Exercise 22.2

**Description:** A flat sheet is in the shape of a rectangle with sides of lengths 0.400 m and 0.600 m. The sheet is immersed in a uniform electric field of magnitude $E$ that is directed at 20 degree(s) from the plane of the sheet. (a) Find the magnitude...

A flat sheet is in the shape of a rectangle with sides of lengths 0.400 m and 0.600 m. The sheet is immersed in a uniform electric field of magnitude 95.0 N/C that is directed at 20° from the plane of the sheet.

**Part A**

Find the magnitude of the electric flux through the sheet.

*Express your answer to two significant figures and include the appropriate units.*

**ANSWER:**
Flux through a Cube

Description: Find the electric flux through a cube given an algebraic formula for the electric field in all space. Then determine the charge enclosed by the cube using Gauss’s law.

A cube has one corner at the origin and the opposite corner at the point \((L, L, L)\). The sides of the cube are parallel to the coordinate planes. The electric field in and around the cube is given by \(\vec{E} = (a + bx)i + cj\).

Part A

Find the total electric flux \(\Phi_E\) through the surface of the cube.

Express your answer in terms of \(a\), \(b\), \(c\), and \(L\).

Hint 1. Definition of flux

The net electric flux \(\Phi_E\) of a field \(\vec{E}\) through a closed surface \(S\) is given by

\[
\Phi_E = \oint \vec{E} \cdot d\vec{A},
\]

where the differential vector \(d\vec{A}\) has magnitude proportional to the differential area and is oriented outward and normal (perpendicular) to the surface. In some cases with simple geometry (like this one), you can break up the integral into manageable pieces. Consider separately the flux coming out of each of the six faces of the cube, and then add the results to obtain the net flux.

Hint 2. Flux through the \(+x\) face

Consider the face of the cube whose outward normal points in the positive \(x\) direction. What is the flux \(\Phi_{+x}\) through this face?

\[
\Phi = E \cdot 0.4 \cdot 0.6 \sin(20) = 7.8 \frac{N \cdot m^2}{C}
\]

Also accepted: \(E \cdot 0.4 \cdot 0.6 \sin(20) = 7.80 \frac{N \cdot m^2}{C}, E \cdot 0.4 \cdot 0.6 \sin(20) = 7.8 \frac{N \cdot m^2}{C}\)
Express your answer in terms of \( a, b, c, \) and \( L. \)

**Hint 1. Simplifying the integral**

The field \( \vec{E} \) depends only on the spatial variable \( x \). On the \( +x \) face of the cube, \( x = L \), so \( \vec{E} \) is constant. Since \( \vec{E} \) is constant over this entire surface, it can be pulled out of the integral:

\[
\oint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A}.
\]

**Hint 2. Evaluate the scalar product**

The scalar (dot) product yields the component of the field that is in the direction of the normal (i.e., perpendicular to the surface). Evaluate the dot product \( \vec{E} \cdot \vec{A} \).

Express your answer in terms of \( a, b, c, x, \) and \( A. \)

**ANSWER:**

\[
\vec{E} \cdot \vec{A} = (a + bx) A
\]

**Hint 3. Find the area of the face of the cube**

This \( +x \) face of the cube is a square with sides of length \( L \). What is the area of this face?

**ANSWER:**

\[
A = L^2
\]

**Hint 3. Flux through the \( +y \) face**

Consider the face of the cube whose outward normal points in the positive \( y \) direction. What is the flux \( \Phi_{+y} \) through this face?

Express your answer in terms of \( a, b, c, \) and \( L. \)

**ANSWER:**

\[
\Phi_{+y} = (a + bL) L^2
\]

**Hint 4. Flux through the \( +z \) face**

Consider the face of the cube whose outward normal points in the positive \( z \) direction. What is the flux \( \Phi_{+z} \) through this face?
**Hint 1. Consider the orientation of the field**

The electric field has no $z$ component (the field vector $\vec{E}$ lies entirely in the $xy$ plane. What is the dot product of a vector in the $xy$ plane and a vector normal to the $+z$ face of the cube?

**ANSWER:**

$$\Phi_{+z} = 0$$

**Hint 5. Flux through the $-x$ face**

Consider the face of the cube whose outward normal points in the negative $x$ direction. What is the flux $\Phi_{-x}$ through this face?

