Exercise 28.2

**Description:** In the Bohr model of the hydrogen atom, the electron moves in a circular orbit of radius $5.3 \times 10^{-11}$ m with a speed of $2.2 \times 10^6$ m/s. (a) If we are viewing the atom in such a way that the electron's orbit is in the plane of the paper...

In the Bohr model of the hydrogen atom, the electron moves in a circular orbit of radius $5.3 \times 10^{-11}$ m with a speed of $2.2 \times 10^6$ m/s.

**Part A**

If we are viewing the atom in such a way that the electron's orbit is in the plane of the paper with the electron moving clockwise, find the magnitude of the electric field that the electron produces at the location of the nucleus (treated as a point).

Express your answer using two significant figures.

**ANSWER:**

$$E = 5.1 \times 10^{11} \text{ N/C}$$

**Part B**

Find the direction of the electric field that the electron produces at the location of the nucleus (treated as a point).

**ANSWER:**

- from the electron
- toward the electron
- out of the page
- into the page
Part C
Find the magnitude of the magnetic field that the electron produces at the location of the nucleus (treated as a point).

Express your answer using two significant figures.

ANSWER:

\[ B = 13 \text{ T} \]

Part D
Find the direction of the magnetic field that the electron produces at the location of the nucleus (treated as a point).

ANSWER:

- from the electron
- toward the electron
- out of the page
- into the page

Direction of the Magnetic Field due to a Wire Conceptual Question

Description: Short conceptual question in which students must indicate the direction of the magnetic field due to long straight wires (one horizontal wire and two parallel horizontal wires).

Find the direction of the magnetic field at each of the indicated points.

For the following two questions consider the wire shown in the figure...

Part A
What is the direction of the magnetic field \( \vec{B}_A \) at Point A?
**Hint 1.** The magnitude of the magnetic field due to a long, straight, current-carrying wire

The magnitude \( B \) of the field is directly proportional to the current \( I \) flowing in the wire and inversely proportional to the distance \( r \) from the wire:

\[
B = \frac{\mu_0 I}{2\pi r}.
\]

**Hint 2.** The direction of the magnetic field due to a long, straight, current-carrying wire

The magnetic field surrounding a long, straight wire encircles the wire, as shown in the figure:

The direction of the field is determined by a right-hand rule: Grasp the wire with the thumb of your right hand in the direction of the current flow. The direction in which your fingers encircle the wire is the direction in which the magnetic field encircles the wire.

**ANSWER:**

- \( \vec{B}_A \) is out of the page.
- \( \vec{B}_A \) is into the page.
- \( \vec{B}_A \) is neither out of nor into the page and \( \vec{B}_A \neq 0 \).
- \( \vec{B}_A = 0 \).

**Part B**

What is the direction of the magnetic field \( \vec{B}_B \) at Point B?

**ANSWER:**
Now consider the wires shown in this figure. Note that the bottom wire carries a current of magnitude $2I$.

Part C

What is the direction of the magnetic field $\vec{B}_C$ at Point C?

**Hint 1. How to approach the problem**

To determine the direction of the magnetic field at Point C, you must determine the contribution to the field from both of the wires. The field at Point C is the vector sum of these two contributions. Keep in mind that if the magnetic fields are in opposite directions, the larger field will decide the direction of the net magnetic field. If they are the same size, the net magnetic field will be zero.

**Hint 2. Find the direction of the magnetic field at Point C due to wire 1**

Is the magnetic field from wire 1 directed out of or into the screen at Point C?

**ANSWER:**

- out of
- into

**Hint 3. Find the direction of the magnetic field at Point C due to wire 2**

Is the magnetic field from wire 2 directed out of or into the screen at Point C?

**ANSWER:**

- out of
- into
Part D

What is the direction of the magnetic field \( \vec{B}_D \) at Point D?

**Hint 1.** Find the direction of the magnetic field at Point D due to wire 1
Is the magnetic field from wire 1 directed out of or into the screen at Point D?

**ANSWER:**

- out of
- into

**Hint 2.** Find the direction of the magnetic field at Point D due to wire 2
Is the magnetic field from wire 2 directed out of or into the screen at Point D?

**ANSWER:**

- out of
- into

**ANSWER:**

- \( \vec{B}_D \) is out of the page.
- \( \vec{B}_D \) is into the page.
- \( \vec{B}_D \) is neither out of nor into the page and \( \vec{B}_D \neq 0 \).
- \( \vec{B}_D = 0 \).
Part E

What is the direction of the magnetic field $\vec{B}_E$ at Point E?

**Hint 1. Find the direction of the magnetic field at Point E due to wire 1**

Is the magnetic field from wire 1 directed out of or into the screen at Point E?

ANSWER:

- [ ] out of
- [ ] into

**Hint 2. Find the direction of the magnetic field at Point E due to wire 2**

Is the magnetic field from wire 2 directed out of or into the screen at Point E?

