

Week 2 homework

IMPORTANT NOTE ABOUT WEBASSIGN:

In the WebAssign versions of these problems, various details have been changed, so that the answers will come out differently. The method to find the solution is the same, but you will need to repeat part of the calculation to find out what your answer should have been.

WebAssign Problem 1: In a football game a kicker attempts a field goal. The ball remains in contact with the kicker's foot for 0.050 s, during which time it experiences an acceleration of 340 m/s². The ball is launched at an angle of 51° above the ground. Determine the horizontal and vertical components of the launch velocity.

REASONING To determine the horizontal and vertical components of the launch velocity, we will use trigonometry. To do so, however, we need to know both the launch angle and the magnitude of the launch velocity. The launch angle is given. The magnitude of the launch velocity can be determined from the given acceleration and the definition of acceleration given in Equation 3.2.

SOLUTION According to Equation 3.2, we have

$$a = \frac{v - v_0}{t - t_0} \quad \text{or} \quad 340 \text{ m/s}^2 = \frac{v - 0 \text{ m/s}}{0.050 \text{ s}} \quad \text{or} \quad v = (340 \text{ m/s}^2)(0.050 \text{ s})$$

Using trigonometry, we find the components to be

$$v_x = v \cos 51^\circ = (340 \text{ m/s}^2)(0.050 \text{ s}) \cos 51^\circ = \boxed{11 \text{ m/s}}$$

$$v_y = v \sin 51^\circ = (340 \text{ m/s}^2)(0.050 \text{ s}) \sin 51^\circ = \boxed{13 \text{ m/s}}$$

WebAssign Problem 2: Michael Jordan, formerly of the Chicago Bulls basketball team, had some fanatic fans. They claimed that he was able to jump and remain in the air for two full seconds from launch to landing. Evaluate this claim by calculating the maximum height that such a jump would attain. For comparison, Jordan's maximum jump height has been estimated at about one meter.

REASONING AND SOLUTION The maximum vertical displacement y attained by a projectile is given by Equation 3.6b ($v_y^2 = v_{0y}^2 + 2a_y y$) with $v_y = 0$:

$$y = -\frac{v_{0y}^2}{2a_y}$$

In order to use Equation 3.6b, we must first estimate his initial speed v_{0y} . When Jordan has reached his maximum vertical displacement, $v_y = 0$, and $t = 1.00$ s. Therefore, according to Equation 3.3b (), with upward taken as positive, we find that

$$v_{0y} = -a_y t = -(-9.80 \text{ m/s}^2)(1.00 \text{ s}) = 9.80 \text{ m/s}$$

Therefore, Jordan's maximum jump height is

$$y = -\frac{(9.80 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{4.90 \text{ m}}$$

This result far exceeds Jordan's maximum jump height, so the claim that he can remain in the air for two full seconds is false.

WebAssign Problem 3: A major-league pitcher can throw a baseball in excess of 41.0 m/s. If a ball is thrown horizontally at this speed, how much will it drop by the time it reaches a catcher who is 17.0 m away from the point of release?

REASONING The vertical displacement y of the ball depends on the time that it is in the air before being caught. These variables depend on the y -direction data, as indicated in the table, where the $+y$ direction is "up."

y-Direction Data

y	a_y	v_y	v_{0y}	t
?	-9.80 m/s^2		0 m/s	?

Since only two variables in the y direction are known, we cannot determine y at this point. Therefore, we examine the data in the x direction, where $+x$ is taken to be the direction from the pitcher to the catcher.

x-Direction Data

x	a_x	v_x	v_{0x}	t
+17.0 m	0 m/s^2		+41.0 m/s	?

Since this table contains three known variables, the time t can be evaluated by using an equation of kinematics. Once the time is known, it can then be used with the y -direction data, along with the appropriate equation of kinematics, to find the vertical displacement y .

