Outline:

- Energy and power in electric circuits.
- Voltage and Current Sources.
- Kirchhoff’s Rules.

Lecture 10:

Connection of resistors in parallel and in series.

Batteries: the potential energy of charge carriers is increased by non-electrostatic (non-conservative) forces.

\[ V_{ab} = \mathcal{E} - Ir \]

Non-ideal batteries: internal resistance.

Potential distribution around a complete circuit.
The second common hour in Physics 227 will be held Thursday, November 12 from 9:50 to 11:10 PM at night in 4 locations on two campuses, Livingston and Busch. You should go to the exam location assigned to the first letters of your last name. Note that the exam locations have changed since the first exam. Make sure you go to the correct exam location where you will find your exam.

<table>
<thead>
<tr>
<th>Aaa-Gzz</th>
<th>BE Aud</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haa-Mzz</td>
<td>LSH Aud</td>
</tr>
<tr>
<td>Naa-Shz</td>
<td>PHY LH</td>
</tr>
<tr>
<td>Sia-Zzz</td>
<td>SEC 111</td>
</tr>
</tbody>
</table>

If you have a conflict for this exam you must send your request for a conflict and your entire schedule for the week of November 8 to Professor Cizewski Cizewski@physics.rutgers.edu no later than 5:00 PM on Wednesday, November 4. If you do not request a conflict exam before the deadline you will have to take the exam as scheduled on November 12.
Each charge carrier going “downhill” from a higher $V_a$ to a lower $V_b$ dissipates the energy $eV_{ab}$ in its environment: the gained kinetic energy is transformed into thermal energy (“heat”) due to inelastic scattering (remember, no acceleration!).

If charge $dq$ passes a circuit element in $dt$, the power generated is:

$$P = \frac{dqV_{ab}}{dt} = V_{ab}I$$

For a resistor $R$:

$$P = VI = RI^2 = \frac{V^2}{R}$$

- Joule (resistive) heating power

Units: 1J/1s = 1Watt (W)
Three identical light bulbs are connected to a battery as shown. Which bulb is brightest?

A. light bulb A
B. light bulb B
C. light bulb C
D. both light bulbs B and C (both are equally bright and are brighter than light bulb A.)
E. All bulbs are equally bright.

\[ P = VI = RI^2 = \frac{V^2}{R} \]
Three identical light bulbs are connected to a battery as shown. Which bulb is brightest?

A. light bulb A
B. light bulb B
C. light bulb C
D. both light bulbs B and C (both are equally bright and are brighter than light bulb A.)
E. All bulbs are equally bright.

\[ P = VI = RI^2 = \frac{V^2}{R} \]
Problem

What is the maximum power one can get (dissipate in the load) from a given \((\mathcal{E}, r)\) battery?

\[ \mathcal{E} = I(r + R) \]

\[ P = RI^2 = \frac{R}{(r + R)^2} \mathcal{E}^2 \]

\[ \frac{dP}{dR} \propto \frac{1}{(r + R)^2} - 2 \frac{R}{(r + R)^3} = 0 \]

\[ r + R - 2R = 0 \quad R = r \]
Light Bulbs

The incandescent light bulb: a resistor, being heated by current, emits black-body radiation in the visible wavelength range.

Bulbs are rated by power. Resistance of a 60W bulb designed for 120V (neglect the fact that this rating is for the AC (not DC) current):

\[
R = \frac{V^2}{P} = \frac{(120V)^2}{60W} = 240\Omega
\]

Note that a light bulb that is rated at 60W actually produces only about 3 W of visible light ☹

The larger the power, the smaller the resistance (we assume that R is much greater than the internal resistance of the voltage source).

The wall outlet 110V can be considered as a voltage source – the internal resistance is smaller than all typical loads.
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in parallel as shown.

Which bulb glows the brightest?

\[ P = VI = RI^2 = \frac{V^2}{R} \]

A. the 60-W light bulb
B. the 120-W light bulb
C. the 240-W light bulb
D. All three light bulbs glow with equal brightness.
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in parallel as shown.

Which bulb glows the brightest?

\[ P = VI = RI^2 = \frac{V^2}{R} \]

A. the 60-W light bulb

B. the 120-W light bulb

C. the 240-W light bulb

D. All three light bulbs glow with equal brightness.
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in series as shown.

Across which bulb is there the greatest voltage drop?

A. the 60-W light bulb
B. the 120-W light bulb
C. the 240-W light bulb
D. All three light bulbs have the same voltage drop.
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in series as shown.

Across which bulb is there the greatest voltage drop?