ANSWER:

- [ ] out of
- [ ] into

ANSWER:

- [ ] $\vec{B}_E$ is out of the page.
- [ ] $\vec{B}_E$ is into the page.
- [ ] $\vec{B}_E$ is neither out of nor into the page and $\vec{B}_E \neq 0$.
- [ ] $\vec{B}_E = 0$.

---

**Magnetic Field from Two Wires**

**Description:** Calculate, at several different locations, the net magnetic field due to two straight infinite wires carrying anti-parallel currents. Students are asked to look for a pattern in the results as the points of interest become more and more remote from the wires.

**Learning Goal:**

To understand how to use the principle of superposition in conjunction with the Biot-Savart (or Ampere's) law.

From the Biot-Savart law, it can be calculated that the magnitude of the magnetic field due to a long straight wire is given by

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d},$$

where $\mu_0 (= 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})$ is the permeability constant, $I$ is the current in the wire, and $d$ is the distance from the wire to the location at which the magnitude of the magnetic field is being calculated.
The same result can be obtained from Ampere's law as well.

The direction of vector $\vec{B}$ can be found using the right-hand rule. (Take care in applying the right-hand rule. Many students mistakenly use their left hand while applying the right-hand rule since those who use their right hand for writing sometimes automatically use their "pencil-free hand" to determine the direction of $\vec{B}$.)

In this problem, you will be asked to calculate the magnetic field due to a set of two wires with antiparallel currents as shown in the diagram. Each of the wires carries a current of magnitude $I$. The current in wire 1 is directed out of the page and that in wire 2 is directed into the page. The distance between the wires is $2d$. The $x$ axis is perpendicular to the line connecting the wires and is equidistant from the wires.

As you answer the questions posed here, try to look for a pattern in your answers.

**Part A**

Which of the vectors best represents the direction of the magnetic field created at point K (see the diagram in the problem introduction) by wire 1 alone?

Enter the number of the vector with the appropriate direction.

**ANSWER:**

3

[Diagram showing vectors and point K]
Part B
Which of the vectors best represents the direction of the magnetic field created at point K by wire 2 alone?
Enter the number of the vector with the appropriate direction.

ANSWER:

3

Part C
Which of these vectors best represents the direction of the net magnetic field created at point K by both wires?
Enter the number of the vector with the appropriate direction.

ANSWER:

3

Part D
Find the magnitude of the magnetic field $B_{1K}$ created at point K by wire 1.

**Express your answer in terms of $I$, $d$, and appropriate constants.**

**ANSWER:**

$$B_{1K} = \frac{\mu_0 I}{2\pi d}$$

Of course, $B_{2K} = B_{1K}$ because point K is equidistant from the wires.

**Part E**

Find the magnitude of the net magnetic field $B_K$ created at point K by both wires.

**Express your answer in terms of $I$, $d$, and appropriate constants.**

**ANSWER:**

$$B_K = \frac{\mu_0 I}{\pi d}$$

This result is fairly obvious because of the symmetry of the problem: At point K, the two wires each contribute equally to the magnetic field. At points L and M you should also consider the symmetry of the problem. However, be careful! The vectors will add up in a more complex way.

**Part F**

Point L is located a distance $d\sqrt{2}$ from the midpoint between the two wires. Find the magnitude of the magnetic field $B_{1L}$ created at point L by wire 1.

**Express your answer in terms of $I$, $d$, and appropriate constants.**

**Hint 1. How to approach the problem**

Use the distances provided and the Pythagorean Theorem to find the distance between wire 1 and point L.

**ANSWER:**

$$B_{1L} = \frac{\mu_0 I}{2\pi d\sqrt{3}}$$

**Part G**

Point L is located a distance $d\sqrt{2}$ from the midpoint between the two wires. Find the magnitude of the net magnetic field $B_L$ created at point L by both wires.

**Express your answer in terms of $I$, $d$, and appropriate constants.**
**Hint 1. How to approach the problem**

Sketch a detailed diagram with all angles marked; draw vectors $\vec{B}_{1L}$ and $\vec{B}_{2L}$; then add them using the parallelogram rule.

**Hint 2. Find the direction of the magnetic field due to wire 1**

Which of the vectors best represents the direction of the magnetic field created at point L (see the diagram in the problem introduction) by wire 1 *alone*?

**Enter the number of the vector with the appropriate direction.**

![Diagram](image1)

**ANSWER:**

2

**Hint 3. Find the direction of the magnetic field due to wire 2**

Which of the vectors best represents the direction of the magnetic field created at point L by wire 2 *alone*?

**Enter the number of the vector with the appropriate direction.**

![Diagram](image2)
**Hint 4. Find the direction of the net magnetic field**

Which of the vectors best represents the direction of the net magnetic field created at point L by both wires? Enter the number of the vector with the appropriate direction.

