Conservation of Linear Momentum

Purpose: To understand conservation of linearl momentum; to investigate whether or not momentum and energy are conserved in elastic and inelastic collisions. To examine the concept of impulse in a real-life situation.

Apparatus: Pasco track, Pasco cart (two), LabPro interace and cables, motion sensor (two), cart weights

Introduction: In the last lab, you tested energy conservation when potential energy was converted to kinetic energy in a system (mass bucket + cart). That system was acted upon by a conservative external net force (gravity), as well as a non-conservative force (friction). This week you will examine a system in which there is again friction, but no conservative external net force. The changes in motion of the constituents of the system (two colliding carts) will arise from the internal forces between them. You will measure the momenta of each cart before and after the collision, and check to see if momentum and energy is conserved.

Newton originally stated his second law using momentum, rather than acceleration :

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} \qquad \text{where} \quad \vec{p} = m\vec{v} \qquad (1)$$

For a **system** of N particles, we can rewrite this as

$$\vec{F}_{net} = \frac{\Delta \vec{p}_{net}}{\Delta t} \qquad \text{where} \quad \vec{p}_{net} = \sum_N \vec{p}_i = \vec{p}_1 + \vec{p}_2 + \vec{p}_{3.} \ldots + \vec{p}_N \qquad (2)$$

In the case we are considering, namely where there are no external forces, the above equation reads

$$\frac{\Delta \vec{p}_{net}}{\Delta t} = 0 \qquad \text{or} \qquad \vec{p}_{net} = constant \qquad (3)$$

which states that the linear momentum of a system is conserved, for no external forces.

For a system composed of two masses along a track, we can relate their momentum values before and after the collison as follows. Here we have dropped the vector notation, and adopted the convention that the only directional information needed comes from positive or negative velocity values:

$$m_1 v_1^i + m_2 v_2^i = m_1 v_1^f + m_2 v_2^f \qquad (4)$$

The experiment consists of colliding two carts along a track, while monitoring their velocities with two sonic rangers mounted at opposite ends of the track. By weighing the carts beforehand, we have Logger Pro output the real-time momentum value.

You may be protesting at this point: "*How do I get the carts to move if there aren't supposed to be any external forces on them? Doesn't my hand, which pushes the cart, represent an external force?*". You will indeed be providing an external force; however, the time interval that we are most interested in is the period *just before* the collision to *just after* the collision. During that interval there are no external forces on the system other than gravity, which we cannot eliminate in the lab. The only forces acting are internal, action-reaction ones, described by the Third Law: $\vec{F}_{12} = -\vec{F}_{21}$. In this lab we will examine two types of collisions: **elastic** collisions, in which energy is ideally conserved, and **inelastic** collisions, in which energy is ideally not conserved. However, linear momentum is ideally conserved for both types of collisions.

Finally, your instructor will demonstrate for you the concept of Impulse, which is defined as $J = \int_{t_1}^{t_2} F \, dt$

In calculus, this quantity is the area under the curve of the F vs. t graph, and represents the change of momentum during a short collision between two objects. Since this is an algebra-based course, we will assume that the force is roughly constant with time, so that the above equation simplifies to:

$$J = F \Delta t = \Delta p$$

The impulse can then be thought of as the change in momentum of an object when acted on by a constant force over a short period of time. Note that a large force acting over a shorter period of time can impart the same impulse as a smaller force acting over a longer period of time. Each can be advantageous in different cases; for instance, the latter is desirable when it comes surviving a car crash – an air bag will lengthen the time of the impulse imparted during the collision, therefore decreasing the force that the passenger experiences, despite the fact that in both situations (with or without airbag), the impulse (the change of momentum) is the same (from velocity v before the collision to 0 after).

Sometimes impulse has the symbol I, which we won't use since it can easily be confused for moment of inertia, also represented by I.

Procedure

You will be colliding two masses (carts) together, while simultaneously monitoring their momenta (calculated from their velocities). You will then identify points immediately before and after the collision, to determine the $p_{initial}$ *and* p_{final} *values. Note that friction will be present throughout the entire motion of the carts (visible in both the momenta and energy plots) but that the effects of friction will be minimal during the brief time interval of the collision.*

A. Preparations

Make sure the left motion sensor is plugged into DIG/SONIC1 on the LabPro interface, and that the right motion sensor is plugged into DIG/SONIC2.

1. Weigh the carts and weights on the lab scale as you did in the last lab. If the masses are already labeled, you can use these figures. Make sure both motion sensors are adjusted for a horizontal beam, and set to narrow beam.