**Express your answer in terms of $a$, $b$, $c$, and $L$.**

**Hint 1. What is the electric field?**

The face of the cube whose outward normal points in the negative $x$ direction lies in the $yz$ plane (i.e., $x = 0$). Find the $x$ component $E_x$ of the electric field across this surface.

**ANSWER:**

$$E_x = a$$

**Hint 2. Direction of flux**

Remember to take note of whether the electric field is going into the surface or out of the surface. Flux is defined as positive when the field is coming out of the surface and negative when the field is going into the surface.

**ANSWER:**

$$\Phi_{-x} = -aL^2$$

**Hint 6. Putting it together**

Using similar calculations to those above, you should be able to find the flux through each of the six faces. Add the six quantities to obtain the net flux.

**ANSWER:**

$$\Phi_E = bL^3$$
Part B

Notice that the flux through the cube does not depend on \( a \) or \( c \). Equivalently, if we were to set \( b = 0 \), so that the electric field becomes

\[
\vec{E}' = a \hat{i} + c \hat{j},
\]

then the flux through the cube would be zero. Why?

ANSWER:

- \( \vec{E}' \) does not generate any flux across any of the surfaces.
- The flux into one side of the cube is exactly canceled by the flux out of the opposite side.
- Both of the above statements are true.

Part C

What is the net charge \( q \) inside the cube?

Express your answer in terms of \( a, b, c, L, \) and \( \epsilon_0 \).

Hint 1. Gauss's law

Gauss's law states that the net flux of an electric field through a surface is proportional to the net charge inside that surface:

\[
\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0}.
\]

ANSWER:

\[
q = \epsilon_0 b L^3
\]

± Calculating Flux for Hemispheres of Different Radii

Description: ± Includes Math Remediation. Compute the electric flux through three surfaces surrounding a point charge: two hemispheres of unequal radii and the circular annulus connecting them.

Learning Goal:

To understand the definition of electric flux, and how to calculate it.

Flux is the amount of a vector field that "flows" through a surface. We now discuss the electric flux through a surface (a quantity needed in Gauss's law): \( \Phi_E = \int \vec{E} \cdot d\vec{A} \), where \( \Phi_E \) is the flux through a surface with differential area element \( d\vec{A} \), and \( \vec{E} \) is the electric field in which the surface lies. There are several important points to consider in this expression:

1. It is an integral over a surface, involving the electric field at the surface.
2. \( d\vec{A} \) is a vector with magnitude equal to the area of an infinitesimal surface element and pointing in a direction normal (and usually outward) to the infinitesimal surface element.
3. The scalar (dot) product $\vec{E} \cdot d\vec{A}$ implies that only the component of $\vec{E}$ normal to the surface contributes to the integral. That is, $\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos(\theta)$, where $\theta$ is the angle between $\vec{E}$ and $d\vec{A}$.

When you compute flux, try to pick a surface that is either parallel or perpendicular to $\vec{E}$, so that the dot product is easy to compute.

Two hemispherical surfaces, 1 and 2, of respective radii $r_1$ and $r_2$, are centered at a point charge and are facing each other so that their edges define an annular ring (surface 3), as shown. The field at position $\vec{r}$ due to the point charge is:

$$\vec{E}(\vec{r}) = \frac{C}{r^2} \hat{r}$$

where $C$ is a constant proportional to the charge, $r = |\vec{r}|$, and $\hat{r} = \vec{r}/r$ is the unit vector in the radial direction.

**Part A**

What is the electric flux $\Phi_3$ through the annular ring, surface 3?

**Express your answer in terms of $C$, $r_1$, $r_2$, and any constants.**

**Hint 1. Apply the definition of electric flux**

The integrand in the equation defining flux (in the problem introduction) can be calculated by noting the following:

$$\vec{E}(\vec{r}) \cdot d\vec{A} = E(r)|d\vec{A}| \cos(\theta).$$

Since the electric field is everywhere parallel to the surface of the annular ring, the element $d\vec{A}$ is normal to the electric field, and thus $\theta = \pi/2$. Therefore, $\cos(\theta) = \cos(\pi/2) = 0$.

**ANSWER:**

$$\Phi_3 = 0$$

**Part B**

What is the electric flux $\Phi_1$ through surface 1?

