Week 7 homework

IMPORTANT NOTE ABOUT WEBASSIGN:
In the WebAssign versions of these problems, various details have been changed, so that the answers will come out differently. The method to find the solution is the same, but you will need to repeat part of the calculation to find out what your answer should have been.

WebAssign Problem 1: The earth spins on its axis once a day and orbits the sun once a year (365¼ days). Determine the average angular velocity (in rad/s) of the earth as it (a) spins on its axis and (b) orbits the sun. In each case, take the positive direction for the angular displacement to be the direction of the earth’s motion.

REASONING The average angular velocity \( \bar{\omega} \) is defined as the angular displacement \( \Delta \theta \) divided by the elapsed time \( \Delta t \) during which the displacement occurs: \( \bar{\omega} = \Delta \theta / \Delta t \) (Equation 8.2). This relation can be used to find the average angular velocity of the earth as it spins on its axis and as it orbits the sun.

SOLUTION
a. As the earth spins on its axis, it makes 1 revolution (2\( \pi \) rad) in a day. Assuming that the positive direction for the angular displacement is the same as the direction of the earth’s rotation, the angular displacement of the earth in one day is \( (\Delta \theta )_{\text{spin}} = +2\pi \) rad. The average angular velocity is (converting 1 day to seconds):

\[
\bar{\omega} = \frac{(\Delta \theta )_{\text{spin}}}{(\Delta t)_{\text{spin}}} = \frac{+2\pi \text{ rad}}{1 \text{ day}} = \frac{24 \text{ h}}{1 \text{ day}} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 7.3 \times 10^{-5} \text{ rad/s}
\]

b. As the earth orbits the sun, the earth makes 1 revolution (2\( \pi \) rad) in one year. Taking the positive direction for the angular displacement to be the direction of the earth’s orbital motion, the angular displacement in one year is \( (\Delta \theta )_{\text{orbit}} = +2\pi \) rad. The average angular velocity is (converting 365¼ days to seconds):

\[
\bar{\omega} = \frac{(\Delta \theta )_{\text{orbit}}}{(\Delta t)_{\text{orbit}}} = \frac{+2\pi \text{ rad}}{365 \frac{1}{4} \text{ days}} = \frac{24 \text{ h}}{1 \text{ day}} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 2.0 \times 10^{-7} \text{ rad/s}
\]
**WebAssign Problem 2:** A CD has a playing time of 74 minutes. When the music starts, the CD is rotating at an angular speed of 480 revolutions per minute (rpm). At the end of the music, the CD is rotating at 210 rpm. Find the magnitude of the average angular acceleration of the CD. Express your answer in rad/s².

**REASONING AND SOLUTION** Using Equation 8.4 and the appropriate conversion factors, the average angular acceleration of the CD in rad/s² is

\[
\vec{\alpha} = \frac{\Delta \omega}{\Delta t} = \left( \frac{210 \text{ rev/min} - 480 \text{ rev/min}}{74 \text{ min}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)^2 = -6.4 \times 10^{-3} \text{ rad/s}^2
\]

The magnitude of the average angular acceleration is \(6.4 \times 10^{-3} \text{ rad/s}^2\).

**WebAssign Problem 3:** A baton twirler throws a spinning baton directly upward. As it goes up and returns to the twirler’s hand, the baton turns through four revolutions. Ignoring air resistance and assuming that the average angular speed of the baton is 1.80 rev/s, determine the height to which the center of the baton travels above the point of release.

**REASONING AND SOLUTION** The baton will make four revolutions in a time \(t\) given by

\[
t = \frac{\theta}{\omega}
\]

Half of this time is required for the baton to reach its highest point. The magnitude of the initial vertical velocity of the baton is then

\[
v_0 = g \left( \frac{1}{2} t \right) = g \left( \frac{\theta}{2\omega} \right)
\]

With this initial velocity the baton can reach a height of

\[
h = \frac{v_0^2}{2g} = \frac{g \theta^2}{8\omega^2} = \left( \frac{9.80 \text{ m/s}^2}{8} \right) \left( \frac{8 \text{ rad}}{1.80 \text{ rev/s}} \right)^2 = 6.05 \text{ m}
\]

**WebAssign Problem 4:** A person is riding a bicycle, and its wheels have an angular velocity of +20.0 rad/s. Then, the brakes are applied and the bike is brought to a uniform stop. During braking, the angular displacement of each wheel is +15.92 revolutions. (a) How much time does it take for the bike to come to rest? (b) What is the angular
acceleration (in rad/s²) of each wheel?

