Lecture 11

• Review: Equilibrium
• Review: Phases of Matter
• Density and Pressure
• Pascal and Archimedes' Principles
• Fluid Dynamics: Equation of continuity, Bernoulli's Equation
A 1 kg ball is hung at the end of a rod 1 m long. If the system balances at a point on the rod 0.25 m from the end holding the mass, what is the mass of the rod?

a) 1/4 kg  
b) 1/2 kg  
c) 1 kg  
d) 2 kg  
e) 4 kg
ICLICKER QUESTION

Which condition(s) are required for equilibrium?

a) \( \Sigma F_{\text{ext}} = 0, \Sigma \tau_{\text{ext}} = 0 \)

b) \( \Sigma F_{\text{ext}} = 0, \Sigma \tau_{\text{ext}} = 0, v = 0, \omega = 0 \)

c) \( \Sigma F_{\text{ext}} = 0, \Sigma \tau_{\text{ext}} = 0, \Sigma F_{\text{internal}} = 0, \Sigma \tau_{\text{internal}} = 0 \)

d) It depends on the situation
An object is in equilibrium if:

\[ \sum F_{ext} = 0 \quad \text{and} \quad \sum \tau_{ext} = 0 \]
Equilibrium

Equilibrium

\[ \Sigma \vec{F}_{ext} = 0 \quad \Sigma \tau \neq 0 \]
Not in equilibrium

\[ \Sigma \vec{F}_{ext} = 0 \quad \Sigma \tau = 0 \]
In equilibrium

\[ 1960 \text{ N} \]

\[ 200 \text{ kg} \]
Statics Equilibrium Problems

Note that if there is a cable or cord in the problem, it can support forces only along its length (direction of the tension force). Forces perpendicular to that would cause it to bend.

\[ \sum \tau_{ext} = 0 \quad (\alpha = 0) \]
\[ \sum F_{ext} = 0 \quad (a = 0) \]
\[ \sum F_x = \cdots = 0, \sum F_y = \cdots = 0 \]
\[ \sum \tau_{ext} = LT \sin \theta - \left( \frac{L}{2} \right) m_{beam} g - L (m_{sign} g) = 0 \]
A box is placed on a ramp in the configurations shown below. Friction prevents it from sliding. The center of mass of the box is indicated by a dot in each case. In which case(s) does the box tip over?

a) all
b) 1 only
c) 2 only
d) 3 only
e) 2 and 3
Phases of Matter

The three common phases of matter are:

- Solid, liquid, and gas.

A solid has a definite shape and size.

A liquid has a fixed volume but can be any shape.

A gas can be any shape and also can be easily compressed.

Liquids and gases both flow, and are called fluids.
Density

The mass density $\rho$ of an object is its mass per unit volume:

$$\rho = \frac{m}{V}$$

- The SI unit for density is kg/m$^3$. Density is also sometimes given in g/cm$^3$; to convert g/cm$^3$ to kg/m$^3$, multiply by 1000.

Water at 4°C has a density of 1 g/cm$^3 = 1000$ kg/m$^3$. 
If one material has a higher density than another, does this mean that the molecules of the first material must be more massive than those of the second?

a) Yes

b) No
ICLICKER QUESTION

Consider what happens when you push both a pin and the blunt end of a pen against your skin with the same force. What will determine whether your skin will be punctured?

a) the pressure on your skin

b) the net applied force on your skin

c) both pressure and net applied force are equivalent

d) neither pressure nor net applied force are relevant here
Pressure in Fluids

Pressure is defined as the force per unit area:

\[ P = \frac{F}{A} \]

Pressure is a scalar; the units of pressure in the SI system are pascals:

1 Pa = 1 N/m²

Pressure is the same in every direction in a fluid at a given depth; if it were not, the fluid would flow.
The pressure at a depth \( h \) below the surface of the liquid is due to the weight of the liquid above it.

We can quickly calculate:

\[
P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho(hA)g}{A}
\]

Then the pressure at a depth “\( h \)” for a fluid of density “\( \rho \)” is given by:

\[
P = \rho gh
\]
Atmospheric Pressure and Gauge Pressure

• At sea level the atmospheric pressure is about $1.013 \times 10^5$ N/m$^2$; this is called one atmosphere (atm).

• Another unit of pressure is the bar:
  
  $1 \text{ bar} = 1.00 \times 10^5 \text{ N/m}^2$

• Standard atmospheric pressure is just over 1 bar.

• This pressure does not crush us, as our cells maintain an internal pressure that balances it.
Atmospheric Pressure and Gauge Pressure

• Most pressure gauges measure the pressure above the atmospheric pressure—this is called the gauge pressure.

