Review: Thin Film Interference
Review: Galilean Relativity
Special Relativity: Time Dilation, Length Contraction,
Relativistic Doppler Effect
Relativistic Energy and Momentum
We can use geometry to find the conditions for constructive and destructive interference:

\[ dsin\theta = m\lambda, \quad m = 0, 1, 2, ... \quad [\text{constructive interference (bright spot)}] \]

\[ dsin\theta = \left( m + \frac{1}{2} \right)\lambda, \quad m = 0, 1, 2, ... \quad [\text{destructive interference (dark spot)}] \]
Diffraction by a Single Slit or Disk

The minima (destructive interference) of the single-slit diffraction pattern occur when

\[ D \sin \theta = m \lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots \]
The maxima (constructive interference) of the diffraction pattern are defined by

\[ \sin \theta = \frac{m \lambda}{d}, \quad m = 0, 1, 2, \ldots \]
Interference in Thin Films

Another way path lengths can differ, and waves interfere, is if the travel through different media.

If there is a very thin film of material—a few wavelengths thick—light will reflect from both the bottom and the top of the layer, causing interference.

This can be seen in soap bubbles and oil slicks, for example.
Interference in Thin Films

Problem Solving: Interference

1. Interference occurs when two or more waves arrive simultaneously at the same point in space.

2. Constructive interference occurs when the waves are in phase.

3. Destructive interference occurs when the waves are out of phase.

4. An extra half-wavelength shift occurs when light reflects from a medium with higher refractive index.
Example: Thin Film Interference

Consider a silicon based material \( n_{Si} = 3.5 \) coated with silicon monoxide \( n_{SiO} = 1.45 \). Determine the minimum non-zero thickness of the SiO coat if the goal to cause destructive interference for incoming EM radiation of wavelength \( \lambda = 600nm \).
A plastic film with index of refraction $n_{film} = 1.70$ is applied to the surface of a car window to increase the reflectivity and thus to keep the car’s interior cooler. The window glass has index of refraction $n_{glass} = 1.52$. What minimum (non-zero) thickness is required if light of wavelength 550 nm in air reflected from the two sides of the film is to interfere constructively?

- a) 275 nm
- b) 81 nm
- c) 91 nm
- d) 137 nm
Resolution: The distance at which a lens can barely distinguish two separate objects.

Resolution is limited by (1) aberrations and by (2) diffraction.

Aberrations can be minimized, but **diffraction is unavoidable**; it is due to the size of the lens compared to the wavelength of the light.
The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.
Limits of Resolution; Circular Apertures

For a circular aperture of diameter $D$, the central maximum has an angular width:

\[ \theta = \frac{1.22\lambda}{D} \]
Resolution of Telescopes and Microscopes; the $\lambda$ Limit

Since the resolution is directly proportional to the wavelength ($\lambda$) and inversely proportional to the diameter (D), radio telescopes are built to be very large.
Resolution of Telescopes and Microscopes; the $\lambda$ Limit

For microscopes, assuming the object is at the focal point, the resolving power is given by:

\[ \text{RP} = s = f \theta = \frac{1.22 \lambda f}{D} \]

Typically, the focal length of a microscope lens is half its diameter, which shows that it is not possible to resolve details smaller than the wavelength being used.

\[ \text{RP} \approx \frac{\lambda}{2} \]
A muon has a mean lifetime of 2.20\(\mu\)s. If a muon is created in the upper atmosphere (by a cosmic ray) with a speed of \(v_\mu = 2.95 \times 10^8 \text{ m/s}\), how deep into the earth’s atmosphere will the muon travel before it decays?

a) 649 km
b) 649 m
c) \(1.4 \times 10^{14} \text{ m}\)
d) 137 nm

The atmosphere is roughly 480 km thick. Yet, on the surface of the Earth, we observe a muon flux of about 1 muon per square centimeter per minute.
ICLICKER QUESTION

You and your friend are playing catch, in a train moving at 60 mph in an eastward direction. Your friend is at the front of the car and throws you the ball at 3 mph (according to him). What velocity does the ball have when you catch it, according to you?

a) 3 mph eastward
b) 3 mph westward
c) 57 mph eastward
d) 57 mph westward
e) 60 mph eastward

