Week 4 homework

IMPORTANT NOTE ABOUT WEBASSIGN:

In the WebAssign versions of these problems, various details have been changed, so that the answers will come out differently. The method to find the solution is the same, but you will need to repeat part of the calculation to find out what your answer should have been.

WebAssign Problem 1: In a skating stunt known as "crack-the-whip," a number of skaters hold hands and form a straight line. They try to skate so that the line rotates about the skater at one end, who acts as the pivot. The skater farthest out has a mass of 80.0 kg and is 6.10 m from the pivot. He is skating at a speed of 6.80 m/s. Determine the magnitude of the centripetal force that acts on him.

REASONING The magnitude F_c of the centripetal force that acts on the skater is given by Equation 5.3 as $F_c = mv^2/r$, where m and v are the mass and speed of the skater, and r is the distance of the skater from the pivot. Since all of these variables are known, we can find the magnitude of the centripetal force.

SOLUTION The magnitude of the centripetal force is

$$F_c = \frac{mv^2}{r} = \frac{(80.0 \text{ kg})(6.80 \text{ m/s})^2}{6.10 \text{ m}} = \boxed{606 \text{ N}}$$

WebAssign Problem 2: A block is hung by a string from the inside roof of a van. When the van goes straight ahead at a speed of 28 m/s, the block hangs vertically down. But when the van maintains this same speed around an unbanked curve (radius = 150 m), the block swings toward the outside of the curve. Then the string makes an angle θ with the vertical. Find θ.

REASONING AND ***SOLUTION***   The centripetal acceleration of the block is

$$a_C = v^2/r = (28 \text{ m/s})^2/(150 \text{ m}) = 5.2 \text{ m/s}^2$$

The angle θ can be obtained from

$$\theta = \tan^{-1}\left(\frac{a_C}{g}\right) = \tan^{-1}\left(\frac{5.2 \text{ m/s}^2}{9.80 \text{ m/s}^2}\right) = \boxed{28°}$$

WebAssign Problem 3: The drawing shows a baggage carousel at an airport. Your suitcase has not slid all the way down the slope and is going around at a constant speed

on a circle ($r = 11.0$ m) as the carousel turns. The coefficient of static friction between the suitcase and the carousel is 0.760, and the angle θ in the drawing is 36.0°. How much time is required for your suitcase to go around once?
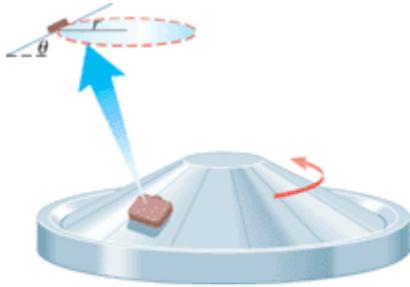

REASONING   The centripetal force F_c required to keep an object of mass m that moves with speed v on a circle of radius r is $F_c = mv^2/r$ (Equation 5.3). From Equation 5.1, we know that $v = 2\pi r/T$, where T is the period or the time for the suitcase to go around once. Therefore, the centripetal force can be written as

$$F_c = \frac{m(2\pi r/T)^2}{r} = \frac{4m\pi^2 r}{T^2} \tag{1}$$

This expression can be solved for T. However, we must first find the centripetal force that acts on the suitcase.

SOLUTION   Three forces act on the suitcase. They are the weight $m\mathbf{g}$ of the suitcase, the force of static friction $\mathbf{f}_s^{MAX}$, and the normal force $\mathbf{F}_N$ exerted on the suitcase by the surface of the carousel. The following figure shows the free body diagram for the suitcase. In this diagram, the y axis is along the vertical direction. The force of gravity acts, then, in the $-y$ direction. The centripetal force that causes the suitcase to move on its circular path is provided by the net force in the $+x$ direction in the diagram. From the diagram, we can see that only the forces $\mathbf{F}_N$ and $\mathbf{f}_s^{MAX}$ have horizontal components. Thus, we have $F_c = f_s^{MAX}\cos\theta - F_N\sin\theta$, where the minus sign indicates that the x component of $\mathbf{F}_N$ points to the left in the diagram. Using Equation 4.7 for the maximum static frictional force, we can write this result as in equation (2).
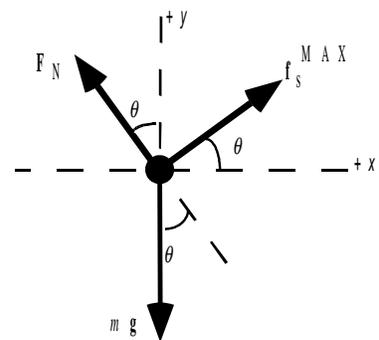
$$F_c = \mu_s F_N \cos\theta - F_N \sin\theta = F_N(\mu_s \cos\theta - \sin\theta) \tag{2}$$