SOLUTION Using the x -direction data, Equation 3.5a can be employed to find the time t that the baseball is in the air:

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t \quad (\text{since } a_x = 0 \text{ m/s}^2)$$

Solving for t gives

$$t = \frac{x}{v_{0x}} = \frac{+17.0 \text{ m}}{+41.0 \text{ m/s}} = 0.415 \text{ s}$$

The displacement in the y direction can now be evaluated by using the y -direction data table and the value of $t = 0.415 \text{ s}$. Using Equation 3.5b, we have

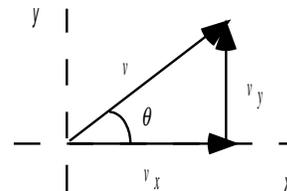
$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (0 \text{ m/s})(0.415 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.415 \text{ s})^2 = -0.844 \text{ m}$$

The distance that the ball drops is given by the magnitude of this result, so Distance = 0.844 m.

WebAssign Problem 4: In the javelin throw at a track-and-field event, the javelin is launched at a speed of 29 m/s at an angle of 36° above the horizontal. As the javelin travels upward, its velocity points above the horizontal at an angle that decreases as time passes. How much time is required for the angle to be reduced from 36° at launch to 18° ?

REASONING As shown in the drawing, the angle that the velocity vector makes with the horizontal is given by

$$\tan \theta = \frac{v_y}{v_x}$$



where, from Equation 3.3b,

$$v_y = v_{0y} + a_y t = v_0 \sin \theta_0 + a_y t$$

and, from Equation 3.3a (since $a_x = 0$),

$$v_x = v_{0x} = v_0 \cos \theta_0$$

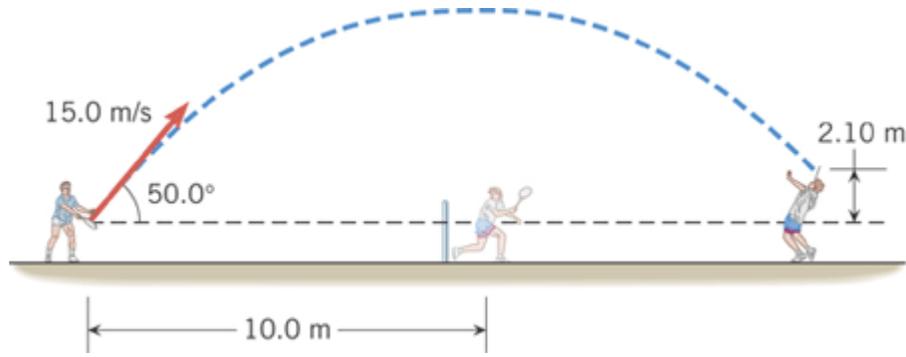
Therefore,

$$\tan \theta = \frac{v_y}{v_x} = \frac{v_0 \sin \theta_0 + a_y t}{v_0 \cos \theta_0}$$

SOLUTION Solving for t , we find

$$t = \frac{v_0 (\cos \theta_0 \tan \theta - \sin \theta_0)}{a_y} = \frac{(29 \text{ m/s}) (\cos 36^\circ \tan 18^\circ - \sin 36^\circ)}{-9.80 \text{ m/s}^2} = \boxed{0.96 \text{ s}}$$

WebAssign Problem 5: The lob in tennis is an effective tactic when your opponent is near the net. It consists of lofting the ball over his head, forcing him to move quickly away from the net (see the drawing). Suppose that you loft the ball with an initial speed of 15.0 m/s, at an angle of 50.0° above the horizontal. At this instant your opponent is 10.0 m away from the ball. He begins moving away from you 0.30 s later, hoping to reach the ball and hit it back at the moment that it is 2.10 m above its launch point. With what minimum average speed must he move? (Ignore the fact that he can stretch, so that his racket can reach the ball before he does.)



REASONING Using the data given in the problem, we can find the maximum flight time t of the ball using Equation 3.5b ($y = v_{0y}t + \frac{1}{2}a_y t^2$). Once the flight time is known, we can use the definition of average velocity to find the minimum speed required to cover the distance x in that time.

