Lecture 12. RC circuits.

Outline:

RC circuits: charging and discharging

Prof. Torgny Gustafsson will be away October 16 -31 and will not be available for office hours.

Prof. Michael Gershenson will be away October 20 - November 13 and will not be available for office hours.

This won’t affect the lecture schedule.
Consider the circuit in the Figure. If you were to unscrew bulb B from its socket (effectively removing it from the circuit and leaving a gap in its place), bulb A would

A. get brighter
B. get dimmer
C. remain the same
D. go out
Consider the circuit in the Figure. If you were to unscrew bulb B from its socket (effectively removing it from the circuit and leaving a gap in its place), bulb A would

A. get brighter
B. get dimmer
C. remain the same
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\[ P_A = R_A I^2 \]

\[ I = \frac{\mathcal{E}}{R_{tot}} \]

\( R_{tot} \) increases if we unscrew bulb B, thus \( P_A \) decreases.
Consider a circuit containing five identical light bulbs and an ideal battery. Assume that the resistance of each light bulb remains constant. Which statement is TRUE for the brightness of the bulbs (A through E)?

A. Bulb A is brighter than bulb B is brighter than bulb C is brighter than bulb D is brighter than bulb E.
B. Bulb A and bulb B are equally bright and brighter than bulb C.
C. Bulb D and bulb E are equally bright and brighter than bulb C.
D. Bulb C is the brightest.
E. All of the bulbs are equally bright.
Consider a circuit containing five identical light bulbs and an ideal battery. Assume that the resistance of each light bulb remains constant. Which statement is TRUE for the brightness of the bulbs (A through E)?

\[ R(A \parallel B) < R(C \parallel D \ldots E) \]

\[ V_1 < V_2 \]

\[ P_C = \frac{V_2^2}{R} \]

A  Bulb A is brighter than bulb B is brighter than bulb C is brighter than bulb D is brighter than bulb E.
B  Bulb A and bulb B are equally bright and brighter than bulb C.
C  Bulb D and bulb E are equally bright and brighter than bulb C.
D  Bulb C is the brightest.
E  All of the bulbs are equally bright.
Initial state (switch open): \( i=0, \ q=0, \ V_{ab}=0, \ V_{bc}=0 \)

How do we know that \( V_{bc}=0 \)?

Switch = capacitor with a very small \( C \)
Capacitor Charging (qualitative), cont’d

**Initial state** (switch open): $i=0$, $q=0$, $V_{ab}=0$, $V_{bc}=0$

At $t=0$, we close the switch: $q(0)=0$, $V_{bc}(0)=0$

- Capacitor initially uncharged
- Charging the capacitor

\[
\tau(C, R, \mathcal{E}) \ ?
\]
Capacitor Charging (qualitative), cont’d

(a) Capacitor initially uncharged

Switch open

\[ i = 0 \quad q = 0 \]

(b) Charging the capacitor

\[ \frac{\varepsilon}{R} \]

\[ l(t) \equiv dq(t)/dt \]

\[ t = 0 \quad \text{- capacitor behaves as a “short”} \]

\[ t = \infty \quad \text{- capacitor behaves as a “break”} \]
**Charging a Capacitor**

**Initial state** (switch open): \( i=0, \ q=0, \ V_{ab}=0, \ V_{bc}=0 \)

\( i(t), \ q(t) \) - instantaneous values

**At t=0, we close the switch:** \( q(0)=0, \ V_{bc}(0)=0 \)

For instantaneous values:

\[
\mathcal{E} - i(t)R - \frac{q(t)}{C} = 0 \quad i(t) \equiv \frac{dq(t)}{dt}
\]

\[
\frac{dq(t)}{dt} + \frac{q(t)}{RC} = \frac{\mathcal{E}}{R} \quad \frac{dq(t)}{dt} = -\frac{[q(t) - \mathcal{E}C]}{RC}
\]

\[
\frac{dq(t)}{q(t) - \mathcal{E}C} = -\frac{dt}{RC} \quad \int_{0}^{t} \frac{dq(t')}{q(t') - \mathcal{E}C} = -\frac{1}{RC} \int_{0}^{t} dt'
\]

\[
\ln \left[ \frac{q(t) - \mathcal{E}C}{-\mathcal{E}C} \right] = -\frac{t}{RC} \quad q(t) = -\mathcal{E}Ce^{-\frac{t}{\tau}} + \mathcal{E}C
\]

\[
q(t) = \mathcal{E}C \left( 1 - e^{-\frac{t}{\tau}} \right) \quad i(t) = \frac{dq(t)}{dt} = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau}} \quad \tau \equiv RC \quad \text{- the time constant (units of time)}
\]
Charging a Capacitor (cont’d)

\[ q(t) = \varepsilon C \left( 1 - e^{-\frac{t}{\tau}} \right) \]

For \( R=1\,\text{k}\Omega \) and \( C=1\,\text{mF} \) → \( \tau \equiv RC = 1\,\text{s} \)

\[ i(t) = \frac{dq(t)}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{\tau}} \]

\[ i_0 = \frac{\varepsilon}{R} \]

\[ i = \frac{1}{e} \frac{\varepsilon}{R} \]

as if there was no capacitor!
A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the *maximum* charge stored on the capacitor?