If we apply Newton's second law in the y direction, we see from the diagram that

$$F_N \cos\theta + f_s^{MAX} \sin\theta - mg = ma_y = 0 \quad \text{or} \quad F_N \cos\theta + \mu_s F_N \sin\theta - mg = 0$$

where we again have used Equation 4.7 for the maximum static frictional force. Solving for the normal force, we find

$$F_N = \frac{mg}{\cos\theta + \mu_s \sin\theta}$$

Using this result in equation (2), we obtain the magnitude of the centripetal force that acts on the suitcase:

$$F_c = F_N (\mu_s \cos\theta - \sin\theta) = \frac{mg(\mu_s \cos\theta - \sin\theta)}{\cos\theta + \mu_s \sin\theta}$$

With this expression for the centripetal force, equation (1) becomes

$$\frac{mg(\mu_s \cos\theta - \sin\theta)}{\cos\theta + \mu_s \sin\theta} = \frac{4m\pi^2 r}{T^2}$$

Solving for the period T, we find

$$T = \sqrt{\frac{4\pi^2 r(\cos\theta + \mu_s \sin\theta)}{g(\mu_s \cos\theta - \sin\theta)}} = \sqrt{\frac{4\pi^2 (11.0 \text{ m})(\cos 36.0° + 0.760 \sin 36.0°)}{(9.80 \text{ m/s}^2)(0.760 \cos 36.0° - \sin 36.0°)}} = \boxed{45 \text{ s}}$$

WebAssign Problem 4: Consult Multiple-Concept Example 14 for background pertinent to this problem. In designing rotating space stations to provide for artificial-gravity environments, one of the constraints that must be considered is motion sickness. Studies have shown that the negative effects of motion sickness begin to appear when the rotational motion is faster than two revolutions per minute. On the other hand, the magnitude of the centripetal acceleration at the astronauts' feet should equal the magnitude of the acceleration due to gravity on earth. Thus, to eliminate the difficulties with motion sickness, designers must choose the distance between the astronauts' feet and the axis about which the space station rotates to be greater than a certain minimum value. What is this minimum value?

REASONING Two pieces of information are provided. One is the fact that the magnitude of the centripetal acceleration a_c is 9.80 m/s². The other is that the space station should not rotate faster than two revolutions per minute. This rate of twice per minute corresponds to thirty seconds per revolution, which is the minimum value for the period T of the motion. With these data in mind, we will base our solution on Equation 5.2, which gives the centripetal acceleration as $a_c = v^2/r$, and on Equation 5.1, which specifies that the speed v on a circular path of radius r is $v = 2\pi r/T$.

SOLUTION From Equation 5.2, we have

$$a_c = \frac{v^2}{r} \qquad \text{or} \qquad r = \frac{v^2}{a_c}$$

Substituting $v = 2\pi\, r/T$ into this result and solving for the radius gives

$$r = \frac{v^2}{a_c} = \frac{(2\pi\, r/T)^2}{a_c} \qquad \text{or} \qquad r = \frac{a_c T^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2)(30.0 \text{ s})^2}{4\pi^2} = \boxed{223 \text{ m}}$$

WebAssign Problem 5: A roller coaster at an amusement park has a dip that bottoms out in a vertical circle of radius r. A passenger feels the seat of the car pushing upward on her with a force equal to twice her weight as she goes through the dip. If $r = 20.0$ m, how fast is the roller coaster traveling at the bottom of the dip?