A. the 60-W light bulb
B. the 120-W light bulb
C. the 240-W light bulb
D. All three light bulbs have the same voltage drop.
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in series as shown. Which bulb glows the brightest?

\[ P = VI = RI^2 = \frac{V^2}{R} \]

A. the 60-W light bulb  
B. the 120-W light bulb  
C. the 240-W light bulb  
D. All three light bulbs glow with equal brightness.
A 60-W light bulb, a 120-W light bulb, and a 240-W light bulb are connected in series as shown.

Which bulb glows the brightest?

\[ P = VI = RI^2 = \frac{V^2}{R} \]

A. the 60-W light bulb
B. the 120-W light bulb
C. the 240-W light bulb
D. All three light bulbs glow with equal brightness.
Kirchhoff’s Junction Rule

Junction Rule (for currents): charge conservation

\[ \sum_{i} I_i = 0 \]

Currents flowing in \( \rightarrow + \)

Currents flowing out \( \rightarrow - \)
Kirchhoff’s Loop Rule

Loop Rule (energy conservation):

\[ \sum_{i} \mathcal{E}_i + \sum_{i} V_i = 0 \]

for any closed loop

\[ \sum_{i} V_i = 0 \]

if we neglect the difference between \( V \)’s and \( \mathcal{E} \)’s, and accept the sign conventions.

(a) Sign conventions for emfs

\(+\mathcal{E}:\) Travel direction from – to +:

\(-\mathcal{E}:\) Travel direction from + to –:

(b) Sign conventions for resistors

\(+IR:\) Travel opposite to current direction:

\(-IR:\) Travel in current direction:
Example

We don’t need to know \textit{a priori} the actual direction of the current: if we get the negative value of $I$, that would mean that the current flows in the direction opposite to the direction of “travel”.

\[
\sum_i \varepsilon_i + \sum_i V_i = 0
\]

\[-4V + 12V - I(7\Omega + 2\Omega + 3\Omega + 4\Omega) = 0
\]

\[
I = \frac{8V}{16\Omega} = 0.5A
\]

(a) Sign conventions for emfs

$+\varepsilon$: Travel direction from $-$ to $+$:

$-\varepsilon$: Travel direction from $+$ to $-$:

(b) Sign conventions for resistors

$+IR$: Travel opposite to current direction:

$-IR$: Travel in current direction:
Example (cont’d)

- Reference voltage = 0

Diagram showing a circuit with resistors and voltages labeled.
11. In this circuit, what is the $EMF$ of the battery? Assume zero internal resistance for the battery.

a) $EMF = 4 \text{ V}$
b) $EMF = 12 \text{ V}$
c) $EMF = 8 \text{ V}$
d) $EMF = 20 \text{ V}$
e) Impossible to determine without knowing the value of $R$. 

![Circuit Diagram]
11. In this circuit, what is the \( EMF \) of the battery? Assume zero internal resistance for the battery.

   a) \( EMF = 4 \text{ V} \)
   b) \( EMF = 12 \text{ V} \)
   c) \( EMF = 8 \text{ V} \)
   d) \( EMF = 20 \text{ V} \)
   e) Impossible to determine without knowing the value of \( R \).

\[
\varepsilon - 4A \cdot 2\Omega - 3A \cdot 4\Omega = 0
\]

\[
\varepsilon = 20V
\]
Problem 26.77:
(a) what is the potential difference $V_{ab}$ when the switch is open?
(b) What is the current through the switch when the switch is closed?
(c) What is the equivalent resistance when the switch is closed?

\[
\begin{align*}
V_a &= 36V - 6\Omega \cdot 4A = 12V \\
V_b &= 36V - 3\Omega \cdot 4A = 24V \\
V_a - V_b &= 12V - 24V = -12V
\end{align*}
\]
Problem 26.77:
(b) What is the current through the switch when the switch is closed?

Choose (arbitrary) directions of currents and travel along the loops. Note that the “current” rule has been taken care of.