ANSWER:

Note that the directions of the magnetic fields created by individual wires at point L are different from each other and from those at point K; however, the direction of the net magnetic field at points K and L is the same.

**Hint 5. The x component of the magnetic field vector due to wire 1**

Use the distances provided and your knowledge of right angle triangle trigonometry to find the x component of the magnetic field due to wire 1 at point L.

Express your answer in terms of the magnitude of the magnetic field vector at point L due to wire 1, $B_{1L}$.

**Hint 1. Calculating the coefficient of the x component**

The x component of the magnetic field vector due to wire 1, can be defined using a trigonometric function and the angle between the magnetic field vector due to wire 1 and the x direction, $\theta$, by $B_{1L}\cos(\theta)$.

Evaluate $\cos(\theta)$ in this instance using the distances provided as a numerical value?

ANSWER:
Hint 6. Net magnetic field

Consider the symmetry of the magnetic field at point L due to wire 1 and the magnetic field due to wire 2. You should note that the y components of these two vectors are of equal magnitude but are opposite in direction. Therefore they will cancel when added together, leaving you only to worry about the x components. Because of symmetry, the x component of the magnetic field at point L due to wire 2 is the same size as that due to wire 1 at point L. To find the net magnetic field at point L you need to add together the x components of the magnetic field at point L due to wire 1 and of the magnetic field due to wire 2.

\[
\frac{B_{1L}}{\sqrt{3}}
\]

If you use your answer to this hint subsequently, use the unrounded expression \( \frac{1}{\sqrt{3}} \), which will assist in simplifying your answers.

Answer:

\[
B_L = \frac{\mu_0 I}{3\pi d}
\]

Part H

Point M is located a distance \(2d\) from the midpoint between the two wires. Find the magnitude of the magnetic field \(B_{1M}\) created at point M by wire 1.

Express your answer in terms of \(I\), \(d\), and appropriate constants.

Answer:

\[
B_{1M} = \frac{\mu_0 I}{2\pi d \sqrt{5}}
\]

Part I

Find the magnitude of the net magnetic field \(B_M\) created at point M by both wires.

Express your answer in terms of \(I\), \(d\), and appropriate constants.
**Hint 1.** How to approach the problem

Sketch a detailed diagram with all angles marked; draw vectors $\vec{B}_{1M}$ and $\vec{B}_{2M}$; then add them using the parallelogram rule.

**Hint 2.** Find the direction of the magnetic field due to wire 1

Which of the vectors best represents the direction of the magnetic field created at point M by wire 1 alone?

Enter the number of the vector with the appropriate direction.

**Answer:**

2

**Hint 3.** The x component of the magnetic field vector due to wire 1

Use the distances provided and your knowledge of right angle triangle trigonometry to find the x component of the magnetic field due to wire 1 at point M.

Express your answer in terms of the magnitude of the magnetic field vector at point M due to wire 1, $B_{1M}$.

**Hint 1. Calculating the coefficient of the x component**

The x component of the magnetic field vector due to wire 1, can be defined using a trigonometric function and the angle between the magnetic field vector due to wire 1 and the x direction, $\theta$, by $B_{1M}\cos(\theta)$.

Evaluate $\cos(\theta)$ in this instance using the distances provided as a numerical value?

**Answer:**

0.447
If you use your answer to this hint subsequently, use the unrounded expression \( \frac{1}{\sqrt{5}} \), which will assist in simplifying your answers.

**ANSWER:**

\[
\frac{B_{1M}}{\sqrt{5}}
\]

If you use your answer to this hint subsequently, use the unrounded expression \( B_{1M}/\sqrt{5} \), which will assist in simplifying your answers.

**Hint 4. Net magnetic field**

Consider the symmetry of the magnetic field at point M due to wire 1 and the magnetic field due to wire 2. You should note that the y components of these two vectors are of equal magnitude but are opposite in direction. Therefore they will cancel when added together, leaving you only to worry about the x components. Because of symmetry, the x component of the magnetic field at point M due to wire 2 is the same size as that due to wire 1 at point M. To find the net magnetic field at point M you need to add together the x components of the magnetic field at point M due to wire 1 and of the magnetic field due to wire 2.

**ANSWER:**

\[
B_M = \frac{\mu_0 I}{5\pi d}
\]

**Part J**

Finally, consider point X (not shown in the diagram) located on the x axis very far away in the positive x direction. Which of the vectors best represents the direction of the magnetic field created at point X by wire 1 alone?

**Enter the number of the vector with the appropriate direction.**
Part K

Which of the vectors best represents the direction of the magnetic field created at point X by wire 2 alone?

Enter the number of the vector with the appropriate direction.