**REASONING**

a. The time $t$ for the wheels to come to a halt depends on the initial and final velocities, $\omega_0$ and $\omega$, and the angular displacement $\theta$: $\theta = \frac{1}{2}(\omega_0 + \omega)t$ (see Equation 8.6). Solving for the time yields

$$t = \frac{2\theta}{\omega_0 + \omega}$$

b. The angular acceleration $\alpha$ is defined as the change in the angular velocity, $\omega - \omega_0$, divided by the time $t$:

$$\alpha = \frac{\omega - \omega_0}{t} \quad (8.4)$$

**SOLUTION**

a. Since the wheel comes to a rest, $\omega = 0$ rad/s. Converting 15.92 revolutions to radians (1 rev = $2\pi$ rad), the time for the wheel to come to rest is

$$t = \frac{2\theta}{\omega_0 + \omega} = \frac{2\left(\frac{15.92 \text{ rev}}{1 \text{ rev}}\right) \frac{2\pi \text{ rad}}{1 \text{ rev}}}{20.0 \text{ rad/s} + 0 \text{ rad/s}} = 10.0 \text{ s}$$

b. The angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 \text{ rad/s} - 20.0 \text{ rad/s}}{10.0 \text{ s}} = -2.00 \text{ rad/s}^2$$

**WebAssign Problem 5:** At the local swimming hole, a favorite trick is to run horizontally off a cliff that is 8.3 m above the water. One diver runs off the edge of the cliff, tucks into a “ball”, and rotates on the way down with an average angular speed of 1.6 rev/s. Ignore air resistance and determine the number of revolutions she makes while on the way down.

**REASONING** According to Equation 3.5b, the time required for the diver to reach the water, assuming free-fall conditions, is $t = \sqrt{2y / a_y}$. If we assume that the "ball" formed by the diver is rotating at the instant that she begins falling vertically, we can use Equation 8.2 to calculate the number of revolutions made on the way down.

**SOLUTION** Taking upward as the positive direction, the time required for the diver to reach the water is
WebAssign Problem 6: An auto race is held on a circular track. A car completes one lap in a time of 18.9 s, with an average tangential speed of 42.6 m/s. Find (a) the average angular speed and (b) the radius of the track.

**REASONING AND SOLUTION**

a. In one lap, the car undergoes an angular displacement of $2\pi$ radians. Therefore, from the definition of average angular speed

\[
\omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi \text{ rad}}{18.9 \text{ s}} = 0.332 \text{ rad/s}
\]

b. Equation 8.9 relates the average angular speed $\omega$ of an object moving in a circle to its average linear speed $\bar{v}_T$ tangent to the path of motion:

\[
\bar{v}_T = r\omega
\]

Solving for $r$ gives

\[
r = \frac{\bar{v}_T}{\omega} = \frac{42.6 \text{ m/s}}{0.332 \text{ rad/s}} = 128 \text{ m}
\]

Notice that the unit "rad," being dimensionless, does not appear in the final answer.

WebAssign Problem 7: Refer to Multiple-Concept Example 7 for insight into this problem. During a tennis serve, a racket is given an angular acceleration of magnitude 160 rad/s$^2$. At the top of the serve, the racket has an angular speed of 14 rad/s. If the distance between the top of the racket and the shoulder is 1.5 m, find the magnitude of the total acceleration of the top of the racket.

**REASONING** The top of the racket has both tangential and centripetal acceleration components given by Equations 8.10 and 8.11, respectively: $a_T = r\alpha$ and $a_c = r\omega^2$. The total acceleration of the top of the racket is the resultant of these two components. Since these acceleration components are mutually perpendicular, their resultant can be found by using the Pythagorean theorem.
**SOLUTION** Employing the Pythagorean theorem, we obtain
\[ a = \sqrt{a_T^2 + a_c^2} = \sqrt{(r \alpha)^2 + (r \omega^2)^2} = r \sqrt{\alpha^2 + \omega^4} \]

Therefore,
\[ a = (1.5 \text{ m}) \sqrt{(160 \text{ rad/s}^2)^2 + (14 \text{ rad/s})^4} = 380 \text{ m/s}^2 \]

**WebAssign Problem 8:** A dentist causes the bit of a high-speed drill to accelerate from an angular speed of \(1.05 \times 10^4\) rad/s to an angular speed of \(3.14 \times 10^4\) rad/s. In the process, the bit turns through \(1.88 \times 10^4\) rad. Assuming a constant angular acceleration, how long would it take the bit to reach its maximum speed of \(7.85 \times 10^4\) rad/s, starting from rest?

**REASONING AND SOLUTION** The angular acceleration is found for the first circumstance.

\[ \alpha = \frac{\omega^2 - \omega_0^2}{2 \theta} = \frac{(3.14 \times 10^4 \text{ rad/s})^2 - (1.05 \times 10^4 \text{ rad/s})^2}{2[1.88 \times 10^4 \text{ rad}]} = 2.33 \times 10^4 \text{ rad/s}^2 \]

For the second circumstance

\[ t = \frac{\omega - \omega_0}{\alpha} = \frac{7.85 \times 10^4 \text{ rad/s} - 0 \text{ rad/s}}{2.33 \times 10^4 \text{ rad/s}^2} = 3.37 \text{ s} \]
Practice conceptual problems:

Chapter 8:

4. The earth rotates once per day about its axis. Where on the earth’s surface should you stand in order to have the smallest possible tangential speed? Justify your answer.