SOLUTION Equation 3.5b is quadratic in t and can be solved for t using the quadratic formula. According to Equation 3.5b, the maximum flight time is (with upward taken as the positive direction)

$$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4\left(\frac{1}{2}\right)a_y(-y)}}{2\left(\frac{1}{2}\right)a_y} = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 2a_y y}}{a_y}$$

$$= \frac{-(15.0 \text{ m/s}) \sin 50.0^\circ \pm \sqrt{[(15.0 \text{ m/s}) \sin 50.0^\circ]^2 + 2(-9.80 \text{ m/s}^2)(2.10 \text{ m})}}{-9.80 \text{ m/s}^2}$$

$$= 0.200 \text{ s} \quad \text{and} \quad 2.145 \text{ s}$$

where the first root corresponds to the time required for the ball to reach a vertical displacement of $y = +2.10 \text{ m}$ as it travels upward, and the second root corresponds to the time required for the ball to have a vertical displacement of $y = +2.10 \text{ m}$ as the ball travels upward and then downward. The desired flight time t is 2.145 s .

During the 2.145 s , the horizontal distance traveled by the ball is

$$x = v_x t = (v_0 \cos \theta) t = [(15.0 \text{ m/s}) \cos 50.0^\circ] (2.145 \text{ s}) = 20.68 \text{ m}$$

Thus, the opponent must move $20.68 \text{ m} - 10.0 \text{ m} = 10.68 \text{ m}$ in $2.145 \text{ s} - 0.30 \text{ s} = 1.845 \text{ s}$. The opponent must, therefore, move with a minimum average speed of

$$\bar{v}_{\min} = \frac{10.68 \text{ m}}{1.845 \text{ s}} = \boxed{5.79 \text{ m/s}}$$

WebAssign Problem 6: Two cars, A and B, are traveling in the same direction, although car A is 186 m behind car B. The speed of A is 24.4 m/s , and the speed of B is 18.6 m/s . How much time does it take for A to catch B?

REASONING Since car A is moving faster, it will eventually catch up with car B. Each car is traveling at a constant velocity, so the time t it takes for A to catch up with B is equal to the displacement between the two cars ($x = +186 \text{ m}$) divided by the velocity \mathbf{v}_{AB} of A relative to B. (If the relative velocity were zero, A would never catch up with B). We can find the velocity of A relative to B by using the subscripting technique developed in Section 3.4 of the text.

\mathbf{v}_{AB} = velocity of car A relative to car B

\mathbf{v}_{AG} = velocity of car A relative to the Ground = $+24.4 \text{ m/s}$

\mathbf{v}_{BG} = velocity of car B relative to the Ground = $+18.6 \text{ m/s}$

We have chosen the positive direction for the displacement and velocities to be the direction in which the cars are moving. The velocities are related by

$$\mathbf{v}_{\text{AB}} = \mathbf{v}_{\text{AG}} + \mathbf{v}_{\text{GB}}$$

SOLUTION The velocity of car A relative to car B is

$$\mathbf{v}_{\text{AB}} = \mathbf{v}_{\text{AG}} + \mathbf{v}_{\text{GB}} = +24.4 \text{ m/s} + (-18.6 \text{ m/s}) = +5.8 \text{ m/s},$$

where we have used the fact that $\mathbf{v}_{\text{GB}} = -\mathbf{v}_{\text{BG}} = -18.6 \text{ m/s}$. The time it takes for car A to catch car B is

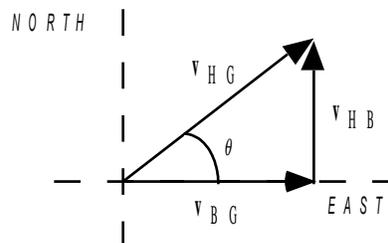
$$t = \frac{x}{v_{AB}} = \frac{+186 \text{ m}}{+5.8 \text{ m/s}} = \boxed{32.1 \text{ s}}$$

WebAssign Problem 7: You are in a hot-air balloon that, relative to the ground, has a velocity of 6.0 m/s in a direction due east. You see a hawk moving directly away from the balloon in a direction due north. The speed of the hawk relative to you is 2.0 m/s. What are the magnitude and direction of the hawk's velocity relative to the ground? Express the directional angle relative to due east.

