Lecture

• Wave Motion
• Mechanical Waves (Transverse, Longitudinal, Standing Waves)
• Characteristics of Sound, The Doppler Effect
• Shock Waves
• Speed of waves on a string of mass m.
• Principle of superposition and interference of sound waves.
• Beats
Consider a wave on a string moving to the right, as shown below. What is the direction of the velocity of a particle at the point labeled B?

a) →
b) ↓
c) ↓
d) Zero
e) ↑
Wave Motion

(a)

(b)

(c)

(d)
Wave Motion

Wave characteristics:

- **Amplitude (A):** The maximum amount of displacement of a particle from its equilibrium (rest) position.

- **Wavelength, $\lambda$:** Distance between successive crests of a wave.

- **Frequency $f$:** Number of crests passing through a given point in one second.

- **Period $T$:** Time elapsed between successive crests.

- **Wave velocity:** $v = \lambda f$
Standing waves occur when both ends of a string are fixed.

In that case, only waves which are motionless at the ends of the string can persist.

There are nodes, where the amplitude is always zero, and antinodes, where the amplitude varies from zero to the maximum value.
Standing Waves; Resonance

• The frequencies of the standing waves on a particular string are called resonant frequencies.

• They are also referred to as the fundamental and harmonics.
Polling Question

What is the wavelength of the first harmonic?

a) $\frac{\ell}{2}$

b) $\ell$

c) $2\ell$

d) $\frac{3\ell}{2}$
Standing Waves; Resonance

• The frequencies of the standing waves on a particular string are called resonant frequencies.

• They are also referred to as the fundamental and harmonics.

\[ \lambda_1 = 2\ell \]

Fundamental or first harmonic, \( f_1 \)

\[ \lambda_2 = \ell \]

First overtone or second harmonic, \( f_2 \)

\[ \lambda_1 = \frac{2}{3}\ell \]

Second overtone or third harmonic, \( f_3 \)
Standing Waves; Resonance

The wavelengths of standing waves are:

$$\lambda_n = \frac{2\ell}{n} \quad n = 1, 2, 3, ...$$

The frequencies of standing waves are:

$$f_n = \frac{v}{\lambda_n} = n \frac{v}{2\ell} = nf_1$$

Frequency in terms of first harmonic $f_1$

$$f_n = nf_1 \quad n = 1, 2, 3, ...$$
Characteristics of Sound

- Sound can travel through any kind of matter, but not through a vacuum.

- The speed of sound is different in different materials; in general, it is slowest in gases, faster in liquids, and fastest in solids.

- The speed depends somewhat on temperature, especially for gases.

<table>
<thead>
<tr>
<th>Material</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>343</td>
</tr>
<tr>
<td>Air (0°C)</td>
<td>331</td>
</tr>
<tr>
<td>Helium</td>
<td>1005</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>1300</td>
</tr>
<tr>
<td>Water</td>
<td>1440</td>
</tr>
<tr>
<td>Sea water</td>
<td>1560</td>
</tr>
<tr>
<td>Iron and steel</td>
<td>≈5000</td>
</tr>
<tr>
<td>Glass</td>
<td>≈4500</td>
</tr>
<tr>
<td>Aluminum</td>
<td>≈5100</td>
</tr>
<tr>
<td>Hardwood</td>
<td>≈4000</td>
</tr>
<tr>
<td>Concrete</td>
<td>≈3000</td>
</tr>
</tbody>
</table>
Speed of Sound

• On an ideal gas:

\[ v_{snd} = \sqrt{\frac{\gamma kT}{m}} \]

\( \gamma = \frac{c_p}{c_v} \) is the ratio of specific heat capacity at constant pressure to that at constant volume, \( k \) is the Boltzmann constant \((1.38 \times 10^{-23} m^2 kg/s^2 K)\) and \( m \) is the mass of a single molecule.

• Liquids:

\[ v_{snd} = \sqrt{\frac{B_{ad}}{\rho}} \]

\( B_{ad} \) is the bulk modulus of the liquid.