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ICLICKER QUESTION

You and your friend are playing catch, in a train moving at 60 mph in an eastward direction. Your friend is at the front of the car and throws you the ball at 3 mph (according to him). What velocity does the ball have when you catch it, according to an observer on the ground?

a) 3 mph eastward
b) 3 mph westward
[c] 57 mph eastward
d) 57 mph westward
e) 60 mph eastward
Galilean-Newtonian Relativity

Definition of an inertial reference frame:

One in which Newton’s first law is valid

Earth is rotating and therefore it is not an inertial reference frame, but we can treat it as one for many purposes

A frame moving with a constant velocity with respect to an inertial reference frame is itself inertial
Relativity principle:

The laws of physics are the same in all inertial reference frames.
Observation: This principle (relativity) works well for mechanical phenomena.

However, Maxwell’s equations yield the constant speed of light in vacuum; it is $3.0 \times 10^8$ m/s.

Question: Which is the reference frame in which light travels at that speed?

Scientists searched for variations in the speed of light depending on the direction of the ray but found none.
ICLICKER QUESTION

You hold an electron in your hand; thus you are at rest with respect to the electron. You can measure the electric field of the electron. Now what would your friend running past you measure?

a) an E field
b) a B field
c) both an E and a B field
d) Impossible to determine
Postulates of the Special Theory of Relativity

1. The laws of physics have the same form in all inertial reference frames.

2. The speed of light in vacuum, “c”, has the same value in all inertial reference frames
   - e.g. Light propagates through empty space with a speed “c” independent of the speed of source or observer.
Simultaneity

One of the implications of special relativity is that time is not absolute.

• Distant observers do not necessarily agree on time intervals between events, or on whether they are simultaneous or not.

• In special relativity, an “event” is defined as occurring at a specific place and time.
Thought experiment #1: Lightning striking at two separate places.

Conclusion: If one observer believes the events are simultaneous, then, a second observer, moving relative to the first, does not since the speed of light is the same for both.
Thought experiment #2: using a clock consisting of a light beam and mirrors.

Conclusion: Moving observers must disagree on the passage of time.
Time Dilation Equation

Calculating the difference between clock “ticks,” we can find the relationship between the interval in the moving frame $\Delta t$ to the interval in the clock’s rest frame $\Delta t_0$.

$\Delta t_0 = \frac{2D}{c}$

- $t_0$ is the proper time, i.e. the time measured by an observer in which the clock is at rest.

$\Delta t = \frac{2}{c} \sqrt{D^2 + \left(\frac{u\Delta t}{2}\right)^2}$
Time Dilation Equation

Solving for D in eq1:

\[ \Delta t_0 = \frac{2D}{c} \implies D = \frac{c}{2} \Delta t_0 \]

Plug into eq2:

\[ \Delta t = \frac{2}{c} \sqrt{D^2 + \left(\frac{u\Delta t}{2}\right)^2} \implies \Delta t = \frac{2}{c} \sqrt{\left(\frac{c}{2} \Delta t_0\right)^2 + \left(\frac{u\Delta t}{2}\right)^2} \]

Time Dilation Equation:

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} \]
The factor multiplying $t_0$ occurs so often in special relativity that it is given its own symbol, $\gamma$.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{[Lorentz Factor]}$$

Then we can write the time dilation equation as:

$$\Delta t = \gamma \Delta t_0$$
# The Lorentz Factor

## Values of $\gamma$

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<th>$\gamma$</th>
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An astronaut moves away from earth at close to the speed of light. How would an observer on Earth measure the astronaut’s pulse rate?

a) It would be faster
b) It would be slower
c) It wouldn’t change
Time Dilation

It has been proposed that space travel could take advantage of time dilation—if an astronaut’s speed is close enough to the speed of light, a trip of 100 light-years could appear to the astronaut as having been much shorter.

The astronaut would return to Earth after being away for a few years, and would find that hundreds of years had passed on Earth.
If any inertial frame is just as good as any other, why doesn’t the astronaut age faster than his twin, on Earth, traveling away from him?

The astronaut’s reference frame has not been continuously inertial—he turns around at some point and comes back. It is impossible to do this without accelerating.
If time intervals are different in different reference frames, lengths must be different as well.

Length contraction is given by:

\[ \ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2}} \]

\[ \ell = \ell_0 / \gamma \]

Note that length contraction occurs only along the direction of motion.
A spaceship moves faster and faster, approaching the speed of light. How would an observer on Earth see the spaceship?

a) It becomes shorter and shorter  
b) It becomes longer and longer  
c) There is no change
Four-Dimensional Space-Time

Further observations from Special Relativity is that space and time are even more intricately connected.

Space has three dimensions, and time is a fourth.

When viewed from different reference frames, the space and time coordinates can mix.

A path through space-time is called a world-line.
Relativistic Doppler Effect

Another consequence of time dilation is a shift in frequency found for light emitted by atoms in motions as opposed to light emitted by atoms at rest. This is known as the Doppler effect.

To analyze this, recall that:

1. Light requires no medium to propagate.
2. There is no method to distinguish the motion of a light source from the motion of the observer.

Relativistic Doppler Equation:  

\[ f_{\text{obs}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{\text{source}} \]

\( v > 0 \) when the source and observer approach each other, \( v < 0 \) otherwise.
Conservation of Momentum Revisited: Relativistic Momentum

In order to define relativistic momentum we must consider two things:

1. The total linear momentum of two particles in an isolated system remains constant.

2. The relativistic value of the momentum must approach the classical value when \( v \ll c \).

Definition of Relativistic Momentum:

\[
p_{\text{rel}} = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv
\]
Relativistic Momentum

**Definition of Relativistic Momentum:**

\[ p_{\text{rel}} = \frac{m_0 v}{\sqrt{1 - v^2 / c^2}} = \gamma m_0 v \]

Sometimes this is interpreted as an increase in mass, so we must make a distinction between relativistic mass and rest mass \((m_0)\):

\[ m_{\text{rel}} = \frac{m_0}{\sqrt{1 - v^2 / c^2}} = \gamma m_0 \]

Then, we can also write the relativistic momentum as:

\[ p_{\text{rel}} = m_{\text{rel}} v \]
A basic result of special relativity is that no object with a mass other than zero can equal or exceed the speed of light in vacuum, \( c = 3 \times 10^8 \text{ m/s} \).

This would require infinite momentum.

\[
\begin{align*}
p_{rel} &= \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv
\end{align*}
\]
What is the rest mass of a photon?

a) Meaningless question, since a photon cannot be brought to rest.

b) Zero

c) It could be zero and non-zero since light is also a wave.

d) It must be nonzero, since the Higgs particle gives mass to all particles.
At relativistic speeds, not only is the equation for momentum modified; that for energy is as well.

The total energy can be written as:

\[ E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = \gamma m_0 c^2 \]

In the particle’s rest frame:

\[ E_0 = m_0 c^2 \]
Combining the relations for energy and momentum gives the relativistic relation between them:

\[ E^2 = p^2 c^2 + m_0^2 c^4 \]
Note that all the equations become the usual Newtonian kinematic equations when the speeds are much smaller than the speed of light (c).

There is no fixed rule for when the speed is high enough that relativistic formulas must be used—it depends on the desired accuracy of the calculation.
Suppose two observers in relative motion with respect to each other are both observing the motion of an object.

For example, consider a rocket traveling at 60% the speed of light ($v = 0.60c$) away from the Earth and a second rocket traveling also at 60% ($u' = 0.60c$) the speed of light but with relative to the first rocket.

In the classical approach we’d say that the speed of rocket 2 with respect to an observer on the Earth is simply $v + u'$ or $1.20c$ which we know is impossible.
Relativistic velocities cannot simply add; the speed of light in vacuum, “c”, is an absolute limit.

Relativistic Velocity Addition: \[ u = \frac{v + u'}{1 + vu'/c^2} \]

- \( u' \) and \( v \) along the same direction

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The predictions of special relativity have been tested thoroughly and verified to great accuracy.

The correspondence principle says that a more general theory must agree with a more restricted theory where their realms of validity overlap. This is why the effects of special relativity are not obvious in everyday life.