Lecture 7

• Review: Newton’s Laws, Friction, Circular Motion
• Work Done by a Constant Force
• The Work–Energy Theorem and Kinetic Energy
• Gravitational Potential Energy
• The Conservation of Mechanical Energy
• Non-conservative Forces and the Work–Energy Theorem
• Power
• Other Forms of Energy and the Conservation of Energy
• Work Done by a Variable Force
Scalar Product of Vectors

A scalar (or dot) product of two vectors is fully defined by:

\[ \hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1 \]

zero otherwise, \( i.e \) \( \hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = 0 \)

An important property of the dot product of vectors is: \( \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta \), where \( \theta \) is the angle that the vectors make with respect to each other \( (\theta \leq 180^\circ) \)
You’re on a Ferris wheel moving in a vertical circle. When the Ferris wheel is at rest, the normal force $N$ exerted by your seat is equal to your weight $mg$. How does $N$ change at the top of the Ferris wheel when you are in motion?

a) $N$ remains equal to $mg$

b) $N$ is smaller than $mg$

c) $N$ is larger than $mg$

d) None of the above
ICLICKER QUESTION

Which is stronger, Earth’s pull on the Moon, or the Moon’s pull on Earth?

a) the Earth pulls harder on the Moon
b) the Moon pulls harder on the Earth
c) they pull on each other equally
d) there is no force between the Earth and the Moon
e) it depends upon where the Moon is in its orbit at that time
The work done by a constant force $\vec{F}$ over a displacement $\vec{s}$ is equal to the magnitude of the displacement ($|\vec{s}|$) multiplied by the component of the force in the direction of displacement:

$$W = \vec{F} \cdot \vec{s} = (F \cos \theta)s$$

$\theta$ is the angle ($< 180^\circ$) that the displacement and force vectors make.
In the SI system, the units of work are joules ($J$):

$$1J = 1N \cdot m$$

**Example:**

A person walking with a bag of groceries at a constant speed. As long as this person does not lift or lower the bag of groceries, he is doing no work on it. The force he exerts has no component in the direction of motion.
ICLICKER QUESTION

Does a centripetal force do work?
(e.g. a tension on a string keeping a ball in circular motion)

a) Yes
b) No

c) It depends on the situation
Does a centripetal force do work?

Centripetal forces do no work, as they are always perpendicular to the direction of motion.
ICLICKER QUESTION

A box is being pulled across a rough floor at a constant speed. What can you say about the work done by friction?

a) friction does no work at all
b) friction does negative work

- friction does positive work
Can friction ever do positive work?

a) Yes
b) No
Work Done by a Varying Force

• For a force that varies, the work can be approximated by dividing the distance up into small pieces.
• Finding the work done during each, and adding them up.
• As the pieces become very narrow, the work done is the area under the force vs. distance curve.
Consider a car of mass $M$, accelerating over a distance $d$ by a constant, horizontal net force of magnitude $F$.

The net work done by the engine is: $W = Fd \cos \theta = Fd$

Using Newton’s 2\textsuperscript{nd} law ($|\vec{F}_{net}| = ma$) the work done by the engine can be written as: $W = mad$

What can we say about the car’s acceleration?
Kinetic Energy and the Work-Energy Principle

The car moves at a constant acceleration (i.e. constant net force “F”).

• Then the equation $v_2^2 = v_1^2 + 2ad$ applies in this case.

• Solving for $ad$ we get: $ad = \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$.

⇒ The work done by a net force is: $W = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$

The quantity $\frac{1}{2} mv^2$ is the Kinetic Energy. $KE = \frac{1}{2} mv^2$
This means that: \textit{the work done by a net force is equal to the change in the kinetic energy.}

\[ W_{\text{net}} = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \]

- If the net work is positive, the kinetic energy increases.
- If the net work is negative, the kinetic energy decreases.
By what factor does the kinetic energy of a car change when its speed is tripled?

a) no change at all
b) factor of 3
c) factor of 6
d) factor of 9
e) factor of 12
Potential Energy

An object can have potential energy by virtue of its surroundings.

Familiar examples of potential energy:

• A wound-up spring
• A stretched elastic band
• An object at some height above the ground
Gravitational Potential Energy

In raising a mass $m$ to a height $h$ ($y_2 - y_1$), the work done by the external force is

$$W_{ext} = \vec{F}_{ext} \cdot \vec{d} = F_{ext}d \cos(0^\circ) = mgh$$

We therefore define the gravitational potential energy:\n\[ PE_g = mgh \]
Observations

• Potential energy can become kinetic energy. For example, if the object is dropped.

• Potential energy is a property of a system as a whole, not just of the object (because it depends on external forces).

• If $PE_G = mgh$, where do we measure $y$ from?

  ➢ It turns out not to matter, as long as we are consistent about where we choose $y = 0$.

Only changes in potential energy can be measured.
Observation: If friction is present, the work done depends not only on the starting and ending points, but also on the path taken.

- Friction is an example of a nonconservative force.
### Examples of Conservative and Nonconservative Forces

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<th>Conservative Forces</th>
<th>Nonconservative Forces</th>
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<tr>
<td>Elastic</td>
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<tr>
<td>Electric</td>
<td>Tension in cord</td>
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<tr>
<td></td>
<td>Motor or rocket propulsion</td>
</tr>
<tr>
<td></td>
<td>Push or pull by a person</td>
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</tbody>
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Potential energy can only be defined for conservative forces.
Mechanical Energy and Its Conservation

If there are no non-conservative forces:

• the sum of the changes in the kinetic energy and in the potential energy is zero

• the kinetic and potential energy changes are equal but opposite in sign.

This allows us to define the total mechanical energy of a system:

\[ E_{mech} = KE + PE \]
Mechanical Energy and Its Conservation

If there are no non-conservative forces:

• The mechanical energy of the system is conserved:

$\Delta E = 0$

• (i.e. $E_1 = E_2 = E_3 = \cdots = E_N$)
Two stones, one twice the mass of the other, are dropped from a cliff. Just before hitting the ground, what is the kinetic energy of the heavy stone compared to the light one? (assume air resistance can be neglected)

a) quarter as much
b) half as much
c) the same
d) twice as much
e) four times as much
In the previous question, just before hitting the ground, what is the final speed of the heavy stone compared to the light one? (assume air resistance can be neglected)

a) quarter as much
b) half as much
c) the same
d) twice as much
e) four times as much
Example

The total mechanical energy is:

\[ E = KE + PE = \frac{1}{2}mv^2 + mgh \]

The energy buckets on the right of the figure show how the energy is transformed from all potential (at the top) to all kinetic (at the bottom).
ICLICKER QUESTION #1

Ignoring friction and air resistance which statement is true about the mechanical energies at points A, B, and C?

a) $E_A = E_B = E_C$

b) $E_A < E_B < E_C$

c) $E_A > E_B > E_C$

d) None of the above.
Example: Roller Coaster

The speed of the roller coaster at point A is $v_A = 1.0 \text{ m/s}$. What is the speed at point B?