ANSWER:

5
As you can see, at a very large distance, the individual magnetic fields (and the corresponding magnetic field lines) created by the wires are directed nearly opposite to each other, thus ensuring that the net magnetic field is very, very small even compared to the magnitudes of the individual magnetic fields, which are also relatively small at a large distance from the wires. Thus, at a large distance, the magnetic fields due to the two wires almost cancel each other out! (That is, if point X is very far from each wire, then the ratio \( B_X / B_{1X} \) is very close to zero.)

Another way to think of this is as follows: If you are really far from the wires, then it's hard to tell them apart. It would seem as if the current were traveling up and down, almost along the same line, thereby appearing much the same as a single wire with almost no net current (because the up and down currents almost cancel each other), and therefore almost no magnetic field. Note that this only works for points very far from the wires; otherwise it's easy to tell that the wires are separated and the currents don't cancel, since they are going up and down at different locations.

It comes as no surprise then that one way to eliminate unnecessary magnetic fields in electric circuits is to twist together the wires carrying equal currents in opposite directions.

\[ ± \text{ Canceling a Magnetic Field} \]

**Description:** ± Includes Math Remediation. The student must find the current needed in a wire to cancel the magnetic field from three other wires at the center of a square. Each wire lies along one edge of the square.

Four very long, current-carrying wires in the same plane intersect to form a square with side lengths of 31.0 cm, as shown in the figure. The currents running through the wires are 8.0 A, 20.0 A, 10.0 A, and \( I \).

**Part A**

Find the magnitude of the current \( I \) that will make the magnetic field at the center of the square equal to zero.

**Express your answer in amperes.**

**Hint 1. How to approach the problem**

Find the magnetic field at the center of the square due to the wires whose current you know. Then, find the current \( I \) whose contribution to the magnetic field will exactly cancel the contribution of the other three wires.
Hint 2. Calculating the contribution from the three known wires

What is the magnitude \( |\vec{B}_c| \) of the magnetic field at the center of the square due to the wires carrying the 8.0-, 20-, and 10-\( \text{A} \) currents? Be careful with signs when you add the contributions from the three different wires.

Express your answer in teslas to three significant figures.

**Hint 1. Ampère's law**

Recall Ampère's law:

\[
\mu_0 I = \oint B \cdot dl.
\]

You can use this to determine the formula for the magnetic field generated by a long wire. Use a circle centered on the wire as your path of integration.

**Hint 2. Getting your signs correct**

Recall the right-hand rule: If your thumb, on your right hand, points in the direction in which the current is flowing, your fingers will curl in the direction of the magnetic field.

\[ \mu_0 I = \frac{10^{-7} \cdot 4.2}{\pi} = 2.58 \times 10^{-6} \text{ T} \]

Good. You should have derived the following equation for the contribution to the magnetic field from one wire:

\[ |B| = \frac{\mu_0 I}{\pi L}, \]

where \( L \) is width of the square. Use this formula and the fact that you want the magnetic field at the center to sum to zero to find the current \( I \).

**Answer:**

\[ I = 2.00 \text{ A} \]

---

**Part B**

What is the direction of the current \( I \)?

**Hint 1. How to approach the problem**

If you were able to master the right-hand rule in Part A, you should be able to get this. Your goal is to cancel the magnetic field contributions from the other three wires.
Exercise 28.30

Description: Two long, parallel wires are separated by a distance of \( x \). The force per unit length that each wire exerts on the other is \( F \), and the wires repel each other. The current in one wire is \( I \). (a) What is the current in the second wire? (b) Are the two...

Two long, parallel wires are separated by a distance of 2.30 cm. The force per unit length that each wire exerts on the other is \( 3.20 \times 10^{-5} \text{ N/m} \), and the wires repel each other. The current in one wire is 0.680 A.

Part A

What is the current in the second wire?

ANSWER:

\[
I = \frac{F \cdot 3.14159 \cdot x}{1.257 \cdot 10^{-6} \cdot I} = 5.41 \text{ A}
\]

Part B

Are the two currents in the same direction or in opposite directions?

ANSWER:

- In the same direction
- In opposite direction

Magnetic Field at the Center of a Wire Loop

Description: Use Biot-Savart law to find field at \( x = y = z = 0 \) of a one turn loop.

A piece of wire is bent to form a circle with radius \( r \). It has a steady current \( I \) flowing through it in a counterclockwise direction as seen from the top (looking in the negative \( z \) direction).
Part A

What is $B_z(0)$, the z component of $\vec{B}$ at the center (i.e., $x = y = z = 0$) of the loop?

Express your answer in terms of $I$, $r$, and constants like $\mu_0$ and $\pi$.

**Hint 1. Specify the integrand**

The Biot-Savart law,

$$\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{I d\vec{l} \times \hat{r}}{r^2},$$

where $\mu_0$ is the permeability of free space, tells us the magnetic field $\vec{B}$ at a point from a length of wire $d\vec{l}$ carrying current $I$. What is $d\vec{B}$ at the center of the loop from a small piece of wire $d\vec{l}$?

