A mass oscillates on a horizontal spring with period $T = 2.0 \text{ s}$. What is the frequency?

a) Impossible to determine.

b) $0.50 \text{ Hz}$

c) $1.0 \text{ Hz}$

d) $2.0 \text{ Hz}$

e) $4.0 \text{ Hz}$
Lecture

- Simple Harmonic Motion
- Review: Spring Potential Energy
- Simple Pendulum
- Damped/Driven Harmonic Motion
Periodic Motion

The motion of an object is called periodic if:

- It vibrates or oscillates back and forth over the same path
- Each cycle takes the same amount of time
Simple Harmonic Motion: Spring Oscillations

- There is a point where the spring is neither stretched nor compressed; this is the equilibrium position.

- We measure displacement from that (equilibrium) point (i.e. $x = 0$ on the figure).

- The force exerted by the spring depends on the displacement:

  $$ \vec{F} = -k\vec{x} $$
Spring Oscillations

• The minus sign on the force indicates that it is a restoring force; it is directed to restore the mass to its equilibrium position.

• $k$ is the spring constant

• The force is not constant (e.g. $F=0$ for $x=0$), so the acceleration is not constant either
Spring Oscillations

- **Displacement** is measured from the equilibrium point
- **Amplitude** (A) is the maximum displacement
- **Period** is the time required to complete one cycle
- **Frequency** is the number of cycles completed per second
A typical earthquake produces vertical oscillations of the earth. Suppose a particular quake oscillates the ground at a frequency of 0.15 Hz. As the earth moves up and down, what time elapses between the highest point of the motion and the lowest point?

a) Impossible to determine.

b) 1.0 s

c) 3.3 s

d) 6.7 s

e) 0.13 s
Example: Hanging mass on a Spring

- If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

At equilibrium Position: 
\[ k\Delta x = mg \]
A block oscillates on a vertical spring. When the block is at the lowest point of the oscillation, the direction of it’s acceleration vector $\vec{a}_y$ is

a) Down
b) Zero
c) Up
Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.
A spring that is either stretched or un-stretched has stored energy that can be released as kinetic energy.
Review: Spring Potential Energy

• The force increases as the spring is stretched or compressed further.

• We find that the potential energy of the compressed or stretched spring, measured from its equilibrium position, can be written as: \[ PE_s = \frac{1}{2} kx^2 \]
Energy in Simple Harmonic Motion

The potential energy of a spring is given by:

\[ P_{Es} = \frac{1}{2} kx^2 \]

The total mechanical energy is then:

\[ E_{Mech} = PE + KE = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]
Energy in Simple Harmonic Motion

- If the mass is at the limits of its motion, the energy is all potential.
- If the mass is at the equilibrium point, the energy is all kinetic.
- The mechanical at the turning points (i.e. where $x=A$) is:

$$E_{Mech} = \frac{1}{2} m(0)^2 + \frac{1}{2} kA^2 = \frac{1}{2} kA^2$$
Energy in Simple Harmonic Motion

Because the mechanical energy is conserved (i.e. in the absence of friction and air resistance). We can write:

\[ E_{Mech} = \frac{1}{2} kA^2 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]

This allows us to solve for the velocity as a function of position:

\[ v = \pm v_{\max} \sqrt{1 - \frac{x^2}{A^2}} \]

where

\[ v_{\max} = A \sqrt{\frac{k}{m}} \]
Observation: If we look at the projection onto the $x$ axis of an object moving in a circle of radius $A$, at a constant speed $v_{\text{max}}$, we find that the $x$ component of its velocity varies as:

$$v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}}$$

This is identical to SHM.
The Period and Sinusoidal Nature of SHM

Therefore, we can use the period and frequency of a particle moving in uniform circular motion (UCM) to find the period and frequency in SHM:

\[ T = \frac{2\pi A}{v_{\text{max}}} \]

\[ T = \frac{2\pi A}{A\sqrt{\frac{k}{m}}} \]

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

\[ f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]
The Period and Sinusoidal Nature of SHM

We can similarly find the position as a function of time:

\[ x(t) = A\cos(\omega t) \]

Or in terms of the frequency (f):

\[ x(t) = A\cos(2\pi f t) \]

Or period (T):

\[ x(t) = A\cos(2\pi t/T) \]
The Period and Sinusoidal Nature of SHM

• The top curve is a graph of the previous equation.

• The bottom curve is the same, but shifted ¼ period so that it is a sine function rather than a cosine.
ICLICKER QUESTION

A mass oscillates on a horizontal spring. It’s velocity is $v$ and the spring exerts force $F$. At the time indicated by the arrow,

a) $v$ is + and $F$ is +
b) $v$ is + and $F$ is –
c) $v$ is – and $F$ is 0
d) $v$ is 0 and $F$ is +
e) $v$ is 0 and $F$ is –
The velocity and acceleration can be calculated as functions of time:

\[ v(t) = -v_{\text{max}} \sin(\omega t) \]

\[ a(t) = \frac{F}{m} = -\frac{kx}{m} = -\left(\frac{kA}{m}\right) \cos(\omega t) \]
The Simple Pendulum

A simple pendulum consists of a mass at the end of a lightweight cord.

We assume that the cord does not stretch, and that its mass is negligible.
A pendulum is pulled to the side and released. The mass swings to the right as shown. The diagram shows positions for half of a complete oscillation. At which point is the speed the highest?
• In order to be in SHM, the restoring force must be proportional to the negative of the displacement.

• Looking at the FBD we see that $|\vec{F}| = -mg \sin \theta$ which is proportional to $\sin \theta$ and not to $\theta$ itself.

• However, if the angle is small, $\sin \theta \approx \theta$. 

The Simple Pendulum
The Simple Pendulum

Therefore, for small angles, the force is approximately proportional to the angular displacement.

The period and frequency are:

\[ T = 2\pi \sqrt{\frac{\ell}{g}} \quad [\theta \text{ small}] \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{\ell}} \quad [\theta \text{ small}] \]
A ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the ball is replaced with another ball having twice the mass, the period will be

a) 1.0 s  
b) 1.4 s  
c) 2.0 s  
d) 2.8 s  
e) 4.0 s

\[ T = 2\pi \sqrt{\frac{\ell}{g}} \quad [\theta \text{ small}] \]
The Simple Pendulum

Observation: As long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.
Damped Harmonic Motion

• Damped harmonic motion is harmonic motion with a frictional or drag force.

• If the damping is small, we can treat it as an “envelope” that modifies the undamped oscillation.
However, if the damping is large, it no longer resembles SHM at all.

- **A: underdamping**: there are a few small oscillations before the oscillator comes to rest.
- **B: critical damping**: this is the fastest way to get to equilibrium.
- **C: overdamping**: the system is slowed so much that it takes a long time to get to equilibrium.
Damped Harmonic Motion

• There are systems where damping is unwanted, such as clocks and watches.

• Then there are systems in which it is wanted, and often needs to be as close to critical damping as possible, such as automobile shock absorbers and earthquake protection for buildings.
Forced Oscillations; Resonance

- Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency of the system.

- If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called resonance.
Forced Oscillations; Resonance

- The sharpness of the resonant peak depends on the damping.
- If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.
- Like damping, resonance can be wanted or unwanted.