2. Open the Logger Pro file "Momentum.CMBL", which is in the same folder as this write-up. You will see a data table and graphs monitoring momentumdata as well as energy data. By convention, the mass on the left is mass **one** and the mass on the right is mass **two**. Individual values such as P1 and P2 will be added to give **total** value P. The only measured values are t, Distance 1 and Distance 2 (not shown on graph); **everything else is calculated**.

3. Double-click on the **P1** column in the Table Window. You will see the definition of the momentum of cart 1. Change the default value of 0.490 to the actual mass of your cart 1. Do the same thing for the **P2 and E total**.

4. Look at the ends of each cart; one end has velcro patches while the other doesn't. When the velcro end of one cart comes in contact with the velcro end of another cart, adhesion occurs and the carts are stuck together - this approximates an **inelastic collision.** Conversely, putting the non-velcro ends together puts internal magnets of each cart near each other to provide a repulsive force - this approximates an **elastic** collision.

5. Do a few practice runs - put Mass 2 halfway along the track, then position Mass 1 about 15cm to the right of the left motion sensor. Press Collect in Logger Pro and push Mass 1 to accelerate and hit Mass 2. Push hard enough such that Mass 2 makes it to the other side of the track. If you are doing an elastic collision **make sure the carts don't actually make contact, and only experience the magnetic repulsive force,** otherwise the collision will be partially inelastic. In all cases be prepared to catch any carts before they hit the right motion sensor.

NOTE: You will probably see a small bump in the Total Momentum graph (and a small dip in the Total Energy graph) during the time that the carts are colliding. This is a spurious (false) bump introduced by the software. The software tells the two sensors not to fire at thes same time, to prevent interference, causing a lag between left and right sensors. Since the collision takes place over a very short time, the software incorrectly calculates the total momentum before both sensors are actually read. It is important to note this slight glitch, but it should not affect your measurements - in subsequent parts of this lab, you will be asked to identify and note values of v, p and K.E. *before* and *after* the collision, at points *outside* this spurious hump.

B. Inelastic collision, Mass 1 roughly equal to Mass 2, Mass 2 initially at rest

1. Try to produce a good clean run for an **inelastic** collision, with no added mass on the carts and pushing Mass 1 into Mass 2 which is initially at rest. Identify on the graph:

> a) the point when you started pushing the cart
> b) the point when you stopped pushing the cart
> c) the region where the carts exerted forces on each other

2. Using the Analyze-->Examine feature, or by examining data in the Table, identify the velocities right **before** the interaction and right **after** the interaction. Write these down in your hand-in sheet. Also record intial and final momenta and kinetic energies. One good run is sufficient. Print out the graph for this run and mark on the printout (a), (b) and (c) as above.

C. Elastic collision, Mass 1 roughly equal to Mass 2, Mass 2 initially at rest

Repeat Part B for an **elastic** collision, with no added mass on the carts. Again, only one good run is required. Write velocities in hand-in sheet. Also record intial and final momenta and kinetic energies. Print out a plot for a good run. Mark on each printout (a), (b) and (c).

D. Elastic collision, Mass1 less than Mass 2, Mass 2 initially at rest

Add two cart weights to Mass 2. Push Mass 1 into Mass 2, initially at rest . **Don't forget to change the weight for Mass 2 in the Logger Pro column definitions.** Write velocities in hand-in sheet. Also record intial and final momenta and kinetic energies. There is no need to print a graph.

E. Elastic collision, Mass 1 roughly equal to Mass 2, both initially heading towards each other

Remove cart weights from Mass 2. Remember **to change back the weight for Mass 2 in the Logger Pro column definitions.** Push both masses towards each other, at roughly the same speed. Write velocities in hand-in sheet. Also record intial and final momenta and kinetic energies.   There is no need to print a graph.

Assessment and Presentation (Hand-in Sheet/Lab Notebook)

Record all calculations and results in the hand-in sheet. Then answer the following questions:

1. For an inelastic collision, was momentum conserved, considering your cart momenta immediately before and after the collision? What about kinetic energy? If either of these is not conserved, explain why.

2. For an elastic collision, was momentum conserved, considering your cart momenta immediately before and after the collision? What about kinetic energy? If either of these is not conserved, explain why.

3. For an elastic collision, why do the carts start interacting even though they never physically touch?

4. For an inelastic collision, what other forms of energy is kinetic energy converted to?

5. Looking at your graphs, what is the main source of energy loss during the entire run (4 seconds)?

6. Can you think of an example where a larger force over a shorter time interval is preferable to a smaller force over a larger time interval?