Lecture

• Review: Resistivity
• Electric Power
• Resistors in Series and Parallel
• Kirchhoff’s Rules
Resistivity

The resistance of a wire is directly proportional to its length and inversely proportional to its cross-sectional area:

\[ R = \rho \frac{\ell}{A} \]

The constant \( \rho \), the resistivity, is characteristic of the material.

For any given material, the resistivity increases with temperature:

\[ \rho_T = \rho_0 [1 + \alpha \Delta T] \]
ICLICKER QUESTION

A wire of resistance $R$ is stretched uniformly (keeping its volume constant) until it is twice its original length. What happens to the resistance?

$$R = \rho \frac{\ell}{A}$$

a) it decreases by a factor 4
b) it decreases by a factor 2
c) it stays the same
d) it increases by a factor 2
e) it increases by a factor 4
Electric Power

• Recall the definition of power: \[ P = \frac{\Delta W}{\Delta t} \]

• That is, power is the energy transformed by a device per unit time:

\[ P = \text{energy transformed per unit time} = \frac{\Delta(qV)}{\Delta t} \]

\[ P = IV \]

For an Ohmic Device, where \( V = IR \) we can write: \[ P = \frac{V^2}{R} \] or \( P = I^2R \)
Electric Power

• What you pay for on your electric bill is not power, but energy—the power consumption multiplied by the time.

• We have been measuring energy in joules, but the electric company measures it in kilowatt-hours, kWh.

\[
1 \text{ kWh} = (1000 \text{ W})(3600 \text{ s}) = 3.60 \times 10^6 \text{ J}
\]
Two lightbulbs operate at 120 V, but one has a power rating of 25 W while the other has a power rating of 100 W. Which one has the greater resistance?

\[ P = IV \quad (Ohm's \ Law \ V = IR) \]

a) the 25 W bulb  
b) the 100 W bulb  
c) both have the same  
d) this has nothing to do with resistance
Power in Household Circuits

• The wires used in homes to carry electricity have very low resistance.

• If the current is high enough, the power will increase \((P=IV)\) and the wires can become hot enough to start a fire.

• To avoid this, we use fuses or circuit breakers, which disconnect when the current goes above a predetermined value.
Power in Household Circuits

Fuses are one-use items—if they blow, the fuse is destroyed and must be replaced.
Circuit breakers, which are now much more common in homes than they once were, are switches that will open if the current is too high; they can then be reset.
EMF and Terminal Voltage

• An electric circuit needs a battery or generator to produce current—these are called sources of emf.

• A battery is a nearly constant voltage source, but does have a small internal resistance, which reduces the actual voltage from the ideal emf ($\mathcal{E}$):

$$V_{ab} = \mathcal{E} - Ir$$
EMF and Terminal Voltage

This resistance behaves as though it were in series with the emf ($\mathcal{E}$).

\[ V_{ab} = \text{terminal voltage} \]
Assume that the voltage of the battery is 9 V and that the three resistors are identical. What is the potential difference across each resistor?

a) 12 V  

b) zero  

c) 3 V  

d) 4 V  

e) you need to know the actual value of R
Resistors in Series

A series connection has a single path from the battery, through each circuit element in turn, then back to the battery.
Resistors in Series

Observations:

- The current through each resistor is the same (I).
- The voltage across each resistor depends on its resistance (IR).
- The sum of the voltage drops across the resistors equals the battery voltage.

\[ V = V_1 + V_2 + V_3 \]

\[ V = IR_1 + IR_2 + IR_3 \quad \text{or} \quad \frac{V}{I} = R_1 + R_2 + R_3 \]
Resistors in Series

From this we get the equivalent resistance (that single resistance that gives the same current in the circuit).

\[ R_{eq} = R_1 + R_2 + R_3. \]
Resistors in Parallel

A parallel connection splits the current.

\[ I = \frac{V}{R_{eq}} \]
Observation: Conservation of charge tells us that the total current is the sum of the currents across each resistor.

\[ I = I_1 + I_2 + I_3 \]

\[ \frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \]
Resistors in Series and in Parallel

**Observation:** In a parallel arrangement, each device (e.g. resistors) have the same potential difference across them.

\[
\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}\\
\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

![Diagram of resistors in parallel with currents I₁, I₂, and I₃ passing through R₁, R₂, and R₃ respectively. The total voltage V is applied across the arrangement.]
In the circuit below, what is the current through $R_1$?

a) 10 A  
b) zero  
c) 5 A  

\boxed{d) 2 A}  
e) 7 A
What happens to the voltage across the resistor $R_4$ when the switch is closed?

a) increases
b) decreases
c) stays the same
Kirchhoff’s Rules

- Some circuits cannot be broken down into series and parallel connections.

- We can use conservation of energy and conservation of charge!
Kirchhoff’s Rules

**Junction rule** (conservation of charge): The sum of currents entering a junction equals the sum of the currents leaving it.

\[ \sum_{\text{in}} I = \sum_{\text{out}} I \]

**Loop rule** (conservation of energy): The sum of the changes in potential around a closed loop is zero.

\[ \sum \Delta V = 0 \]
ICLICKER QUESTION

What is the current in branch P?

a)  2 A  
b)  3 A  
c)  5 A  
d)  6 A  
e)  10 A  

Diagram: 
- Current entering: 5 A down, 8 A up, 2 A right
- Current leaving: 6 A down, 10 A right
Problem Solving Using Kirchhoff’s Rules

1. Label each current (if the label direction is wrong you will get a negative current after solving the equations which means the current flows in the opposite direction).

2. Identify unknowns.

3. Apply junction (conservation of charge) and loop (conservation of energy) rules (for a full solution you will need as many independent equations as there are unknowns).

4. Solve the equations, being careful with signs.
Iclicker Question

Which of the equations is valid for the circuit below?

a) \(2 - I_1 - 2I_2 = 0\)
b) \(2 - 2I_1 - 2I_2 - 4I_3 = 0\)
c) \(2 - I_1 - 4 - 2I_2 = 0\)
d) \(I_3 - 4 - 2I_2 + 6 = 0\)
e) \(2 - I_1 - 3I_3 - 6 = 0\)
The circuit shown is connected to a 25 A circuit breaker. If all devices are turned on at the same time will it trip the circuit breaker?

a) Yes  
b) No  
c) More information is needed.
ICLICKER QUESTION

What happens to the voltage across the resistor R1 when the switch is closed? The voltage will:

a) increase
b) decrease
c) stay the same
Example: Find the current through each resistor

\[
\sum_{\text{in}} I = \sum_{\text{out}} I
\]

At c: \( I_1 + I_2 = I_3 \)

\[
\sum_{\text{Closed Loop}} \Delta V = 0
\]

febadcf loop: \( 14 + 4I_2 + 2I_3 = 0 \)

bcdab loop: \( 10 - 6I_1 - 2I_3 = 0 \)
The lightbulbs in the circuit are identical. When the switch is closed, what happens?

a) both bulbs go out
b) intensity of both bulbs increases
c) intensity of both bulbs decreases
d) A gets brighter and B gets dimmer
e) nothing changes