\[
\begin{align*}
\text{loop 1:} & \quad 36 - 6 \cdot I_1 - 3 \cdot (I_1 - I_3) = 0 \\
\text{loop 2:} & \quad -6 \cdot I_1 - 3 \cdot I_3 + 3 \cdot I_2 = 0 \\
\text{loop 3:} & \quad -3 \cdot (I_1 - I_3) + 6 \cdot (I_2 + I_3) + 3 \cdot I_3 = 0 \\
\end{align*}
\]

\[
I_3 = I_2 - 2I_1 \quad \text{(from Eq.2)}
\]

\[
-3 \cdot I_1 + 12 \cdot I_3 + 6 \cdot I_2 = 0 \quad \text{(from Eq.3)}
\]

\[
-3 \cdot I_1 + 12 \cdot I_2 - 24 \cdot I_1 + 6 \cdot I_2 = 0 \quad I_2 = \frac{3}{2}I_1
\]

\[
36 - 6 \cdot I_1 - 3 \cdot I_1 + 3(I_2 - 2I_1) = 36 - 10.5 \cdot I_1 = 0
\]

\[
I_1 = \frac{36}{10.5} \quad I_3 = \frac{3}{2}I_1 - 2I_1 = -0.5I_1 \quad I_3 = -\frac{18}{10.5} = -1.71
\]

"-" means that our initial direction of \(I_3\) has to be reversed.
Problem 26.77:
(a) what is the potential difference $V_{ab}$ when the switch is open?
(b) What is the current through the switch when the switch is closed?
(c) What is the equivalent resistance when the switch is closed?

\[
\begin{align*}
&\text{(c)} \quad R_{eq} = \frac{\mathcal{E}}{I} \\
&I = I_1 + I_2 \\
&I_1 = \frac{36}{10.5} \quad I_2 = \frac{3}{2} I_1 \quad I \approx 8.6A \\
&I \approx 8.6A \\
&R_{eq} = \frac{36V}{8.6A} = 4.2\Omega
\end{align*}
\]
Conclusion

Resistors in Series and Parallel
Voltmeters and Ammeters
Kirchhoff’s Rules

Next time: Lecture 12: RC circuits
§§ 26.4
Voltmeters

**The goal:** to measure the voltage difference across an element (ideally, without affecting the circuit due to the voltmeter connection).

An ideal voltmeter:

A. has $R_{in} = 0$ and should be connected in parallel with the circuit element being measured.

B. has $R_{in} = 0$ and should be connected in series with the circuit element being measured.

C. has $R_{in} = \infty$ and should be connected in parallel with the circuit element being measured.

D. has $R_{in} = \infty$ and should be connected in series with the circuit element being measured.
**Voltmeters**

*The goal:* to measure the voltage difference across an element (ideally, without affecting the circuit due to the voltmeter connection).

**An ideal voltmeter:**

A. has $R_{in} = 0$ and should be connected in parallel with the circuit element being measured.

B. has $R_{in} = 0$ and should be connected in series with the circuit element being measured.

C. has $R_{in} = \infty$ and should be connected in parallel with the circuit element being measured.

D. has $R_{in} = \infty$ and should be connected in series with the circuit element being measured.

Voltmeter: high internal resistance
The goal: to measure the current in a circuit element (ideally, without affecting the current due to the ammeter connection).

An ideal ammeter:

A. has $R_{in} = 0$ and should be connected in parallel with the circuit element being measured.

B. has $R_{in} = 0$ and should be connected in series with the circuit element being measured.

C. has $R_{in} = \infty$ and should be connected in parallel with the circuit element being measured.

D. has $R_{in} = \infty$ and should be connected in series with the circuit element being measured.
The goal: to measure the current in a circuit element (ideally, without affecting the current due to the ammeter connection).

An ideal ammeter:

A. has $R_{in} = 0$ and should be connected in parallel with the circuit element being measured.

B. has $R_{in} = 0$ and should be connected in series with the circuit element being measured.

C. has $R_{in} = \infty$ and should be connected in parallel with the circuit element being measured.

D. has $R_{in} = \infty$ and should be connected in series with the circuit element being measured.

Ammeter: low internal resistance
Appendix 1: Built-in Battery Tester

Bi-layer structure: the thermochromic (non-conductive) ink deposited on top of the conductive ink.

\[ R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} \]

\[ I = \frac{\mathcal{E}}{r + R_{eq}} \quad V_{ab} = \mathcal{E} - Ir = \mathcal{E} \frac{R_{eq}}{r + R_{eq}} \]

\[ P_1 = \frac{V_{ab}^2}{R_1} \quad P_2 = \frac{V_{ab}^2}{R_2} \quad P_3 = \frac{V_{ab}^2}{R_3} \]

\( \Delta T \) of the thermochromic ink is proportional to \( \frac{P}{\text{area}} \):
(e.g., \( P_3 \) is \( \sim \)2 times smaller than \( P_1 \), and its area is 2 times greater)

\[ \frac{P_1}{A_1} : \frac{P_2}{A_2} : \frac{P_3}{A_3} = 4: 2: 1 \]
\[ \mathcal{E} > 63V = 75V - 12\Omega \cdot 1A \]