electron beam hits the face. You bring a permanent magnet toward the CRT with its north pole oriented upward. Which direction will the spot be deflected across the screen?

A. up
B. down
C. the spot does not deflect
D. right
E. left
Imagine that you are looking at the face of a CRT. The bright spot indicating where the electron beam hits the face. You bring a permanent magnet toward the CRT with its north pole oriented upward. Which direction will the spot be deflected across the screen?

A. up 
B. down 
C. the spot does not deflect 
D. right 
E. left
\[ \vec{F} = q(\vec{v} \times \vec{B}) \quad \Rightarrow \quad \vec{F} \perp d\vec{l} \quad \Rightarrow \quad dW = \vec{F} \cdot d\vec{l} = 0 \]

The work done by the magnetic field on a moving charge = 0

You cannot increase the speed of charged particles using magnetic field. However, the $B$-induced acceleration is non-zero, it’s just perpendicular to the velocity \( \left( \vec{a} \equiv \frac{d\vec{v}}{dt} \right) \). An accelerated charge (e.g. moving with constant speed along a curved trajectory) loses its energy by radiating electromagnetic waves.

However, there are many situations in which this statement appears to be false: in a non-uniform magnetic field, a current-carrying wire loop would accelerate, two permanent magnets would accelerate towards one another, etc. In these cases, the kinetic energy of objects increases. Who does the work? For discussion, see

http://van.physics.illinois.edu/qa/listing.php?id=17176
Cross Product

\[ \vec{F} = -e(\vec{v} \times \vec{B}) \]

An electron moves along the z axis, the B field is in the x-y plane.

\[ \vec{v} = 1\hat{k} \text{ m/s} \quad \vec{B} = (2\hat{i} + 3\hat{j})T \]

\[ \vec{F} = -e(\vec{v} \times \vec{B}) = -e \left( 1\hat{k} \times (2\hat{i} + 3\hat{j}) \right) \]

\[ = -e(1\hat{k} \times 2\hat{i} + 1\hat{k} \times 3\hat{j}) \]

\[ = -e(2\hat{j} - 3\hat{i}) = e(3\hat{i} - 2\hat{j}) \]
A particle with charge \( q = -1 \text{ C} \) is moving in the positive \( z \)-direction at 5 m/s. The magnetic field at its position is

\[
\vec{B} = \left( 3\hat{i} - 4\hat{j} \right) \text{ T}
\]

What is the magnetic force on the particle?

A. \( \left( 20\hat{i} + 15\hat{j} \right) \text{ N} \)

B. \( \left( 20\hat{i} - 15\hat{j} \right) \text{ N} \)

C. \( \left( -20\hat{i} + 15\hat{j} \right) \text{ N} \)

D. \( \left( -20\hat{i} - 15\hat{j} \right) \text{ N} \)

E. none of these

\[
\vec{F} = q(\vec{v} \times \vec{B})
\]
A particle with charge \( q = -1 \text{ C} \) is moving in the positive \( z \)-direction at 5 m/s. The magnetic field at its position is

\[
\vec{B} = (3\hat{i} - 4\hat{j}) \text{ T}
\]

What is the magnetic force on the particle?

A. \((20\hat{i} + 15\hat{j}) \text{ N}\)

B. \((20\hat{i} - 15\hat{j}) \text{ N}\)

C. \((-20\hat{i} + 15\hat{j}) \text{ N}\)

D. \((-20\hat{i} - 15\hat{j}) \text{ N}\)

E. none of these

\[
\vec{F} = q(\vec{v} \times \vec{B}) = (-1) \left( 5\hat{k} \times (3\hat{i} - 4\hat{j}) \right) = -15\hat{j} - 20\hat{i}
\]
When a charged particle moves through a magnetic field, the trajectory of the particle at a given point is

A. parallel to the magnetic field line that passes through that point.

B. perpendicular to the magnetic field line that passes through that point.

C. neither parallel nor perpendicular to the magnetic field line that passes through that point.

D. any of the above, depending on circumstances
When a charged particle moves through a magnetic field, the trajectory of the particle at a given point is

A. parallel to the magnetic field line that passes through that point.

B. perpendicular to the magnetic field line that passes through that point.

C. neither parallel nor perpendicular to the magnetic field line that passes through that point.

D. any of the above, depending on circumstances
A positively charged particle moves in the positive $z$-direction. The magnetic force on the particle is in the positive $y$-direction. What can you conclude about the $z$-component of the magnetic field at the particle’s position?