**Express $\Phi_1$ in terms of $C$, $r_1$, $r_2$, and any needed constants.**

**Hint 1. Apply the definition of electric flux**

The integrand in the equation defining flux (in the problem introduction) can be calculated by noting the
Since the electric field is everywhere perpendicular to surface 2, \( \cos(\theta) = \cos(0) = 1 \). Therefore, the integral simply becomes the magnitude of the electric field multiplied by the surface area of surface 2.

**Hint 2. Find the area of surface 1**

Find the area \( A_1 \) of the hemisphere that is surface 1.

Express \( A_1 \) in terms of \( r_1 \) and other given or known quantities.

ANSWER:

\[ A_1 = 2\pi r_1^2 \]

**Part C**

What is the electric flux \( \Phi_2 \) passing outward through surface 2?

Express \( \Phi_2 \) in terms of \( r_1 \), \( r_2 \), \( C \), and any constants or other known quantities.

**Hint 1. Apply the definition of electric flux**

The integrand in the equation defining flux (in the problem introduction) can be calculated by noting the following:

\[ \vec{E}(\vec{r}) \cdot d\vec{A} = E(\vec{r})|d\vec{A}| \cos \theta \]

Since the electric field is everywhere perpendicular to surface 2, \( \cos(\theta) = \cos(0) = 1 \). Therefore, the integral simply becomes the magnitude of the electric field multiplied by the surface area of surface 2.

**Hint 2. Find the area of surface 2**

Find the area \( A_2 \) of the hemisphere that is surface 2.

Express your answer in terms of \( r_2 \), and any needed constants.

ANSWER:

\[ A_2 = 2\pi r_2^2 \]

ANSWER:
Observe that the electric flux through surface 1 is the same as that through surface 2, despite the fact that surface 2 has a larger area. If you think in terms of field lines, this means that there is the same number of field lines passing through both surfaces. This is because of the inverse square, \( \frac{1}{r^2} \), behavior of the electric field surrounding a point particle. A good rule of thumb is that the flux through a surface is proportional to the number of field lines that pass through the surface.

**Exercise 22.10**

**Description:** A point charge \( q_1 \) is located on the x-axis at \( x \), and a second point charge \( q_2 \) is on the y-axis at \( y \).

(a) What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius \( r_1 \)?

(b)...

A point charge \( q_1 = 4.20 \text{ nC} \) is located on the x-axis at \( x = 2.20 \text{ m} \), and a second point charge \( q_2 = -6.30 \text{ nC} \) is on the y-axis at \( y = 1.30 \text{ m} \).

**Part A**

What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius \( r_1 = 0.745 \text{ m} \)?

**ANSWER:**

\[
\Phi = 0 \text{ N} \cdot \text{m}^2/\text{C}
\]

**Part B**

What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius \( r_2 = 1.60 \text{ m} \)?

**ANSWER:**

\[
\Phi = \frac{q_2}{8.85 \times 10^{-12}} = -712 \text{ N} \cdot \text{m}^2/\text{C}
\]

**Part C**

What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius \( r_3 = 2.85 \text{ m} \)?

**ANSWER:**

\[
\Phi = \frac{q_1 + q_2}{8.85 \times 10^{-12}} = -237 \text{ N} \cdot \text{m}^2/\text{C}
\]
The Electric Field of a Ball of Uniform Charge Density

Description: Use Gauss's law to find the electric field inside and outside a uniformly charged ball.

A solid ball of radius \( r_b \) has a uniform charge density \( \rho \).

Part A

What is the magnitude of the electric field \( E(r) \) at a distance \( r > r_b \) from the center of the ball? Express your answer in terms of \( \rho, r_b, r, \) and \( \epsilon_0 \).

**Hint 1. Gauss's law**

Gauss's law can be written as

\[
\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0},
\]

where \( d\vec{A} \) refers to an infinitesimal element of an imaginary Gaussian surface, \( q_{\text{encl}} \) is the net charge enclosed in the Gaussian surface, and \( \epsilon_0 \) is the permittivity of free space. Always choose a Gaussian surface that matches the symmetry of the problem. Implicit in the question "what is \( E(r) \)?" is the assumption that the electric field depends only on distance from the origin, which is also the center of the charged ball. Also, the ball has uniform charge density. Therefore, the electric field must point either radially outward or radially inward, since by symmetry there is no possibility for the electric field to point in any other direction. Given the symmetry of this problem, the best Gaussian surface to use is a sphere centered at the origin. Since the electric field is the same at all points on this surface, the constant \( E(r) \) can be "pulled out" of the integrand. The left side of Gauss's law reduces to \( E(r) A(r) \), where \( A(r) \) is the surface area of a sphere with radius \( r \).