Passenger

r

$2\,m\mathbf{g}$

$m\mathbf{g}$

REASONING According to Equation 5.3, the magnitude F_c of the centripetal force that acts on each passenger is $F_c = mv^2/r$, where m and v are the mass and speed of a passenger and r is the radius of the turn. From this relation we see that the speed is given by $v = \sqrt{F_c\, r/m}$. The centripetal force is the net force required to keep each passenger moving on the circular path and points toward the center of the circle. With the aid of a free-body diagram, we will evaluate the net force and, hence, determine the speed.
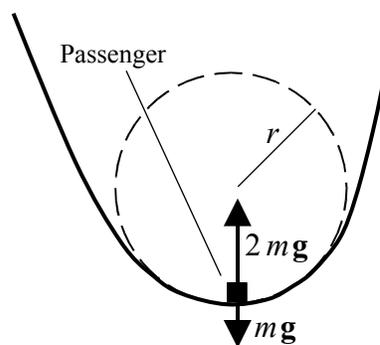
SOLUTION The free-body diagram shows a passenger at the bottom of the circular dip. There are two forces acting: her downward-acting weight $m\mathbf{g}$ and the upward-acting force $2m\mathbf{g}$ that the seat exerts on her. The net force is $+2mg - mg = +mg$, where we have taken "up" as the positive direction. Thus, $F_c = mg$. The speed of the passenger can be found by using this result in the equation above.

Substituting $F_c = mg$ into the relation $v = \sqrt{F_c\, r/m}$ yields

$$v = \sqrt{\frac{F_c\, r}{m}} = \sqrt{\frac{(mg)\, r}{m}} = \sqrt{g\, r} = \sqrt{(9.80 \text{ m/s}^2)(20.0 \text{ m})} = \boxed{14.0 \text{ m/s}}$$


WebAssign Problem 6: . In an automatic clothes dryer, a hollow cylinder moves the clothes on a vertical circle (radius $r = 0.32$ m), as the drawing shows. The appliance is designed so that the clothes tumble gently as they dry. This means that when a

piece of clothing reaches an angle of θ above the horizontal, it loses contact with the wall of the cylinder and falls onto the clothes below. How many revolutions per second should the cylinder make in order that the clothes lose contact with the wall when $\theta = 70.0°$?
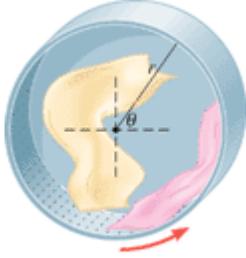
REASONING   The drawing at the right shows the two forces that act on a piece of clothing *just before* it loses contact with the wall of the cylinder. At that instant the centripetal force is provided by the normal force $\mathbf{F}_N$ and the radial component of the weight. From the drawing, the radial component of the weight is given by

$$mg \cos\phi = mg \cos(90° - \theta) = mg \sin\theta$$
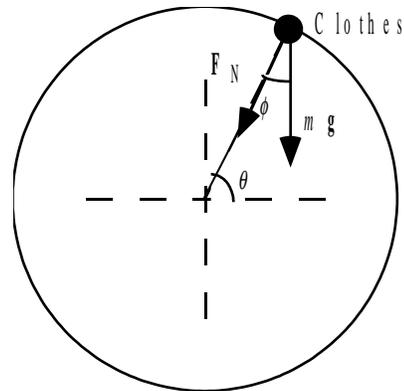
Therefore, with inward taken as the positive direction, Equation 5.3 ($F_c = m v^2 / r$) gives

$$F_N + mg \sin\theta = \frac{mv^2}{r}$$

At the instant that a piece of clothing loses contact with the surface of the drum, $F_N = 0$, and the above expression becomes

$$mg \sin\theta = \frac{mv^2}{r}$$

According to Equation 5.1, $v = 2\pi r / T$, and with this substitution we obtain

$$g \sin\theta = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

This expression can be solved for the period T. Since the period is the required time for one revolution, the number of revolutions per second can be found by calculating $1/T$.

SOLUTION Solving for the period, we obtain

$$T = \sqrt{\frac{4\pi^2 r}{g \sin\theta}} = 2\pi\sqrt{\frac{r}{g \sin\theta}} = 2\pi\sqrt{\frac{0.32 \text{ m}}{(9.80 \text{ m/s}^2)\sin 70.0°}} = 1.17 \text{ s}$$

Therefore, the number of revolutions per second that the cylinder should make is

$$\frac{1}{T} = \frac{1}{1.17 \text{ s}} = \boxed{0.85 \text{ rev/s}}$$

WebAssign Problem 7: The earth rotates once per day about an axis passing through the north and south poles, an axis that is perpendicular to the plane of the equator. Assuming the earth is a sphere with a radius of 6.38×10^6 m, determine the speed and centripetal acceleration of a person situated (a) at the equator and (b) at a latitude of 30.0° north of the equator.