**Give your answer in terms of $\mu_0$, $I$, $\pi$, radius $r$, $dl$, and $\hat{k}$.**

**Hint 1. Find the magnitude of the magnetic field**

What is the magnitude of $d\vec{B}$ at the center of the loop from a small piece of wire $d\vec{l}$?

**Give your answer in terms of $\mu_0$, $I$, $\pi$, radius $r$, and $dl$.**

**ANSWER:**

$$|d\vec{B}| = \frac{\mu_0 I dl}{4\pi r^3}$$

**Hint 2. Direction of the magnetic field**

The field direction is determined from the cross product in the Biot-Savart law. You can use the right-hand rule to find the direction of the cross-product vector.

**ANSWER:**
Hint 2. Perform the integration

Now you have to integrate the Biot-Savart expression for the differential field $d\vec{B}$ along the length of the wire to find the total magnetic field at the point. If you keep in mind that $I$, $r$, $\mu_0$, and $\pi$ are constants in this case, the integral isn't too difficult. What is the value of $\int dl$ around the loop?

Express your answer in terms of variables given in the problem introduction.

Hint 1. Help with the integral

The value of this integral is just the length of the circumference of the loop.

Hint 2. Reexpress $dl$ to enable integration

The expression is easiest to integrate with respect to the angle $\theta$ (the angle from the $x$ axis to the differential piece of the wire $dl$). Express $dl$ using this idea.

Express the magnitude of $dl$ in terms of variables such as $d\theta$ and $r$.

ANSWER:

\[ dl = r\,d\theta \]

Now do the integral over $d\theta$. Recall that $\theta$ goes from 0 to $2\pi$ as you go around the loop.

ANSWER:

\[ \int dl \text{ around the loop} = 2\pi r \]

ANSWER:

\[ B_z(0) = \frac{\mu_0 I}{2r} \]

Ampère’s Law Explained

Description: Discusses terms in and use of Ampère’s law, and whether it can be used in a number of specific examples.

Learning Goal:
To understand Ampère’s law and its application.

Ampère’s law is often written \( \oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_0 I_{encl} \).

**Part A**

The integral on the left is

ANSWER:

- the integral throughout the chosen volume.
- the surface integral over the open surface.
- the surface integral over the closed surface bounded by the loop.
- the line integral along the closed loop.
- the line integral from start to finish.

**Part B**

What physical property does the symbol \( I_{encl} \) represent?

ANSWER:

- The current along the path in the same direction as the magnetic field.
- The current in the path in the opposite direction from the magnetic field.
- The total current passing through the loop in either direction.
- The net current through the loop.

The positive direction of the line integral and the positive direction for the current are related by the right-hand rule:

Wrap your right-hand fingers around the closed path, then the direction of your fingers is the positive direction for \( d\vec{l} \) and the direction of your thumb is the positive direction for the net current.

Note also that the angle the current-carrying wire makes with the surface enclosed by the loop doesn't matter. (If the wire is at an angle, the normal component of the current is decreased, but the area of intersection of the wire and the surface is correspondingly increased.)

**Part C**

The circle on the integral means that \( \vec{B}(\vec{r}) \) must be integrated

ANSWER:

- over a circle or a sphere.
- along any closed path that you choose.
- along the path of a closed physical conductor.
- over the surface bounded by the current-carrying wire.
Part D

Which of the following choices of path allow you to use Ampère’s law to find \( \mathbf{B}(\mathbf{r}) \)?

a. The path must pass through the point \( \mathbf{r} \).

b. The path must have enough symmetry so that \( \mathbf{B}(\mathbf{r}) \cdot d\mathbf{l} \) is constant along large parts of it.

c. The path must be a circle.

ANSWER:

- [ ] a only
- [ ] a and b
- [ ] a and c
- [ ] b and c

Part E

Ampère’s law can be used to find the magnetic field around a straight current-carrying wire.

Is this statement true or false?

ANSWER:

- [ ] true
- [ ] false

In fact Ampère’s law can be used to find the magnetic field inside a cylindrical conductor (i.e., at a radius \( r \) less than the radius of the wire, \( R \)). In this case \( I_{\text{enc}} \) is just that current inside \( r \), not the current inside \( R \) (which is the total current in the wire).

Part F

Ampère’s law can be used to find the magnetic field at the center of a square loop carrying a constant current.

Is this statement true or false?

ANSWER:

- [ ] true
- [ ] false

The key point is that to be able to use Ampère’s law, the path along which you take the line integral of \( \mathbf{B} \) must have sufficient symmetry to allow you to pull the magnitude of \( B \) outside the integral. Whether the current distribution has symmetry is incidental.
Part G

Ampère’s law can be used to find the magnetic field at the center of a circle formed by a current-carrying conductor.