**Hint 2. Find \( q_{\text{encl}} \)**

The \( q_{\text{encl}} \) in Gauss's law refers to the net charge enclosed inside the Gaussian surface. What is \( q_{\text{encl}} \) here? Express your answer in terms of \( \rho, \pi, \) and \( r_b \).

**Hint 1. What is the volume of the sphere?**

If a body has uniform charge density \( \rho \), the charge in a volume \( V \) is \( \rho V \) (this formula is the same as that for the mass of a sphere of uniform mass density). What is the volume of a sphere with radius \( r \)?

Express your answer in terms of \( \pi \) and \( r \).

**ANSWER:**

\[
V = \frac{4}{3} \pi r^3
\]

**ANSWER:**

\[
q_{\text{encl}} = \frac{4}{3} \pi r_b^3 \rho
\]
Notice that this result is identical to that reached by applying Coulomb's law to a point charge centered at the origin with \( q = \rho V \). The field outside of a uniformly charged sphere does not depend on the size of the sphere, only on its charge. A uniformly charged sphere generates an electric field as if all the charge were concentrated at its center.

**Part B**

What is the magnitude of the electric field \( E(r) \) at a distance \( r < r_b \) from the center of the ball?

Express your answer in terms of \( \rho, r, r_b, \) and \( \epsilon_0 \).

**Hint 1. How does this situation compare to that of the field outside the ball?**

Now you are asked to find the electric field inside the ball, as opposed to outside the ball. What is different in the physical situation when you move inside the ball?

**ANSWER:**

- The electric field now depends on spatial variables besides the radius.
- The direction of \( E \) is different.
- The shape of the appropriate Gaussian surface is no longer a sphere.
- The net charge \( q_{\text{encl}} \) enclosed by the Gaussian surface is different.
- The charge density \( \rho \) is different.

Since the Gaussian surface is inside the ball, the surface encloses only a fraction of the ball's charge. This is why Gauss's law is so useful: As long as it is symmetrically distributed, the charge outside the Gaussian surface is irrelevant in calculating the field!

**ANSWER:**

\[
E(r) = \frac{\rho r}{3\epsilon_0}
\]

**Part C**

Let \( E(r) \) represent the electric field due to the charged ball throughout all of space. Which of the following statements about the electric field are true?

Check all that apply.
Problem 22.41

Description: A very long, solid cylinder with radius $R$ has positive charge uniformly distributed throughout it, with charge per unit volume $\rho$. (a) Derive the expression for the electric field inside the volume at a distance $r$ from the axis of the cylinder in...

A very long, solid cylinder with radius $R$ has positive charge uniformly distributed throughout it, with charge per unit volume $\rho$.

Part A

Derive the expression for the electric field inside the volume at a distance $r$ from the axis of the cylinder in terms of the charge density $\rho$.

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:
Part B
What is the electric field at a point outside the volume in terms of the charge per unit length $\lambda$ in the cylinder?

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

Also accepted: $\frac{2k\lambda}{r}$

Problem 22.46

Description: A small conducting spherical shell with inner radius $a$ and outer radius $b$ is concentric with a larger conducting spherical shell with inner radius $c$ and outer radius $d$. The inner shell has total charge $+2q$, and the outer shell has charge $-2q$. (a) ...

A small conducting spherical shell with inner radius $a$ and outer radius $b$ is concentric with a larger conducting spherical shell with inner radius $c$ and outer radius $d$. The inner shell has total charge $+2q$, and the outer shell has charge $-2q$.

Part A

Calculate the magnitude of the electric field in terms of $q$ and the distance $r$ from the common center of the two shells for $r < a$.

Express your answer in terms of the given quantities, and appropriate constants.
Part B

Calculate the magnitude of the electric field for $a < r < b$.

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

$$E_1 = 0$$

Part C

Calculate the magnitude of the electric field for $b < r < c$.

