Outline:

- Intro to Magnetostatics.
- Magnetic Field Flux, Absence of Magnetic Monopoles.
- Force on charges moving in magnetic field.
Structure of the Course

Electromagnetism

$E$ - electric fields
$B$ – magnetic fields
$Q$ – charges
$I$ – currents
$\Phi$ - fluxes

$d \frac{d}{dt} = 0$

Ch. 21 - 26

Electrostatics

Electric fields generated by charges at rest

Ch. 27 - 31

Magnetostatics

Magnetic fields generated by time-independent currents

Ch. 32

Electromagnetic Waves

$\frac{d}{dt} \neq 0$

- covered in detail in more advanced courses on electrodynamics and optics.
Our goal: to describe the magnetostatic field the same way we’ve described the electrostatic field.

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \text{- always true}
\]

\[
\oint \vec{E} \cdot d\vec{l} = 0 \quad \text{- true only if the fields are static}
\]

\[
\vec{F} = q\vec{E}
\]

Do you know a vector field \( \vec{a} \) that has both \( \oint \vec{a} \cdot d\vec{A} = 0 \) and \( \oint \vec{a} \cdot d\vec{l} = 0 \)?
Charges at rest do not generate $\mathbf{B}$. What are the sources of the magnetic field?

- “magnetic point charge” (a magnetic monopole): has not been observed yet (though its existence doesn’t contradict anything)
- ferromagnetic materials (electron spins – purely quantum phenomenon).
- Time-dependent electric fields (we’ll consider this source later in Electrodynamics).
- Charges in motion: currents, orbital motion of electrons in atoms.

**In magnetostatics:**

The $\mathbf{B}$ field a moving single charge is **non-stationary**, not good for magnetostatics

Steady (time-independent) current: our best bet.
Characteristic Magnetic Fields

**Units: tesla, T**

\[ B \approx 10^{-4}T \]

- **Rare-earth magnets**
  \[ B \text{ up to } 1.4T \]

- **Superconducting solenoids in LHC**
  \[ B \text{ up to } 8T \]

- **Superconducting solenoids**
  \[ B \text{ up to } 30T \]
Flux of the Magnetic Field

Flux of the magnetic field:

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

compare with

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

Consequence:
for ANY closed surface

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]

- always true, not only in electrostatics but also in electrodynamics

No “magnetic point charges” (magnetic monopoles): have not been observed yet.

Breaking a magnet in two ...

... yields two magnets, not two isolated poles.
Magnetic Field Lines

Magnetic field lines are *closed loops* (no mag. monopoles):

Magnetic field is a *non-conservative* vector field.

\[ \oint \vec{B} \cdot d\vec{l} \neq 0 \]

to be specified later
Force on a Charge Moving in Magnetic Field

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]

\[ F = qvB \sin \phi \]

\( \vec{F} = 0 \) for charges moving along \( \vec{B} \)

\( F \) is max when \( \vec{v} \perp \vec{B} \)

Magnetic field lines are not lines of force!

demonstration
Imagine that you are looking at the face of a CRT. The bright spot indicating where the electron beam hits the face. You bring a permanent magnet toward the CRT with its north pole oriented upward. Which direction will the spot be deflected across the screen?

A. up  \[ \vec{F} = q(\vec{v} \times \vec{B}) \]
B. down
C. the spot does not deflect
D. right
E. left
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When does a magnetic field exert a force on a charged particle?

A. Always.

B. Only when the particle moves exactly perpendicular to the magnetic field lines.

C. When the particle is moving at a non-zero angle with respect to the magnetic field lines.

D. When the particle is moving along the magnetic field lines.

E. When the particle is moving.

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\[ \vec{F} = q(\vec{v} \times \vec{B}) \]
No Work Done by Magnetic Force!

\[ \vec{F} = q(\vec{v} \times \vec{B}) \quad \Rightarrow \quad \vec{F} \perp d\vec{l} \quad \Rightarrow \quad dW = \vec{F} \cdot d\vec{l} = 0 \]

The work done by the magnetic field on a moving charge is zero.

You cannot increase the speed of charged particles using magnetic field. However, the \( B \)-induced acceleration is non-zero, it’s just perpendicular to the velocity \( \vec{a} \equiv \frac{d\vec{v}}{dt} \).