Is this statement true or false?

**ANSWER:**

- [ ] true
- [ ] false

Part H

Ampère’s law can be used to find the magnetic field inside a toroid. (A toroid is a doughnut shape wound uniformly with many turns of wire.)

Is this statement true or false?

**ANSWER:**

- [ ] true
- [ ] false

Therefore, though Ampère’s law holds quite generally, it is useful in finding the magnetic field only in some cases, when a suitable path through the point of interest exists, typically such that all other points on the path have the same magnetic field through them.

Alternative Exercise 28.87

**Description:** A solid conductor with radius \( a \) is supported by insulating disks on the axis of a conducting tube with inner radius \( b \) and outer radius \( c \) (see the figure). The central conductor and tube carry equal currents \( I \) in opposite directions. The currents are...

A solid conductor with radius \( a \) is supported by insulating disks on the axis of a conducting tube with inner radius \( b \) and outer radius \( c \) (see the figure). The central conductor and tube carry equal currents \( I \) in opposite directions. The currents are distributed uniformly over the cross sections of each conductor.
Part A

Derive an expression for the magnitude of the magnetic field at points outside the central, solid conductor, but inside the tube \((a < r < b)\).

Express your answer in terms of the variables \(I\), \(r\), and appropriate constants.

\[
B = \frac{\mu_0 I}{2\pi r}
\]

Also accepted: \(\frac{2\cdot10^{-7}I}{r}\)

Part B

Derive an expression for the magnitude of the magnetic field at points outside the tube \((r > c)\).

\[
B = 0
\]

Magnetic Field inside a Very Long Solenoid

**Description:** Leads through steps of using Ampère’s law to find field inside solenoid a long solenoid.

**Learning Goal:**
To apply Ampère’s law to find the magnetic field inside an infinite solenoid.

In this problem we will apply Ampère’s law, written

\[
\oint B(\vec{r}) \cdot d\vec{l} = \mu_0 I_{encl}
\]

to calculate the magnetic field inside a very long solenoid (only a relatively short segment of the solenoid is shown in the pictures). The solenoid has length \(L\), diameter \(D\), and \(n\) turns per unit length with each carrying current \(I\). It is usual to assume that the component of the current along the z axis is negligible. (This may be assured by winding two layers of closely spaced wires that spiral in opposite directions.)

From symmetry considerations it is possible to show that far from the ends of the solenoid, the magnetic field is axial.
Part A

Which figure shows the loop that the must be used as the Ampèrean loop for finding $B_{in}(r)$ for $r$ inside the solenoid?

![Diagram of four loop options](image)

**Hint 1. Choice of path for loop integral**

Which of the following choices are a requirement of the Ampèrean loop that would allow you to use Ampère's law to find $\vec{B}(\vec{r})$?

- a. The path must pass through the point $\vec{r}$.
- b. The path must have enough symmetry so that $\vec{B}(\vec{r}) \cdot d\vec{l}$ is constant along large parts of it.
- c. The path must be a circle.

**ANSWER:**

- a only
- a and b
- a and c
- b and c

If possible, choose the loop so that the desired field component runs parallel to the loop and other sides of the loop have zero field component along them.

**ANSWER:**

- A
- B
- C
- D
Part B
Assume that loop B (in the Part A figure) has length $L$ along $\hat{k}$ (the $z$ direction). What is the loop integral in Ampère’s law? Assume that the top end of the loop is very far from the solenoid (even though it may not look like it in the figure), so that the field there is assumed to be small and can be ignored.

Express your answer in terms of $B_{in}$, $L$, and other quantities given in the introduction.

ANSWER:
$$\oint \vec{B} \cdot d\vec{l} = B_{in}L$$

Part C
What physical property does the symbol $I_{encl}$ represent?

ANSWER:
- The current along the path in the same direction as the magnetic field
- The current in the path in the opposite direction from the magnetic field
- The total current passing through the Ampèrean loop in either direction
- The net current through the Ampèrean loop

The positive direction of the line integral and the positive direction for the current are related by the right-hand rule:
Wrap your right-hand fingers around the closed path, then the direction of your fingers is the positive direction for $dl$ and the direction of your thumb is the positive direction for the net current.
Note also that the angle the current-carrying wire makes with the surface enclosed by the loop doesn't matter.
(If the wire is at an angle, the normal component of the current is decreased, but the area of intersection of the wire and the surface is correspondingly increased.)

Part D
What is $I_{encl}$, the current passing through the chosen loop?

Express your answer in terms of $L$ (the length of the Ampèrean loop along the axis of the solenoid) and other variables given in the introduction.

ANSWER:
$$I_{encl} = lnL$$

Part E
Find $B_{in}$, the $z$ component of the magnetic field inside the solenoid where Ampère’s law applies.

Express your answer in terms of $L$, $D$, $n$, $I$, and physical constants such as $\mu_0$.