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$$

Also accepted: $\frac{k \cdot 2q}{r^2}$

Part D

What is the direction of the electric field for $b < r < c$?

ANSWER:

- toward the center
- outward the center

Part E

Calculate the magnitude of the electric field for $c < r < d$.

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

$$E_4 = 0$$
Part F
Calculate the magnitude of the electric field for \( r > d \).

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

\[ E_5^r = 0 \]

Part G
What's the total charge on the inner surface of the small shell?

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

\[ q_i = 0 \]

Part H
What's the total charge on the outer surface of the small shell?

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

\[ q_o = 2q \]

Part I
What's the total charge on the inner surface of the large shell?

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

\[ q_i = -2q \]

Part J
What's the total charge on the outer surface of the large shell?

Express your answer in terms of the given quantities, and appropriate constants.

ANSWER:

\[ q_o = 0 \]
The Electric Field and Surface Charge at a Conductor

Description: After two conceptual questions about the charge and field inside a conductor, find the surface charge on a conductor, given the field just above the surface.

Learning Goal:
To understand the behavior of the electric field at the surface of a conductor, and its relationship to surface charge on the conductor.

A conductor is placed in an external electrostatic field. The external field is uniform before the conductor is placed within it. The conductor is completely isolated from any source of current or charge.

Part A
Which of the following describes the electric field inside this conductor?
ANSWER:

- It is in the same direction as the original external field.
- It is in the opposite direction from that of the original external field.
- It has a direction determined entirely by the charge on its surface.
- It is always zero.

The net electric field inside a conductor is always zero. If the net electric field were not zero, a current would flow inside the conductor. This would build up charge on the exterior of the conductor. This charge would oppose the field, ultimately (in a few nanoseconds for a metal) canceling the field to zero.

Part B
The charge density inside the conductor is:
ANSWER:

- 0
- non-zero; but uniform
- non-zero; non-uniform
- infinite

You already know that there is a zero net electric field inside a conductor; therefore, if you surround any internal point with a Gaussian surface, there will be no flux at any point on this surface, and hence the surface will enclose zero net charge. This surface can be imagined around any point inside the conductor with the same result, so the charge density must be zero everywhere inside the conductor. This argument breaks down at the surface of the conductor, because in that case, part of the Gaussian surface must lie outside the conducting object, where there is an electric field.

Part C
Assume that at some point just outside the surface of the conductor, the electric field has magnitude $E$ and is directed toward the surface of the conductor. What is the charge density $\eta$ on the surface of the conductor at that point?

Express your answer in terms of $E$ and $\epsilon_0$.

**Hint 1. How to approach the problem**

Which of the following is the best way to solve this problem?

**ANSWER:**

- Use Coulomb's law for the electric field from each charge element.
- Use Gauss's law with a short and flat cylindrical surface (picture a coin) with one end just below (inside) and the other just above (outside) the surface of the conductor.
- Use Gauss's law with a long and thin cylindrical surface (picture a straw capped at both ends) with one end far below (but inside the object) and the other far above the surface of the conductor.
- None of these: You need to know the surface charge distribution everywhere on the surface.
- None of these: You need to know the electric field at all points on the surface.

A straightforward way to solve this problem is to choose a Gaussian surface one end of which is just above and the other just below the surface of the conductor.

![Diagram of electric field and Gaussian surface](image)

**Hint 2. Calculate the flux through the top of the cylinder**

Using a flat cylinder with large top and bottom each of area $A$ just above and just below the surface of the conductor, find the flux $\Phi_{\text{top}}$ generated through the top surface of the cylinder by the electric field of magnitude $E$ that points into the surface.

Express your answer in terms of $A$, $E$, and any needed constants.

**ANSWER:**

$$\Phi_{\text{top}} = -EA$$
Hint 3. Calculate the flux through the bottom of the box

Using a flat cylinder with large top and bottom of area $A$ just above and just below the surface of the conductor, find the flux $\Phi_{\text{bot}}$ generated through the bottom surface of the cylinder by the electric field inside the conductor (keep in mind that positive flux is outward through the cylinder’s surface, which is downward into the conductor).

Answer in terms of $A$, $E$, and any needed constants.

**ANSWER:**

\[ \Phi_{\text{bot}} = 0 \]

Hint 4. What is the charge inside the Gaussian surface?

Find the net charge $q_{\text{in}}$ inside this Gaussian surface.

Express your answer in terms of the charge density $\eta$ and other given quantities.

**ANSWER:**

\[ q_{\text{in}} = \eta A \]

Notice that the surface charge density $\eta$ has dimensions of charge per area. Therefore, when $\eta$ is multiplied by an area, the result is charge, which is needed for Gauss’s law.

Hint 5. Apply Gauss's law

Now apply Gauss's law, neglecting any contribution to the flux due to the very short sides of the cylinder. Gauss’s law states that $\epsilon_0 \Phi_E = q_{\text{in}}$. The area $A$ should cancel out of your result.

**ANSWER:**

\[ \eta = -\epsilon_0 E \]

The Charge Inside a Conductor

**Description:** A spherical conductor has a cavity containing a fixed charge. Conceptual questions about how charge is arranged on the surfaces of a conductor to cancel the field due to the fixed charge and how the charge arrangement would change if an additional external charge were brought near the conductor.
A spherical cavity is hollowed out of the interior of a neutral conducting sphere. At the center of the cavity is a point charge, of positive charge \( q \).

**Part A**

What is the total surface charge \( q_{\text{int}} \) on the interior surface of the conductor (i.e., on the wall of the cavity)?

**Hint 1. Gauss's law and properties of conductors**

The net electric field in the interior of the conducting material must be zero. (The electric field in the cavity, however, need not be zero.) Knowing this, you can use Gauss's law to find the net charge on the interior surface of the cavity. Use the following Gaussian surface: an imaginary sphere, centered at the cavity, that has an *infinitesimally* larger radius than that of the cavity, so that it encompasses the inner surface of the cavity. This Gaussian surface lies within the conductor, so the field on the Gaussian surface must be zero. Thus, by Gauss's law, the net charge inside the Gaussian surface must be zero as well. But you know that there is a point charge \( q \) within the Gaussian surface. If the *net* charge within the Gaussian surface must be zero, how much charge must be present on the surface of the cavity?

**ANSWER:**

\[ q_{\text{int}} = -q \]

**Part B**

What is the total surface charge \( q_{\text{ext}} \) on the exterior surface of the conductor?

**Hint 1. Properties of the conductor**

In the problem introduction you are told that the conducting sphere is neutral. Furthermore, recall that the free charges within a conductor always accumulate on the conductor's surface (or surfaces, in this case). You found the net charge on the conductor's interior surface in Part A. If the conductor is to have zero net charge (as it must, since it is neutral), how much charge must be present on its exterior surface?

**ANSWER:**
Part C

What is the magnitude $E_{\text{int}}$ of the electric field inside the cavity as a function of the distance $r$ from the point charge? Let $k$, as usual, denote $\frac{1}{4\pi\varepsilon_0}$.

**Hint 1. How to approach the problem**

The net electric field inside the conductor has three contributions:

1. from the charge $q$;
2. from the charge on the cavity's walls $q_{\text{int}}$;
3. from the charge on the outer surface of the spherical conductor $q_{\text{ext}}$.

However, the net electric field inside the conductor must be zero. How must $q_{\text{int}}$ and $q_{\text{ext}}$ be distributed for this to happen?

Here's a clue: the first two contributions above cancel each other out, outside the cavity. Then the electric field produced by $q_{\text{ext}}$ inside the spherical conductor must separately be zero also. How must $q_{\text{ext}}$ be distributed for this to happen?

After you have figured out how $q_{\text{int}}$ and $q_{\text{ext}}$ are distributed, it will be easy to find the field in the cavity, either by adding field contributions from all charges, or using Gauss's Law.

**Hint 2. Charge distributions and finding the electric field**

$q_{\text{int}}$ and $q_{\text{ext}}$ are both uniformly distributed. Unfortunately there is no easy way to determine this, that is why a clue was given in the last hint. You might hit upon it by assuming the simplest possible distribution (i.e., uniform) or by trial and error, and check that it works (gives no net electric field inside the conductor).

If $q_{\text{ext}}$ is distributed uniformly over the surface of the conducting sphere, it will not produce a net electric field inside the sphere. What are the characteristics of the field $q_{\text{int}}$ produces inside the cavity?