However, there are many situations in which this statement appears to be false: in a non-uniform magnetic field, a current-carrying wire loop would accelerate, two permanent magnets would accelerate towards one another, etc. In these cases, the kinetic energy of objects increases. Who does the work? For discussion, see

http://van.physics.illinois.edu/qa/listing.php?id=17176
When a charged particle moves through a magnetic field, the trajectory of the particle at a given point is

A. parallel to the magnetic field line that passes through that point.

B. perpendicular to the magnetic field line that passes through that point.

C. neither parallel nor perpendicular to the magnetic field line that passes through that point.

D. any of the above, depending on circumstances
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\( \vec{F} = -e(\vec{v} \times \vec{B}) \)

An electron moves along the z axis, the \( B \) field is in the \( x-y \) plane.

\[
\vec{v} = 1\hat{k} \text{ m/s} \quad \vec{B} = (2\hat{i} + 3\hat{j}) T
\]

\[
\vec{F} = -e(\vec{v} \times \vec{B}) = -e \left( 1\hat{k} \times (2\hat{i} + 3\hat{j}) \right)
\]

\[
= -e(1\hat{k} \times 2\hat{i} + 1\hat{k} \times 3\hat{j})
\]

\[
= -e(2\hat{j} - 3\hat{i}) = e(3\hat{i} - 2\hat{j})
\]
A particle with charge $q = -1$ C is moving in the positive $z$-direction at 5 m/s. The magnetic field at its position is

$$\vec{B} = (3\hat{i} - 4\hat{j}) \text{ T}$$

What is the magnetic force on the particle?

A. $(20\hat{i} + 15\hat{j}) \text{ N}$
B. $(20\hat{i} - 15\hat{j}) \text{ N}$
C. $(-20\hat{i} + 15\hat{j}) \text{ N}$
D. $(-20\hat{i} - 15\hat{j}) \text{ N}$
E. none of these

$$\vec{F} = q(\vec{v} \times \vec{B})$$

\[
\begin{align*}
\hat{i} \times \hat{j} &= \hat{k} \\
\hat{j} \times \hat{k} &= \hat{i} \\
\hat{k} \times \hat{i} &= \hat{j}
\end{align*}
\]
A particle with charge \( q = -1 \, \text{C} \) is moving in the positive \( z \)-direction at 5 m/s. The magnetic field at its position is

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\vec{B} = \left(3 \hat{i} - 4 \hat{j}\right) \, \text{T}
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B. \( \left(20 \hat{i} - 15 \hat{j}\right) \, \text{N} \)

C. \( \left(-20 \hat{i} + 15 \hat{j}\right) \, \text{N} \)

D. \( \left(-20 \hat{i} - 15 \hat{j}\right) \, \text{N} \)

E. none of these

\[
\vec{F} = q \left(\vec{v} \times \vec{B}\right) = (-1) \left(5 \hat{k} \times (3 \hat{i} - 4 \hat{j})\right) = -15 \hat{j} - 20 \hat{i}
\]
A positively charged particle moves in the positive $z$-direction. The magnetic force on the particle is in the positive $y$-direction. What can you conclude about the $z$-component of the magnetic field at the particle’s position?

A. $B_z > 0$
B. $B_z = 0$
C. $B_z < 0$
D. not enough information given to decide
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A. $B_z > 0$
B. $B_z = 0$
C. $B_z < 0$
D. not enough information given to decide

\[ \vec{F} = q(\vec{v} \times \vec{B}) \]

\( \hat{i} \times \hat{j} = \hat{k} \)
\( \hat{j} \times \hat{k} = \hat{i} \)
\( \hat{k} \times \hat{i} = \hat{j} \)
Cyclotron Motion in Magnetic Field

Motion along a circular orbit:

\[
\vec{F} = q(\vec{v} \times \vec{B}) \rightarrow qvB = m \frac{v^2}{R}
\]

\[
R = \frac{mv}{qB}
\]

alternatively, by measuring \(R\), one can determine the ratio \(m/q\) if \(v\) (the kinetic energy) is known.

\[
T = \frac{2\pi R}{v} = \frac{2\pi m}{qB} \quad \text{- the period } T \text{ is independent of } v
\]

If a charge has a velocity component along \(\vec{B}\):
Large Hadron Collider

What’s wrong? In this form, the equation works only for non-relativistic motion. To correct, you need to replace the “classical” momentum $mv$ with the relativistic one $mv/\sqrt{1 - (v/c)^2}$. 

\[ R = \frac{mv}{qB} \]

\[ m \approx 1.6 \cdot 10^{-27} \text{kg} \]

\[ v = c \]

\[ B = 8 \text{T} \]

\[ R = \frac{1.6 \cdot 10^{-27} \text{kg} \cdot 3 \cdot 10^8 \text{m/s}}{1.6 \cdot 10^{-19} \text{C} \cdot 8 \text{T}} \approx 0.38 \text{m} \]
Mass Spectrometer

\[ \frac{mv^2}{2} = qV \]
\[ v = \sqrt{\frac{2qV}{m}} \]
\[ qE = qvB \]
\[ v = \frac{E}{B} \]

Ionization

Accelerating voltage applied

Velocity selector

Detector

\[ R = \frac{mv}{qB} \]
Magnetostatics: $B \neq B(t)$

Sources of $B$: motion of charges (currents), orbital motion of electrons in atoms, electron spins, time-dependent electric fields.

Sources of $B$ in “classical” magnetostatics: steady currents.

Absence of Magnetic Monopoles: $\oint B \cdot d\vec{A} = 0$

Force on charges moving in magnetic field: $\vec{F} = q(\vec{v} \times \vec{B})$.

Next time: Lecture 14: Magnetic Forces on Currents.
§§ 27.6- 27.9