REASONING AND ***SOLUTION***

a. At the equator a person travels in a circle whose radius equals the radius of the earth,

$r = R_e = 6.38 \times 10^6$ m, and whose period of rotation is $T = 1$ day = 86 400 s. We have

$$v = 2\pi R_e/T = 464 \text{ m/s}$$

The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(464 \text{ m/s})^2}{6.38\times 10^6 \text{ m}} = \boxed{3.37\times 10^{-2} \text{ m/s}^2}$$

b. At 30.0° latitude a person travels in a circle of radius,

$$r = R_e \cos 30.0° = 5.53 \times 10^6 \text{ m}$$

Thus,

$$v = 2\pi r/T = 402 \text{ m/s} \qquad \text{and} \qquad a_c = v^2/r = 2.92 \times 10^{-2} \text{ m/s}^2$$

WebAssign Problem 8: At amusement parks, there is a popular ride where the floor of a rotating cylindrical room falls away, leaving the backs of the riders "plastered" against the wall. Suppose the radius of the room is 3.30 m and the speed of the wall is 10.0 m/s when the floor falls away. (a) What is the source of the centripetal force

acting on the riders? (b) How much centripetal force acts on a 55.0-kg rider? (c) What is the minimum coefficient of static friction that must exist between a rider's back and the wall, if the rider is to remain in place when the floor drops away?

REASONING AND ***SOLUTION***

a.   The   centripetal   force   is   provided   by
[the norm al force exerted on the rider by the w all] .

b. Newton's second law applied in the horizontal direction gives

$$F_N = mv^2/r = (55.0 \text{ kg})(10.0 \text{ m/s})^2/(3.30 \text{ m}) = \boxed{1670 \text{ N}}$$

c. Newton's second law applied in the vertical direction gives $\mu_s F_N - mg = 0$ or

$$\mu_s = (mg)/F_N = \boxed{0.323}$$

Practice conceptual problems:

3. The equations of kinematics (Equations (3.3a) (3.4a) (3.5a) (3.6b)) describe the motion of an object that has a constant acceleration. These equations cannot be applied to uniform circular motion. Why not?

REASONING AND SOLUTION   The equations of kinematics (Equations 3.3 - 3.6) cannot be applied to uniform circular motion because an object in uniform circular motion does not have a constant acceleration. While the acceleration vector is constant in magnitude $\left(a = v^2/r\right)$, its direction changes constantly -- it always points toward the center of the circle. As the object moves around the circle the direction of the acceleration must constantly change. Because of this changing direction, the condition of constant acceleration that is required by Equations 3.3 – 3.6 is violated.

8. What is the chance of a light car safely rounding an unbanked curve on an icy road as compared to that of a heavy car: worse, the same, or better? Assume that both cars have the same speed and are equipped with identical tires. Account for your answer.

REASONING AND SOLUTION   From Example 7, the maximum safe speed with which a car can round an unbanked curve of radius r is given by $v = \sqrt{\mu_s gr}$. This expression is independent of the mass (and therefore the weight) of the car. Thus, the chance of a light car safely rounding an unbanked curve on an icy road is the same as that for a heavier car (assuming that all other factors are the same).

12. Explain why a real airplane must bank as it flies in a circle, but a model airplane on a guideline can fly in a circle without banking.

REASONING AND SOLUTION A model airplane on a guideline can fly in a circle because the tension in the guideline provides the horizontal centripetal force necessary to pull the plane into a horizontal circle. A real airplane has no such horizontal forces. The air on the wings on a real plane exerts an upward lifting force that is perpendicular to the wings. The plane must bank so that a component of the lifting force can be oriented horizontally, thereby providing the required centripetal force to cause the plane to fly in a circle.

14. A stone is tied to a string and whirled around in a circle at a constant speed. Is the string more likely to break when the circle is horizontal or when it is vertical? Account for your answer, assuming the constant speed is the same in each case.

REASONING AND SOLUTION When the string is whirled in a horizontal circle, the tension in the string, F_T, provides the centripetal force which causes the stone to move in a circle. Since the speed of the stone is constant, $mv^2/r = F_T$ and the tension in the string is constant.

When the string is whirled in a vertical circle, the tension in the string and the weight of the stone both contribute to the centripetal force, depending on where the stone is on the circle. Now, however, the tension increases and decreases as the stone traverses the vertical circle. When the stone is at the lowest point in its swing, the tension in the string pulls the stone upward, while the weight of the stone acts downward. Therefore, the centripetal force is $mv^2/r = F_T - mg$. Solving for the tension shows that $F_T = mv^2/r + mg$. This tension is larger than in the horizontal case. Therefore, the string has a greater chance of breaking when the stone is whirled in a vertical circle.