ANSWER:
Part F

What is $B_{out}$, the z component of the magnetic field outside the solenoid?

**Hint 1. Find the Ampèrean loop to use**

Which figure shows the loop that the must be used as the Ampèrean loop for finding $B_{out}$ outside the solenoid? Note: From symmetry considerations, the field outside (if non-zero) must also be axial and opposite to the field inside.

![Ampèrean loop options](image)

**Hint 1. Choice of path for loop integral**

Which of the following choices are a requirement of the Ampèrean loop that would allow you to use Ampère's law to find $\vec{B}(\vec{r})$?

- a. The path must pass through the point $\vec{r}$.
- b. The path must have enough symmetry so that $\vec{B}(\vec{r}) \cdot d\vec{l}$ is constant along large parts of it.
- c. The path must be a circle.

**ANSWER:**

- [ ] a only
- [x] a and b
- [ ] a and c
- [ ] b and c

If possible, choose the loop so that the desired field component runs parallel to the loop and other sides of the loop have zero field component along them. From symmetry considerations, the magnetic field outside the solenoid, if it exists, must also be axial.
Part G

The magnetic field inside a solenoid can be found exactly using Ampère's law only if the solenoid is infinitely long. Otherwise, the Biot-Savart law must be used to find an exact answer. In practice, the field can be determined with very little error by using Ampère's law, as long as certain conditions hold that make the field similar to that in an infinitely long solenoid.

Which of the following conditions must hold to allow you to use Ampère's law to find a good approximation?

a. Consider only locations where the distance from the ends is many times $D$.
b. Consider any location inside the solenoid, as long as $L$ is much larger than $D$ for the solenoid.
c. Consider only locations along the axis of the solenoid.

Hint 1. Implications of symmetry

Imagine that the the solenoid is made of two equal pieces, one extending from $z = 0$ to $z = -\infty$ and the other from $z = 0$ to $z = +\infty$. If both were present the field would have its normal value, but if either is removed the field at $z = 0$ drops to one-half of its previous value. This shows that the field drops off significantly near the ends of the solenoid (relative to its value in the middle). However, in doing this calculation, you assumed that the field is constant along the length of the Ampèrean loop. So where would this assumption break down?

Hint 2. Off-axis field dependence

You also used symmetry considerations to say that the magnetic field is purely axial. Where would this symmetry argument not hold?

Note that far from the ends there cannot be a radial field, because it would imply a nonzero magnetic charge along the axis of the cylinder and no magnetic charges are known to exist (Gauss's Law for magnetic fields...
and charges). In conjunction with Ampère’s law, this allows us to conclude that the $z$ component of the field cannot depend on $r$ inside the solenoid.

**Exercise 28.51**

**Description:** A wooden ring whose mean diameter is $d$ is wound with a closely spaced toroidal winding of $N$ turns. (a) Compute the magnitude of the magnetic field at the center of the cross section of the windings when the current in the windings is $I$.

A wooden ring whose mean diameter is 13.0 cm is wound with a closely spaced toroidal winding of 580 turns.

**Part A**

Compute the magnitude of the magnetic field at the center of the cross section of the windings when the current in the windings is 0.680 A.

**ANSWER:**

\[
B = \frac{4.314159 \times 10^{-7} NI}{2\pi \times 0.5d} = 1.21 \times 10^{-3} \text{T}
\]

**Exercise 28.53**

**Description:** A long solenoid with $n$ turns of wire per centimeter carries a current of $I$. The wire that makes up the solenoid is wrapped around a solid core of silicon steel $K_m=5200$. (The wire of the solenoid is jacketed with an insulator so that none of the...

A long solenoid with 68 turns of wire per centimeter carries a current of 0.20 A. The wire that makes up the solenoid is wrapped around a solid core of silicon steel ($K_m = 5200$). (The wire of the solenoid is jacketed with an insulator so that none of the current flows into the core.)

**Part A**

For a point inside the core, find the magnitude of the magnetic field $\vec{B}_0$ due to the solenoid current.

**Express your answer using two significant figures.**

**ANSWER:**
Part B

For a point inside the core, find the magnitude of the magnetization \( \vec{M} \).

Express your answer using two significant figures.

ANSWER:

\[
M = (5200 - 1) nI = 7.1 \times 10^6 \quad \text{A/m}
\]

Part C

For a point inside the core, find the magnitude of the total magnetic field \( \vec{B} \).

Express your answer using two significant figures.

ANSWER:

\[
B = 5200 \cdot 4.31459 \cdot 10^{-7} nI = 8.9 \quad \text{T}
\]

Magnetic Materials

**Description:** Given description of a materials response to an external field, identify the type of material (diamagnetic, paramagnetic, ferromagnetic).