• The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

\[ P = P_A + P_G \]
Pascal’s Principle

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

This principle is used, for example, in hydraulic lifts and hydraulic brakes.
Example: Pascal’s Principle

You exert force on a piston with circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston with a radius of 5.0 m. What force must you apply to lift a car weighing 13.3 kN?

\[
\frac{F_{\text{in}}}{A_{\text{in}}} = \frac{F_{\text{out}}}{A_{\text{out}}} \rightarrow F_{\text{in}} = F_{\text{out}} \frac{A_{\text{in}}}{A_{\text{out}}}
\]

\[
F_{\text{in}} = 13300N \frac{\pi (0.05m)^2}{\pi (5.0m)^2} = 1.33N
\]
Measurement of Pressure; Gauges

Here are two more devices for measuring pressure: the aneroid gauge and the tire pressure gauge.

(b) Aneroid gauge (used mainly for air pressure, and then called an aneroid barometer)

(c) Tire gauge
Measurement of Pressure; The Barometer

This is a mercury barometer, developed by Torricelli to measure atmospheric pressure.

- The height of the column of mercury is such that the pressure in the tube at the surface level is 1 atm.
- Therefore, pressure is often quoted in millimeters (or inches) of mercury.
Buoyancy and Archimedes’ Principle

This is an object submerged in a fluid. There is a net force on the object because the pressures at the top and bottom of it are different.

The buoyant force is found to be the upward force on the same volume of fluid:

\[ F_B = F_2 - F_1 = P_1A - P_2A \]
\[ = \rho_F g h_1 A - \rho_F g h_2 A \]
\[ = \rho_F g A \Delta h = \rho_F g V \]
\[ F_B = m_F g \]
The net force on the object is then the difference between the buoyant force and the gravitational force.
Buoyancy and Archimedes’ Principle

If the object’s density is less than that of water, there will be an upward net force on it, and it will rise until it is partially out of the water.
Buoyancy and Archimedes’ Principle

For a floating object, the fraction that is submerged is given by the ratio of the object’s density to that of the fluid.

\[ \Sigma F_y = F_B - m_O g = 0 \]
\[ \Sigma F_y = \rho_F V_{displaced} g - \rho_O V_O g = 0 \]

\[ F_B = \rho_F V_{displ} g \]
\[ m_O g = \rho_O V_O g \]
Buoyancy and Archimedes’ Principle

This principle also works in the air; this is why hot-air and helium balloons rise.
When a hole is made in the side of a Coke can holding water, water flows out and follows a parabolic trajectory. If the container is dropped in free fall, the water flow will:

a) diminish
b) stop altogether
c) go out in a straight line
d) curve upwards
If the flow of a fluid is smooth, it is called streamline or laminar flow (a).

Above a certain speed, the flow becomes turbulent (b).

Turbulent flow has eddies; the viscosity of the fluid is much greater when eddies are present.
We will deal with laminar flow:

• *The mass flow rate is the mass that passes a given point per unit time.*

• *The flow rates at any two points must be equal, as long as no fluid is being added or taken away.*

This gives us the equation of continuity:

\[
\rho_1 A_1 v_1 = \rho_2 A_2 v_2
\]
Fluids in Motion; Flow Rate and the Equation of Continuity

If the density doesn’t change—typical for liquids—this simplifies to \( A_1 v_1 = A_2 v_2 \).

\( \rightarrow \) Where the pipe is wider, the flow is slower.
Bernoulli’s Equation

- A fluid can also change its height.
- By looking at the work done as it moves, we find:

\[
P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1
\]

This is Bernoulli’s equation. One thing it tells us is that as the speed goes up, the pressure goes down.
Using Bernoulli’s principle, we find that the speed of fluid coming from a spigot on an open tank is:

\[ \frac{1}{2} \rho v_1^2 + \rho g y_1 = \rho g y_2 \]

or

\[ v_1 = \sqrt{2g(y_2 - y_1)} \]

This is called Torricelli’s theorem.
Lift on an airplane wing is due to the different air speeds and pressures on the two surfaces of the wing.

Applications of Bernoulli’s Principle: Airplanes
Applications of Bernoulli’s Principle: Torricelli, Airplanes, Baseballs, Blood Flow

A ball’s path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal.
Applications of Bernoulli’s Principle: Blood Flow

A person with constricted arteries will find that they may experience a temporary lack of blood to the brain as blood speeds up to get past the constriction, thereby reducing the pressure.