A. the emf $\varepsilon$ of the battery  
B. the capacitance $C$ of the capacitor  
C. the resistance $R$ of the resistor  
D. both $\varepsilon$ and $C$  
E. all three of $\varepsilon$, $C$, and $R$
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C. the resistance $R$ of the resistor  

D. both $\mathcal{E}$ and $C$  

E. all three of $\mathcal{E}$, $C$, and $R$
A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the rate at which the capacitor charges?

A. the emf $\varepsilon$ of the battery
B. the capacitance $C$ of the capacitor
C. the resistance $R$ of the resistor
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E. all three of $\mathcal{E}$, $C$, and $R$
Discharging a Capacitor

\[ \frac{dq(t)}{dt} + \frac{q(t)}{RC} = 0 \quad q(0) = q_0 = \mathcal{E}C \]

\[ q(t) = \mathcal{E}Ce^{-\frac{t}{\tau}} \]

\[ \int_{\mathcal{E}C}^{q(t)} \frac{dq(t')}{q(t')} = -\frac{1}{RC} \int_{0}^{t} dt' \]

\[ \ln \left[ \frac{q(t)}{\mathcal{E}C} \right] = -\frac{t}{RC} \]

\[ i(t) = \frac{dq(t)}{dt} = -\frac{\mathcal{E}}{R}e^{-\frac{t}{\tau}} \]

"-" means that the current flows in the direction opposite to the charging current.
Pr. 26.43:

Initially the capacitors are charged to 45V. At $t=0$ the switch is closed. Find $t_1$ at which the voltage across the capacitors is reduced to 10V. What is the current at this time?

The potential difference between points $a$ and $b$:

$$V(t) = V(0)e^{\frac{-t}{R_C}}$$

$$t_1 = R_C C \ln \left[ \frac{V(0)}{V(t_1)} \right] = 80\Omega \cdot 35\mu F \cdot \ln 4.5 = 0.0042s$$

$$i(t_1) = \frac{V(t_1)}{R_C} = \frac{10V}{80\Omega} = 0.125A$$
Originally the left-hand switch is closed and the voltage source charges a parallel-plate capacitor to a charge of magnitude $Q$ on both plates. Then the left-hand switch is opened and the right hand switch is closed so that the capacitor is disconnected from the voltage source $V$, allowing the capacitor to discharge through a resistor of resistance $R$. Immediately after the right-hand switch is closed, which of the following is true?

a) The voltage drop across the resistor is $V$.
b) The current through the resistor is 0.
c) The voltage drop across the capacitor is $V/2$.
d) The current through the resistor is $V/RC$.
e) The voltage drop across the capacitor is 0.
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d) The current through the resistor is $V/RC$.

e) The voltage drop across the capacitor is 0.
Energy Loss in Charging a Capacitor

Energy conservation:

\[ \mathcal{E} i = \mathcal{R} i^2 + iV_{bc} \]

- \( i(t) = \frac{\mathcal{E}}{\mathcal{R}} e^{-\frac{t}{\tau}} \)

Total work by the battery:

\[
\int_{0}^{\infty} \mathcal{E} i(t) dt = \int_{0}^{\infty} \frac{\mathcal{E}^2}{\mathcal{R}} e^{-t/\tau} dt = \frac{\mathcal{E}^2}{\mathcal{R}} (-\tau)(e^{-\infty/\tau} - e^{-0/\tau}) = \mathcal{E}^2 \mathcal{C} = \mathcal{E} q_{\infty}
\]

Energy dissipated in the resistor:

\[
\int_{0}^{\infty} \mathcal{R} i^2(t) dt = \int_{0}^{\infty} \frac{\mathcal{E}^2}{\mathcal{R}} e^{-2t/\tau} dt = \frac{\mathcal{E}^2}{\mathcal{R}} \left(-\frac{\tau}{2}\right)(e^{-\infty/\tau} - e^{-0/\tau}) = \frac{\mathcal{E}^2 \mathcal{C}}{2}
\]

- Half of the work done by the battery is wasted as heat (doesn’t depend on \( \mathcal{R} \)).

Solution: slowly (in comparison with \( \tau \)) ramp up the emf in the process of charging.
Outline:

RC circuits: charging and discharging.

Next time: Lecture 13: Magnetic Field, Magnetic Forces on Moving Charges

§§ 27.1- 27.4
Let’s consider AC current (it will be considered in greater detail later in the course).

The voltage drop build-up across the capacitor during one AC period:

\[
\tau = RC
\]

Thus, if \( RCf \gg 1 \), \( V \ll \varepsilon \) and we can ignore the presence of the “break” in the circuit. On the other hand, when \( RCf \ll 1 \), \( V \approx \varepsilon \), and the capacitor indeed “breaks” the circuit.