Part A

You are given a material which produces no initial magnetic field when in free space. When it is placed in a region of uniform magnetic field, the material produces an additional internal magnetic field parallel to the original field. However, this induced magnetic field disappears when the external field is removed.

What type of magnetism does this material exhibit?

ANSWER:

- [ ] diamagnetism
- [ ] paramagnetism
- [ ] ferromagnetism

When a paramagnetic material is placed in a magnetic field, the field helps align the magnetic moments of the atoms. This produces a magnetic field in the material that is parallel to the applied field.
Part B

Once again, you are given an unknown material that initially generates no magnetic field. When this material is placed in a magnetic field, it produces a strong internal magnetic field, parallel to the external magnetic field. This field is found to remain even after the external magnetic field is removed. Your material is which of the following?

ANSWER:

- diamagnetic
- paramagnetic
- ferromagnetic

Very good! Materials that exhibit a magnetic field even after an external magnetic field is removed are called ferromagnetic materials. Iron and nickel are the most common ferromagnetic elements, but the strongest permanent magnets are made from alloys that contain rare earth elements as well.

Part C

What type of magnetism is characteristic of most materials?

ANSWER:

- ferromagnetism
- paramagnetism
- diamagnetism
- no magnetism

Almost all materials exhibit diamagnetism to some degree, even materials that also exhibit paramagnetism or ferromagnetism. This is because a magnetic moment can be induced in most common atoms when the atom is placed in a magnetic field. This induced magnetic moment is in a direction opposite to the external magnetic field. The addition of all of these weak magnetic moments gives the material a very weak magnetic field overall. This field disappears when the external magnetic field is removed. The effect of diamagnetism is often masked in paramagnetic or ferromagnetic materials, whose constituent atoms or molecules have permanent magnetic moments and a strong tendency to align in the same direction as the external magnetic field.

Magnetic Force on a Bent Wire Conceptual Question

**Description:** Short conceptual question on finding the direction of the magnetic force on a current-carrying wire in a uniform magnetic field.

The bent wire circuit shown in the figure is in a region of space with a uniform magnetic field in the +z direction. Current flows through the circuit in the direction indicated. Note that segments 2 and 5 are oriented parallel to the z axis; the other pieces are parallel to either the x or y axis.
Part A

Determine the direction of the magnetic force along segment 1, which carries current in the \(-x\) direction.

Enter the direction of the force as a sign (+ or -) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

**Hint 1. Magnetic force on a current-carrying wire**

Electric current, by convention, is considered to be the flow of positively charged particles. Therefore, to determine the direction of the magnetic force on a current-carrying wire, simply determine the direction of the force on a positive charge moving in the direction of the current flow.

**Hint 2. Magnetic force on segment 1**

1. Point the fingers of your right hand in the direction of the current flow (to the left).
2. Rotate your hand until you can curl your fingers in the +z direction (directly out from the computer screen).

At this point, is your palm facing the computer screen?

ANSWER:

- [ ] yes
- [ ] no

The direction of the magnetic force on the wire segment is the direction your thumb is pointing.

ANSWER:

+ y
Part B

Determine the direction of the magnetic force along segment 2, which carries current in the -z direction.

Enter the direction of the force as a sign (+ or −) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

**Hint 1. Magnetic force on segment 2**

The magnetic force is proportional to the sine of the angle between the current flow and the magnetic field. What is the angle between the direction of current flow and the magnetic field along segment 2?

Express your answer in degrees.

ANSWER:

180 degrees

Also accepted: -180

\[ \sin(180 \text{ degrees}) = 0. \] Use this fact to determine the magnetic force along segment 2.

ANSWER:

0

Part C

Determine the direction of the magnetic force along segment 3, which carries current in the +y direction.

Enter the direction of the force as a sign (+ or −) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

**Hint 1. Magnetic field on segment 3**

1. Point the fingers of your right hand in the direction of the current flow (upward).
2. Rotate your hand until you can curl your fingers in the +z direction (directly out from the computer screen).

At this point, is your palm facing the computer screen?

ANSWER:

- yes
- no

The direction of the magnetic force on the wire segment is the direction your thumb is pointing.
Part D
Determine the direction of the magnetic force along segment 4, which carries current in the +x direction.

Enter the direction of the force as a sign (+ or -) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

ANSWER:

Part E
Determine the direction of the magnetic force along segment 5, which carries current in the +z direction.

Enter the direction of the force as a sign (+ or -) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

ANSWER:

Part F
Determine the direction of the magnetic force along segment 6, which carries current in the +x direction.

Enter the direction of the force as a sign (+ or -) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

ANSWER:

Part G
Determine the direction of the magnetic force along segment 7, which carries current in the -y direction.

Enter the direction of the force as a sign (+ or -) followed by a coordinate direction (x, y, or z) without spaces. For instance, if you think that the force points in the positive y direction, enter +y. If there is no magnetic force, enter 0.

ANSWER: