Inductors and Inductance

I. Inductance of a single loop

Consider a single loop of radius \( a \) as shown in the figure to the right. If the loop is very small, we can consider the field \( \vec{B} \) to be constant in the full area of the loop.

1. Calculate the magnetic field \( \vec{B} \) in the center of the loop.

2. Calculate the magnetic flux through the loop.

3. Using Faraday’s Law, calculate the emf induced if the current is changing over time and everything else stays constant. Explain in words what this induced emf means.

We can identify everything other than the \( \frac{dl}{dt} \) as a constant depending only on the geometry of the loop. This is \( L \), the inductance of the loop. It is a quantity used to determine how large the induced emf will be for a given change in current.

4. What is the inductance \( L \) of the single loop?
II. Inductance of a solenoid

Now we will consider a long solenoid of length $\ell$, radius $a$, and $n$ turns per unit length. This is more like the inductors you would find in a conventional electronic circuit.

1. Calculate the magnetic field $\vec{B}$ in the solenoid.

2. Calculate the magnetic flux through one turn of the solenoid.

3. Calculate the magnetic flux through the entire solenoid.

4. Using Faraday’s Law, calculate the emf induced if the current is changing over time and everything else stays constant. Explain in words what this induced emf means.

5. What is the inductance $L$ of the solenoid?
III. Mutual inductance (Larger Loop)

To the right is shown a solenoid of radius $a$ with a current $I_1(t)$ that has a larger loop around it with radius $R, R>a$. The two do not touch. However, an induced emf could be generated in the outer loop of wire.

1. Calculate the magnetic field $\vec{B}$ in the solenoid.

2. Calculate the magnetic flux through the outer loop.

3. Using Faraday’s Law, calculate the emf induced in the outer loop if the current in the solenoid is changing over time and everything else stays constant. Explain in words what this induced emf means.

*The mutual inductance is defined as the proportionality constant $M$ which determines how the emf in the outer coil is related to the change in current in the solenoid (or vice versa if the situation was reversed). In general it can be shown that $M_{12}=M_{21}=M$.*

4. Calculate the mutual inductance $M$ for the above system.
IV. Mutual Inductance (Smaller Loop)

To the right is shown a solenoid with radius $a$ with a current $I_1(t)$ that has a smaller loop of radius $R$ inside of it, i.e., $R < a$. The two do not touch, however, an induced emf could be generated in the inner coil of wire.

1. Calculate the magnetic field $\mathbf{B}$ in the solenoid.

2. Calculate the magnetic flux through the inner loop.

3. Using Faraday’s Law, calculate the emf induced in the inner loop if the current in the solenoid is changing over time and everything else stays constant. Explain in words what this induced emf means.

4. Calculate the mutual inductance $M$ for the above system.

*In the first case the radius of the solenoid was the only radius that mattered and in the second case the radius of the inner loop was the only radius that mattered. In general, however, the situation can be more complicated and can depend on all of the characteristics of both of the inductors.*