• Solid bar:

\[ v_{snd} = \sqrt{\frac{Y}{\rho}} \]

\( Y \) is Young’s modulus
Characteristics of Sound

• **Loudness:** related to the intensity of the sound wave

• **Pitch:** related to frequency (frequency depends on source).

• **Audible range:** about 20 Hz to 20,000 Hz; upper limit decreases with age

  Ultrasound: **above 20,000 Hz**
  Infrasound: **below 20 Hz**
When a sound wave passes from air into water, what properties of the wave will change?

a) the frequency $f$

b) the wavelength $\lambda$

c) the speed of the wave $v_{wave}$

d) both $f$ and $\lambda$

e) both $v_{wave}$ and $\lambda$
Energy Transported by Waves

• The energy transported by a wave is proportional to the square of the amplitude.

• **Intensity:** Power per unit area. \[ I = \frac{Power}{Area} \]

• Intensity is also proportional to the square of the amplitude: \( I \propto A^2 \)
Energy Transported by Waves

• If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is spherical.

Just from geometrical considerations, as long as the power output is constant, we see: \( I \propto \frac{1}{r^2} \)

\[
I = \frac{P}{4\pi r^2} \quad \text{spherical wave}
\]
Intensity of Sound and Sound Level (Decibels)

- The loudness of a sound is much more closely related to the logarithm of the intensity.
- Sound level is measured in decibels (dB) and is defined:

\[ \beta (\text{in } dB) = 10 \log \frac{I}{I_0} \]

- \( I_0 \) is taken to be the threshold of hearing:

\[ I_0 = 1.0 \times 10^{-12} \text{ W/m}^2 \]

<table>
<thead>
<tr>
<th>Source of Sound</th>
<th>( \beta ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby jet airplane</td>
<td>150</td>
</tr>
<tr>
<td>Jackhammer; machine gun</td>
<td>130</td>
</tr>
<tr>
<td>Siren; rock concert</td>
<td>120</td>
</tr>
<tr>
<td>Subway; power mower</td>
<td>100</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>80</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>70</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>50</td>
</tr>
<tr>
<td>Mosquito buzzing</td>
<td>40</td>
</tr>
<tr>
<td>Whisper</td>
<td>30</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
</tbody>
</table>
Example:

Consider two identical machines positioned at equal distances from a person. The intensity of sound measured for each machine at the person’s location is $2.0 \times 10^{-7} \text{W/m}^2$. What is the sound level heard by the person when one machine is operating?

$$\beta (\text{in dB}) = 10 \log \frac{I}{I_0}$$

$I_0 = 1.0 \times 10^{-12} \text{W/m}^2$

$$\beta = 10 \log \left( \frac{2.0 \times 10^{-7} \text{W/m}^2}{1.0 \times 10^{-12} \text{W/m}^2} \right) = 53\text{dB}$$
Consider two identical machines positioned at equal distances from a person. The sound level heard at the person’s position when one machine is operating is 53 dB.

What is the sound level heard by the person when both machines are operating?

\[
\beta (\text{in dB}) = 10 \log \frac{I}{I_0}
\]

\[
I_0 = 1.0 \times 10^{-12} \text{ W/m}^2
\]

\[
\beta = 10 \log \left( \frac{2 \times 2.0 \times 10^{-7} \text{ W/m}^2}{1.0 \times 10^{-12} \text{ W/m}^2} \right) = 56 \text{dB}
\]

a) 106 dB
b) 53 dB
c) 56 dB
d) 82 dB
e) 3 dB
The Doppler effect occurs when a source of sound is moving with respect to an observer.
As can be seen in the image:

• A source moving toward an observer has a higher frequency and shorter wavelength.

• The opposite is true when a source is moving away from an observer.
If we can figure out what the change in the wavelength ($\lambda$) is, we also know the change in the frequency.

Doppler Effect

- Crest emitted when source was at point 1.
- Next crest emitted when source was at point 2.

\[ d_{\text{source}} = v_{\text{source}} T \]

(b) Source moving
Doppler Effect

The change in the wavelength is given by:

\[ \lambda' = d - d_{\text{source}} \]

\[ \lambda' = \lambda - v_{\text{source}} T \]

\[ \lambda' = \lambda - v_{\text{source}} \frac{\lambda}{v_{\text{snd}}} \]

\[ \lambda' = \lambda \left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}} \right) \]
Doppler Effect

And the change in the frequency:

$$f' = \frac{f}{1 - \frac{v_{\text{source}}}{v_{\text{snd}}}}.$$  
[source moving toward stationary observer]

If the source is moving away from the observer:

$$f' = \frac{f}{1 + \frac{v_{\text{source}}}{v_{\text{snd}}}}.$$  
[source moving away from stationary observer]
If the observer is moving with respect to the source, things are a bit different. The wavelength remains the same, but the wave speed is different for the observer.
Doppler Effect

We find, for an observer moving towards a stationary source:

\[ f' = \frac{(v_{snd} + v_{obs})f}{v_{snd}}, \]

or

\[ f' = \left(1 + \frac{v_{obs}}{v_{snd}}\right)f. \]

[observer moving toward stationary source]

And if it is moving away:

\[ f' = \left(1 - \frac{v_{obs}}{v_{snd}}\right)f. \]

(observer moving away from stationary source)
General Doppler Effect Expression

\[ f' = \frac{f}{1 - \frac{v_{\text{source}}}{v_{\text{snd}}}}. \quad \text{[source moving toward stationary observer]} \]

\[ f' = \frac{f}{1 + \frac{v_{\text{source}}}{v_{\text{snd}}}}. \quad \text{[source moving away from stationary observer]} \]

\[ f' = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f. \quad \text{[observer moving toward stationary source]} \]

\[ f' = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f. \quad \text{[observer moving away from stationary source]} \]

**General D.S. Eq.**

\[ f' = \left(\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}} - v_{\text{source}}}\right)f \]

The signs for \( v_{\text{observer}} \) and \( v_{\text{source}} \) depend on the direction of the velocity. A positive value used for motion of the observer or source toward the other and a negative for motion away from the other.
Example:
You physics professor is awoken by the irritating sound (f = 600Hz) of his alarm clock. Tired of the annoying sound he throws the clock of a 10th floor window 40.0m off the ground.
What frequency will he hear at the moment just before the clock hits the ground? (Assume the speed of sound is 343.0m/s)

\[ f' = f \left( \frac{v_{snd} + v_{obs}}{v_{snd} - v_{src}} \right) \]

\[ f' = f \left( \frac{343.0 + 0}{343.0 - (-\sqrt{2g\Delta y})} \right) = 555\text{Hz} \]
Microwaves travel with the speed of light, \( c = 3 \times 10^8 \text{ m/s}. \) At a frequency of 10 GHz these waves cause the water molecules in your burrito to vibrate. What is their wavelength?

a) 0.3 mm  

b) 3 cm  

c) 30 cm  

d) 300 m  

e) 3 km

- Amplitude, \( A \)
- Wavelength, \( \lambda \)
- Frequency \( f \) and period \( T \)
- Wave velocity \( v = \lambda f \).
What is the mode number of this standing wave?

- a) 4
- b) 5
- c) 6
- d) Can’t say without knowing what kind of wave it is.

(b) 5
A speaker emits a 400-Hz tone. The air temperature increases. The frequency of the sound ...
A speaker emits a 400-Hz tone. The air temperature increases. The wavelength of the sound ...

(a) Increases  
(b) Does not change  
(c) Decreases
Shock Waves and the Sonic Boom

- If a source is moving faster than the wave speed in a medium, waves cannot keep up and a shock wave is formed.

- The angle of the cone is:

\[
\sin \theta = \frac{v_{snd}}{v_{obj}}
\]
Shock Waves and the Sonic Boom

• Shock waves are analogous to the bow waves produced by a boat going faster than the wave speed in water.
Shock Waves and the Sonic Boom

- Aircraft exceeding the speed of sound in air will produce two sonic booms
- One from the front and one from the tail.
Speed of Waves on Strings

\[ v = \sqrt{\frac{T}{m/L}} \]

where the small angle approximation is used \( \sin \theta \approx \theta \) for small \( \theta \)

e.g. \( \sin(0.05\text{rad}) = 0.04998 \)