**ANSWER:**

- zero
- the same as the field produced by a point charge $q$ located at the center of the sphere
- the same as the field produced by a point charge located at the position of the charge in the cavity

**ANSWER:**

- 0
- $k \frac{q}{r^2}$
- $2k \frac{q}{r^2}$
Part D

What is the electric field $E_{\text{ext}}$ outside the conductor?

**Hint 1. How to approach the problem**

The net electric field inside the conductor has three contributions:

1. from the charge $q$;
2. from the charge on the cavity's walls $q_{\text{int}}$;
3. from the charge on the outer surface of the spherical conductor $q_{\text{ext}}$.

However, the net electric field inside the conductor must be zero. How must $q_{\text{int}}$ and $q_{\text{ext}}$ be distributed for this to happen?

Here's a helpful clue: the first two contributions above cancel each other out, outside the cavity. Then the electric field produced by $q_{\text{ext}}$ inside the spherical conductor must be separately be zero also. How must $q_{\text{ext}}$ be distributed for this to happen? What sort of field would such a distribution produce outside the conductor?

**Hint 2. The distribution of $q_{\text{ext}}$**

If $q_{\text{ext}}$ is distributed uniformly over the surface of the conducting sphere, it will not produce a net electric field inside the sphere. What are the characteristics of the field it produces outside the sphere?

**ANSWER:**

- zero
- the same as the field produced by a point charge $q$ located at the center of the sphere
- the same as the field produced by a point charge located at the position of the charge in the cavity

Now a second charge, $q_2$, is brought near the outside of the conductor. Which of the following quantities would change?

Part E

The total surface charge on the wall of the cavity, $q_{\text{int}}$:

**Hint 1. Canceling the field due to the charge $q$**

The net electric field inside a conductor is always zero. The charges on the inner conductor cavity will always arrange themselves so that the field lines due to charge $q$ do not penetrate into the conductor.

**ANSWER:**
Part F

The total surface charge on the exterior of the conductor, $q_{\text{ext}}$:

**Hint 1. Canceling the field due to the charge $q_{2}$**

The net electric field inside a conductor is always zero. The charges on the outer surface of the conductor will rearrange themselves to shield the external field completely. Does this require the net charge on the outer surface to change?

**ANSWER:**

- [ ] would change
- [x] would not change

Part G

The electric field within the cavity, $E_{\text{cav}}$:

**ANSWER:**

- [ ] would change
- [x] would not change

Part H

The electric field outside the conductor, $E_{\text{ext}}$:

**ANSWER:**

- [ ] would change
- [x] would not change

Exercise 22.25

**Description:** A conductor with an inner cavity, like that shown in Fig.22.23c from the textbook, carries a total charge of $+q_{1}$. The charge within the cavity, insulated from the conductor, is $q_{2}$. How much charge is on (a) How much charge is on the inner surface ...
A conductor with an inner cavity, like that shown in Fig.22.23c from the textbook, carries a total charge of +5.80 \( \text{nC} \). The charge within the cavity, insulated from the conductor, is -8.10 \( \text{nC} \). How much charge is on

**Part A**

How much charge is on the inner surface of the conductor?

**ANSWER:**

\[
q_{\text{inner}} = q_2 = 8.10 \ \text{nC}
\]

**Part B**

How much charge is on the outer surface of the conductor?

**ANSWER:**

\[
q_{\text{outer}} = q_1 + q_2 = -2.30 \ \text{nC}
\]

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**Charge Distribution on a Conductor with a Cavity**

**Description:** Conceptual problem. Positive charge sits outside a conducting shell. What is the resulting charge distribution on the inside and outside of the shell.

A positive charge is brought close to a fixed neutral conductor that has a cavity. The cavity is neutral; that is, there is no net charge inside the cavity.

**Part A**

Which of the figures best represents the charge distribution on the inner and outer walls of the conductor?

**Hint 1. Conductors have no internal field**

At steady state, conductors have no internal electric field (otherwise, charge would flow). Therefore, the
arrangement of charges on the surfaces of the conductor must exactly cancel out any external electric field to ensure that the internal field is zero.

**Hint 2. Charges on the cavity walls**

Think about what the answer would be for a conductor without a cavity. Would there be net charges on the surface of some imaginary sphere drawn inside of the conductor? Would this change if you removed all of the material inside of that sphere?

ANSWER:

- 1
- 2
- 3