Example

Two identical charges spheres of mass \( m = 3.0 \times 10^{-2} \text{kg} \), hang in equilibrium by two strings of equal length \( \ell = 0.15 \text{ m} \). Each hanging sphere makes an angle \( \theta = 5^\circ \) (i.e. \( \theta_1 = \theta_2 = 5^\circ \)) with the vertical. Find the magnitude of the charge on each sphere (assume they are of equal magnitude).

\[
\begin{align*}
\Sigma F_x &= F_E - T \sin \theta = 0 \\
\Sigma F_y &= T \cos \theta - mg = 0 \\
r_{12} &= 2\ell \sin \theta \\
q &= 2\ell \sin \theta \sqrt{\frac{mg \tan \theta}{k}}
\end{align*}
\]
Lecture

• Review: Coulomb’s Law
• Electric Fields in Conductors
• Gauss’s Law
• Electric Potential Energy and Potential Difference
• Equipotential Lines and Surfaces
• Electric Potential Due to Point Charges
A metal ball hangs from the ceiling by an insulating thread. The ball is attracted to a positive-charged rod held near the ball. The charge of the ball must be:

a) positive
b) negative
c) neutral
d) positive or neutral
e) negative or neutral
ICLICKER QUESTION

What is the magnitude of the force $F_2$?

- a) 1.0 N
- b) 1.5 N
- c) 3.0 N
- d) 6.0 N
- e) 9.0 N

$F_1 = 3N$  
$F_2 = ?$
A proton and an electron are held apart a distance of 1 m and then released. Which particle has the larger acceleration at any one moment?

a) proton
b) electron
c) both the same
Electric Fields and Conductors

• The static electric field inside a conductor is zero—if it were not, the charges would move.

• The net charge on a conductor must be on its surface.
The electric field is perpendicular to the surface of a conductor—again, if it were not, charges would move.
Electric Flux

- Electric flux is proportional to the total number of field lines penetrating some surface:

\[ \Phi_E \propto |\vec{E}||\vec{A}| \]

The Symbol \( \propto \) means "proportional to"

- If the area is perpendicular to the electric field then:

\[ \Phi_E = |\vec{E}||\vec{A}| \]
• If the surface under consideration is not perpendicular to the field, the flux through it must be less than the product \( \Phi_E < EA \).

• To understand this consider the figure:

• **Observation:** The number of field lines going through the area \( A \) (front) is the same as the number of lines going through area \( A' \) (back).
Conclusion: Using the Pythagorean theorem $A' = A \cos \theta$. Then a general expression for the Electric Flux can be written as:

$$\Phi_E = E A \cos \theta = \vec{E} \cdot \vec{A}$$
Electric Flux through a Closed Surface

\[ \Phi_E = E_1 \Delta A_1 \cos \theta_1 + E_1 \Delta A_1 \cos \theta_1 + \cdots \]

\[ \Phi_E = \sum_{\text{All } i} E_i \Delta A_i \cos \theta_i \]
Gauss’s Law

The net number of field lines through the surface is proportional to the charge enclosed, and to the flux.

$$\Phi_{E_{\text{closed surface}}} = \sum_{\text{Closed Surface}} E_i \Delta A_i \cos \theta_i$$

Gauss Law: $$\Phi_{E_{\text{closed surface}}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

This law can be used to find the electric field in situations with a high degree of symmetry.
Note that $E$ does not depend on $x$! (i.e. $E$ will have the same magnitude at any $x$).

This equation can also be written in terms of the charge per unit area $\sigma = Q/A$:  
$$E = \frac{\sigma}{\varepsilon_0}$$
Electric Potential Energy and Potential Difference

- The electrostatic force is conservative so potential energy can be defined.
- Change in electric potential energy is negative of work done by electric force ($W_E = F_E d \cos \theta$):

$$PE_E = -F_E d \cos \theta$$

$$PE_{Eb} - PE_{Ea} = -qEd$$
Electric Potential

Electric potential is defined as potential energy per unit charge; analogous to definition of electric field as force per unit charge:

\[ V_a = \frac{P E_a}{q} \]

Unit of electric potential: the volt (V).

\[ 1 \text{ V} = 1 \text{ J/C}. \]
Only changes in potential can be measured, allowing free assignment of $V = 0$.

$$V_{ba} = V_b - V_a = \frac{PE_b - PE_a}{q} = -\frac{W_{ba}}{q}$$
Electric Potential Energy and Potential Difference

Analogy between gravitational and electrical potential energy. Just as a more massive rock has more potential energy than a small one under the same gravitational potential, so does the larger charge under the same electric potential.
In general, the electric field in a given direction at any point in space is equal to the rate at which the electric potential decreases over distance in that direction.

\[ |\vec{E}| = -\frac{V_{ba}}{d} \]  

[uniform \( \vec{E} \)]
Equipotential Lines and Surfaces

• An equipotential is a line or surface over which the potential is constant.

• Electric field lines are perpendicular to equipotentials.

• The surface of a conductor is an equipotential.
Equipotential Lines and Surfaces

Equipotential lines of an electric dipole:
The Electron Volt, a Unit of Energy

One electron volt (eV) is the energy gained by an electron moving through a potential difference of one volt.

\[ 1\text{eV} = 1.602 \times 10^{-19}\text{J} \]
Electric Potential Due to Point Charges

The electric potential due to a point charge can be derived using calculus.

\[ V = k \frac{Q}{r} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \]
ICLICKER QUESTION

What is the electric potential at point B?

Electric Potential for a point charge: \( V = k \frac{Q}{r} \)

a) \( V > 0 \)

b) \( V = 0 \)

c) \( V < 0 \)
ICLICKER QUESTION

Four point charges are arranged at the corners of a square. Find the electric field $E$ and the potential $V$ at the center of the square.

a) $E = 0$  $V = 0$

b) $E = 0$  $V \neq 0$

c) $E \neq 0$  $V \neq 0$

d) $E \neq 0$  $V = 0$

e) $E = V$ regardless of the value
Example

Find the total potential energy of the system shown.

\[ P_E = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \]
Example: Flux Through a Cube of Side “L”

A point charge $q=6.0\, \text{C}$ is placed at the center of a cube of side $L=2.0\, \text{cm}$. Determine the electric flux one side of the cube.

$$\Phi_E = \frac{6.0\, \text{C}}{\varepsilon_0}$$

**Flux through a single side:**
$$\Phi_{\text{one side}} = \frac{1.0\, \text{C}}{\varepsilon_0}$$