Lecture

• Review: Simple Harmonic Motion
• Mechanical Waves (Transverse, Longitudinal, Standing Waves)
• Characteristics of Sound
• The Doppler Effect
• Some Applications
A poorly design device has two bolts that almost touch each other in the interior. One made of steel and one made of brass as shown in the figure. If the initial gap, at a temperature 80.6°F is 5.0µm, at what temperature will the bolts touch? ($\alpha_{\text{Brass}} = 19 \times 10^{-6}/^\circ C$, $\alpha_{\text{Steel}} = 11 \times 10^{-6}/^\circ C$)

\[ \ell = \ell_0 + \ell_0 \alpha \Delta T \]

or

\[ \Delta \ell = \ell_0 \alpha \Delta T \]

a) 7.4 °C
b) 13.3 °F

\[ \boxed{c) \ 93.9 \ °F} \]

d) 88.0 °C
e) 0 K
The Period and Sinusoidal Nature of SHM

\[ x(t) = A\cos(\omega t) \]
\[ v(t) = -A\omega \sin(\omega t) \]
\[ a(t) = -A\omega^2 \cos(\omega t) \]

Note that \( A\omega = A2\pi f = A\frac{2\pi}{T} = v_{\text{max}} \), and \( A\omega^2 = a_{\text{max}} \).
Damped harmonic motion is harmonic motion with a frictional or drag force.

If the damping is small, we can treat it as an “envelope” that modifies the undamped oscillation.
However, if the damping is large, it no longer resembles SHM at all.

- **A: underdamping**: there are a few small oscillations before the oscillator comes to rest.
- **B: critical damping**: this is the fastest way to get to equilibrium.
- **C: overdamping**: the system is slowed so much that it takes a long time to get to equilibrium.
Forced Oscillations; Resonance

- **Forced vibrations occur when there is a periodic driving force.** This force may or may not have the same period as the natural frequency of the system.

- **If the frequency is the same as the natural frequency,** the amplitude becomes quite large. This is called resonance.
Forced Oscillations; Resonance

• The sharpness of the resonant peak depends on the damping.

• If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.

• Like damping, resonance can be wanted or unwanted.
A wave travels along its medium, but the individual particles just move up and down.
Types of Waves: Transverse and Longitudinal

We identify two types of waves depending on the motion of particles in the wave.

- **Transverse (a):** particle motion is perpendicular to the wave direction
- **Longitudinal (b):** particle motion is parallel to the wave.
Types of Waves: Transverse and Longitudinal

Sound waves are longitudinal waves.

Drum membrane  Compression  Expansion
All types of traveling waves transport energy.

• Study of a single wave pulse shows that it is begun with a vibration and transmitted through internal forces in the medium.

• Continuous waves start with vibrations too. If the vibration is Simple Harmonic Motion, then the wave will be sinusoidal.
Consider a wave on a string moving to the right, as shown below. What is the direction of the velocity of a particle at the point labeled A?

a) 

b) 

c) 

d) 

e) zero
ICLICKER QUESTION

Consider a wave on a string moving to the right, as shown below. What is the direction of the velocity of a particle at the point labeled A?

a) 

b) 

c) 

d) 

e) zero
Wave Motion

Wave characteristics:

- **Amplitude (A):** The maximum amount of displacement of a particle from its equilibrium (rest) position.

- **Wavelength, \( \lambda \):** Distance between successive crests of a wave.

- **Frequency \( f \):** Number of crests passing through a given point in one second.

- **Period \( T \):** Time elapsed between successive crests.

- **Wave velocity:** \( \nu = \lambda f \)
Standing Waves; Resonance

• Standing waves occur when both ends of a string are fixed.

• In that case, only waves which are motionless at the ends of the string can persist.

• There are nodes, where the amplitude is always zero, and antinodes, where the amplitude varies from zero to the maximum value.
Standing Waves; Resonance

The frequencies of the standing waves on a particular string are called resonant frequencies.

They are also referred to as the fundamental and harmonics.
ICLICKER QUESTION

What is the wavelength of the first harmonic?

a) $\frac{\ell}{2}$

b) $\ell$

c) $2\ell$

d) $\frac{3\ell}{2}$
The frequencies of the standing waves on a particular string are called resonant frequencies. They are also referred to as the fundamental and harmonics.
Standing Waves; Resonance

The wavelengths of standing waves are:

\[ \lambda_n = \frac{2\ell}{n} \quad n = 1, 2, 3, ... \quad \text{[string fixed at both ends]} \]

The frequencies of standing waves are:

\[ f_n = \frac{v}{\lambda_n} \]

\[ \frac{v}{\lambda_n} = n \frac{v}{2\ell} = nf_1 \]

Frequency in terms of first harmonic \( f_1 \)

\[ f_n = nf_1 \quad n = 1, 2, 3, ... \]
Characteristics of Sound

• Sound can travel through any kind of matter, but not through a vacuum.

• The speed of sound is different in different materials; in general, it is slowest in gases, faster in liquids, and fastest in solids.

• The speed depends somewhat on temperature, especially for gases.

<table>
<thead>
<tr>
<th>Material</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>343</td>
</tr>
<tr>
<td>Air (0°C)</td>
<td>331</td>
</tr>
<tr>
<td>Helium</td>
<td>1005</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>1300</td>
</tr>
<tr>
<td>Water</td>
<td>1440</td>
</tr>
<tr>
<td>Sea water</td>
<td>1560</td>
</tr>
<tr>
<td>Iron and steel</td>
<td>≈ 5000</td>
</tr>
<tr>
<td>Glass</td>
<td>≈ 4500</td>
</tr>
<tr>
<td>Aluminum</td>
<td>≈ 5100</td>
</tr>
<tr>
<td>Hardwood</td>
<td>≈ 4000</td>
</tr>
<tr>
<td>Concrete</td>
<td>≈ 3000</td>
</tr>
</tbody>
</table>
Speed of Sound

• On an ideal gas:

\[ v_{snd} = \sqrt{\frac{\gamma k T}{m}} \]

\( \gamma = \frac{c_p}{c_v} \) is the ratio of specific heat capacity at constant pressure to that at constant volume, k is the Boltzmann constant \((1.38 \times 10^{-23} \text{ } m^2 \text{kg/s}^2 \text{K})\) and m is the mass of a single molecule.

• Liquids:

\[ v_{snd} = \sqrt{\frac{B_{ad}}{\rho}} \]

\( B_{ad} \) is the bulk modulus of the liquid.

• Solid bar:

\[ v_{snd} = \sqrt{\frac{Y}{\rho}} \]

Y is Young’s modulus
Characteristics of Sound

**Loudness:** related to the intensity of the sound wave

**Pitch:** related to frequency (frequency depends on source).

**Audible range:** about 20 Hz to 20,000 Hz; upper limit decreases with age

**Ultrasound:** above 20,000 Hz

**Infrasound:** below 20 Hz
When a sound wave passes from air into water, what properties of the wave will change?

a) the frequency $f$

b) the wavelength $\lambda$

c) the speed of the wave $v_{\text{wave}}$

d) both $f$ and $\lambda$

e) both $v_{\text{wave}}$ and $\lambda$
Energy Transported by Waves

- The energy transported by a wave is proportional to the square of the amplitude.

- **Intensity:** Power per unit area. \[ I = \frac{P}{A} \]

- Intensity is also proportional to the square of the amplitude: \( I \propto A^2 \)
Energy Transported by Waves

- If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is spherical.

Just from geometrical considerations, as long as the power output is constant, we see: $I \propto \frac{1}{r^2}$

$$I = \frac{P}{4\pi r^2} \text{ spherical wave}$$
Intensity of Sound: Decibels

The loudness of a sound is much more closely related to the logarithm of the intensity.

Sound level is measured in decibels (dB) and is defined:

$$\beta (in\ dB) = 10 \log \frac{I}{I_0}$$

$I_0$ is taken to be the threshold of hearing: $I_0 = 1.0 \times 10^{-12}$ W/m$^2$
**Example: Sound Level**

Consider two **identical machines** positioned at equal distances from a person. The intensity of sound measured for each machine at the person’s location is $2.0 \times 10^{-7} W/m^2$.

What is the sound level heard by the person when one machine is operating?

\[
\beta (in \ dB) = 10 \log \frac{I}{I_0}
\]

\[
I_0 = 1.0 \times 10^{-12} \ W/m^2
\]

\[
\beta = 10 \log \left( \frac{2.0 \times 10^{-7} \ W/m^2}{1.0 \times 10^{-12} \ W/m^2} \right) = 53 dB
\]
Consider two identical machines positioned at equal distances from a person. The sound level heard at the person’s position when one machine is operating is 53 dB.

What is the sound level heard by the person when both machines are operating?

\[ \beta (in \, dB) = 10 \log \frac{I}{I_0} \]

\[ I_0 = 1.0 \times 10^{-12} \, W/m^2 \]

\[ \beta = 10 \log \left( \frac{2 \times 2.0 \times 10^{-7} \, W/m^2}{1.0 \times 10^{-12} \, W/m^2} \right) = 56 dB \]

a) 106 dB  
b) 53 dB  
c) **56 dB**  
d) 82 dB  
e) 3 dB
Doppler Effect

The Doppler effect occurs when a source of sound is moving with respect to an observer.
Doppler Effect

As can be seen in the image:

• A source moving toward an observer has a higher frequency and shorter wavelength.

• The opposite is true when a source is moving away from an observer.
If we can figure out what the change in the wavelength ($\lambda$) is, we also know the change in the frequency.
The change in the wavelength is given by:

1. \[ \lambda' = d - d_{source} \]
2. \[ \lambda' = \lambda - v_{source}T \]
3. \[ \lambda' = \lambda - v_{source} \frac{\lambda}{v_{snd}} \]
4. \[ \lambda' = \lambda \left(1 - \frac{v_{source}}{v_{snd}} \right) \]
Doppler Effect

And the change in the frequency:

If the source is moving away from the observer:

\[
f' = \frac{f}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)}.
\]

[ source moving away from stationary observer ]

If the source is moving toward the observer:

\[
f' = \frac{f}{\left(1 + \frac{v_{\text{source}}}{v_{\text{snd}}}\right)}.
\]

[ source moving toward stationary observer ]
If the observer is moving with respect to the source, things are a bit different. The wavelength remains the same, but the wave speed is different for the observer.
Doppler Effect

We find, for an observer moving towards a stationary source:

\[ f' = \left(\frac{v_{\text{snd}} + v_{\text{obs}}}{v_{\text{snd}}}\right)f. \]

or

\[ f' = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f. \]

[observer moving toward stationary source]

And if it is moving away:

\[ f' = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f. \]

[observer moving away from stationary source]
General Doppler Effect Expression

\[ f' = \frac{f}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}} \right)} \text{ [source moving toward stationary observer]} \]

\[ f' = \frac{f}{\left(1 + \frac{v_{\text{source}}}{v_{\text{snd}}} \right)} \text{ [source moving away from stationary observer]} \]

\[ f' = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}} \right)f \text{ [observer moving toward stationary source]} \]

\[ f' = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}} \right)f \text{ [observer moving away from stationary source]} \]

**General D.S.Eq.**  
\[ f' = \left(\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}} - v_{\text{source}}} \right)f \]

The signs for \( v_{\text{observer}} \) and \( v_{\text{source}} \) depend on the direction of the velocity. A positive value used for motion of the observer or source toward the other and a negative for motion away from the other.
Example: Doppler Effect

You physics professor is awoken by the irritating sound (f = 600Hz) of his alarm clock. Tired of the annoying sound he throws the clock of a 10\textsuperscript{th} floor window 40.0m off the ground.

What frequency will he hear at the moment just before the clock hits the ground? (Assume the speed of sound is 343.0m/s)

\[ f' = f \left( \frac{v_{snd} + v_{obs}}{v_{snd} - v_{src}} \right) \]

\[ f' = f \left( \frac{343.0 + 0}{343.0 - (-\sqrt{2g\Delta y})} \right) = 555\text{Hz} \]
ICLICKER QUESTION

Microwaves travel with the speed of light, \( c = 3 \times 10^8 \text{ m/s} \). At a frequency of 10 GHz these waves cause the water molecules in your burrito to vibrate. What is their wavelength?

a) 0.3 mm  

b) 3 cm  

c) 30 cm  

d) 300 m  

e) 3 km

- Amplitude, \( A \)
- Wavelength, \( \lambda \)
- Frequency \( f \) and period \( T \)
- Wave velocity \( v = \lambda f \).