**REASONING AND SOLUTION** The tangential speed, $v_T$, of a point on the earth's surface is related to the earth's angular speed $\omega$ according to $v_T = r\omega$, Equation 8.9, where $r$ is the perpendicular distance from the point to the earth's rotation axis. At the equator, $r$ is equal to the earth's radius. As one moves away from the equator toward the north or south geographic pole, the distance $r$ becomes smaller. Since the earth's rotation axis passes through the geographic poles, $r$ is effectively zero at those locations. Therefore, your tangential speed would be a minimum if you stood as close as possible to either the north or south geographic pole.

6. A car is up on a hydraulic lift at a garage. The wheels are free to rotate, and the drive wheels are rotating with a constant angular velocity. Does a point on the rim of a wheel have (a) a tangential acceleration and (b) a centripetal acceleration? In each case, give your reasoning.

**REASONING AND SOLUTION** The wheels are rotating with a constant angular velocity.

a. Since the angular velocity is constant, each wheel has zero angular acceleration, $\alpha = 0$ rad/s. Since the tangential acceleration $a_T$ is related to the angular acceleration through Equation 8.10, $a_T = r\alpha$, every point on the rim has zero tangential acceleration.

b. Since the particles on the rim of the wheels are moving along a circular path, they must have a centripetal acceleration. This can be supported by Equation 8.11, $a_c = r\omega^2$, where $a_c$ is the magnitude of the centripetal acceleration. Since $\omega$ is nonzero, $a_c$ is nonzero.

11. Explain why a point on the rim of a tire has an acceleration when the tire is on a car that is moving at a constant linear velocity.

**REASONING AND SOLUTION** Any point on a rotating object possesses a centripetal acceleration that is directed radially toward the axis of rotation. This also applies to a tire on a moving car and is true regardless of whether the car has a constant linear velocity or whether it is accelerating.
Chapter 9:

2. Explain (a) how it is possible for a large force to produce only a small, or even zero, torque, and (b) how it is possible for a small force to produce a large torque.

**REASONING AND SOLUTION** The torque produced by a force $F$ is given by $\tau = F \ell$, where $\ell$ is the lever arm.

a. A large force can be used to produce a small torque by choosing a small lever arm. Then the product $F \ell$ is small even though the magnitude of the force, $F$, is large. If the line of action passes through the axis of rotation, $\ell = 0$ m, and the torque is zero regardless of how large $F$ is.

b. A small force may be used to produce a large torque by choosing a large lever arm $\ell$. Then, the product $F \ell$ can be large even though the magnitude of the force $F$ is small.

9. An A-shaped stepladder is standing on frictionless ground. The ladder consists of two sections joined at the top and kept from spreading apart by a horizontal crossbar. Draw a free-body diagram showing the forces that keep one section of the ladder in equilibrium.

**REASONING AND SOLUTION** The forces that keep the right section of the ladder in equilibrium are shown in the free-body diagram at the right.

They are: the weight of the section, $mg$, the normal force exerted on the ladder by the floor, $F_N$, the tension in the crossbar, $F_T$, and the reaction force due to the left side of the ladder, $F_R$.

11. A flat triangular sheet of uniform material is shown in the drawing. There are three possible axes of rotation, each perpendicular to the sheet and passing through one corner, A, B, or C. For which axis is the greatest net external torque required to bring the triangle up to an angular speed of 10.0 rad/s in 10.0 s, starting from rest? Explain, assuming that the net torque is kept constant while it is being applied.
The torque required to give the triangular sheet an angular acceleration $\alpha$ is given by the rotational analog of Newton's second law for a rigid body: $\sum \tau = I \alpha$. The greatest net torque will be required about the axis for which the moment of inertia $I$ is the greatest. The greatest value for $I$ occurs when the greatest portion of the mass of the sheet is far from the axis. This will be for axis B. Therefore, for axis B, the greatest net external torque is required to bring the rectangular plate up to 10.0 rad/s in 10.0 s, starting from rest.

15. For purposes of computing the translational kinetic energy of a rigid body, its mass can be considered as concentrated at the center of mass. If one wishes to compute the body’s moment of inertia, can the mass be considered as concentrated at the center of mass? If not, why not?

REASONING AND SOLUTION The translational kinetic energy of a rigid body depends only on the mass and the speed of the body. It does not depend on how the mass is distributed. Therefore, for purposes of computing the body's translational kinetic energy, the mass of a rigid body can be considered as concentrated at its center of mass.

Unlike the translational kinetic energy, the moment of inertia of a body does depend on the location and orientation of the axis relative to the particles that make up the rigid body. Therefore, for purposes of computing the body's moment of inertia, the mass of a rigid body cannot be considered as concentrated at its center of mass. The distance of any individual mass particle from the axis is important.