A. $B_z > 0$
B. $B_z = 0$
C. $B_z < 0$
D. not enough information given to decide
A positively charged particle moves in the positive $z$-direction. The magnetic force on the particle is in the positive $y$-direction. What can you conclude about the $z$-component of the magnetic field at the particle’s position?

A. $B_z > 0$
B. $B_z = 0$
C. $B_z < 0$
D. not enough information given to decide
Cyclotron Motion in Magnetic Field

Motion along a circular orbit:

\[ \vec{F} = q(\vec{v} \times \vec{B}) = qvB = m \frac{v^2}{R} \]

\[
R = \frac{mv}{qB}
\]

alternatively, by measuring \( R \), one can determine the ratio \( m/q \) if \( v \) (the kinetic energy) is known.

\[
T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}
\]

- the period \( T \) is independent of \( v \)

If a charge has a velocity component along \( \vec{B} \):
What’s wrong? In this form, the equation works only for non-relativistic motion. To correct, you need to replace the “classical” momentum $mv$ with the relativistic one $mv/\sqrt{1-(v/c)^2}$. 
\[
\frac{mv^2}{2} = qV \\
v = \sqrt{\frac{2qV}{m}} \\
qE = qvB \\
v = \frac{E}{B}
\]

Ionization  \quad \text{Accelerating voltage applied}  \quad \text{Velocity selector}  \quad \text{Magnetic field region}

\[ R = \frac{mv}{qB} \]
Magnetostatics: $B \neq B(t)$

Sources of $B$: motion of charges (currents), orbital motion of electrons in atoms, electron spins, time-dependent electric fields.

Absence of Magnetic Monopoles: $\oint \mathbf{B} \cdot d\mathbf{A} = 0$

Force on charges moving in magnetic field: $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. 

Next time: Lecture 14: Magnetic Forces on Currents.
§§ 27.6- 27.9
Structure of the Course

Electromagnetism

- Electric fields
- Magnetic fields
- Charges
- Currents

Electrostatics

Electric fields generated by charges at rest

Ch. 21 - 26

Magnetostatics

Magnetic fields generated by time-independent currents

Ch. 27 - 31

Electromagnetic Waves

\[ \frac{d}{dt} \neq 0 \]

- Covered in detail in more advanced courses on electrodynamics and optics.
Which of the following statements describes an observation that represents strong evidence that magnetic poles are not the same as electric charges?

A. Poles are described as being north and south, while charges are described as being positive and negative

B. A magnet does not exert a force on a motionless charge

C. Magnetic poles always come in pairs

D. A and B

E. B and C
Iclicker Question

Which of the following statements describes an observation that represents strong evidence that magnetic poles are not the same as electric charges?

A. Poles are described as being north and south, while charges are described as being positive and negative

B. A magnet does not exert a force on a motionless charge

C. Magnetic poles always come in pairs

D. A and B

E. B and C
Protons with energy 1 MeV move \( \perp \) earth B field of 0.5 gauss or \( 5 \times 10^{-5} \) T. Find radius & frequency of orbit.

\[
K = \frac{1}{2} m v^2 \implies v = \sqrt{\frac{2K}{m}}
\]

\[
K = \left(10^6\right)\left(1.6 \times 10^{-19}\right) = 1.6 \times 10^{-13} \text{ J}
\]

\[
m = 1.67 \times 10^{-27} \text{ kg}
\]

\[
R = \frac{mv}{eB} = \frac{\sqrt{2mK}}{eB}
\]

\[
R = 2900 \text{ m}
\]

\[
f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{v}{2\pi \left(\frac{mv}{eB}\right)} = \frac{eB}{2\pi m}
\]

\[
f = 760 \text{ Hz}
\]

Frequency is independent of \( v \)!
7. What is the relative magnitude of the electric and magnetic forces between two equal charges?

a) The electric force is always greater.
b) The magnetic force is always greater.
c) The two forces are always equal.
d) The electric force is never less than the magnetic force.
e) The magnetic force is never less than the electric force.