REASONING REASONING Let v_{HB} represent the velocity of the hawk relative to the balloon and v_{BG} represent the velocity of the balloon relative to the ground. Then, as indicated by Equation 3.7, the velocity of the hawk relative to the ground is $v_{HG} = v_{HB} + v_{BG}$. Since the vectors v_{HB} and v_{BG} are at right angles to each other, the vector addition can be carried out using the Pythagorean theorem.

SOLUTION Using the drawing at the right, we have from the Pythagorean theorem,

$$v_{HG} = \sqrt{v_{HB}^2 + v_{BG}^2} = \sqrt{(2.0 \text{ m/s})^2 + (6.0 \text{ m/s})^2} = \boxed{6.3 \text{ m/s}}$$



The angle θ is

$$\theta = \tan^{-1} \left(\frac{v_{HB}}{v_{BG}} \right) = \tan^{-1} \left(\frac{2.0 \text{ m/s}}{6.0 \text{ m/s}} \right) = \boxed{18^\circ, \text{ north of east}}$$

WebAssign Problem 8: A person looking out the window of a stationary train notices that raindrops are falling vertically down at a speed of 5.0 m/s relative to the ground. When the train moves at a constant velocity, the raindrops make an angle of 25° when they move past the window, as the drawing shows. How fast is the train moving?



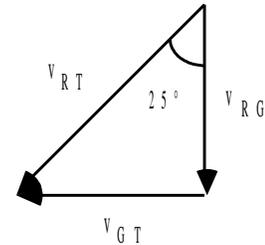
REASONING AND SOLUTION The velocity of the raindrops relative to the train is given by

$$v_{RT} = v_{RG} + v_{GT}$$

where v_{RG} is the velocity of the raindrops relative to the ground and v_{GT} is the velocity of the ground relative to the train.

Since the train moves horizontally, and the rain falls vertically, the velocity vectors are related as shown in the figure at the right. Then

$$v_{GT} = v_{RG} \tan \theta = (5.0 \text{ m/s}) (\tan 25^\circ) = 2.3 \text{ m/s}$$



The train is moving at a speed of 2.3 m/s

WebAssign Problem 9: Concept Questions (a) For a projectile that follows a trajectory like that in Figure 3.12 the range is given by $R = v_{0x}t$, where v_{0x} is the horizontal component of the launch velocity and t is the time of flight. Is v_{0x} proportional to the launch speed v_0 ? (b) Is the time of flight t proportional to the launch speed v_0 ? (c) Is the range R proportional to v_0 or v_0^2 ? Explain each answer.

Problem In the absence of air resistance, a projectile is launched from and returns to ground level. It follows a trajectory similar to that in Figure 3.12 and has a range of 23 m. Suppose the launch speed is doubled, and the projectile is fired at the same angle above the ground. What is the new range?

CONCEPT QUESTIONS a. The horizontal component v_{0x} of the launch velocity is proportional to the launch speed v_0 , because $v_{0x} = v_0 \cos \theta$, where θ is the launch angle.

b. The time of flight t is also proportional to the launch speed v_0 . The greater the launch speed, the longer the projectile is in flight. More exactly, we know that, for a projectile that is launched from and returns to ground level, the vertical displacement is $y = 0$ m. Using Equation 3.5b, we have

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 \text{ m} = v_{0y}t + \frac{1}{2}a_y t^2 \quad \text{or} \quad 0 \text{ m} = v_{0y} + \frac{1}{2}a_y t \quad \text{or} \quad t = \frac{-2v_{0y}}{a_y}$$

The flight time is proportional to the vertical component of the launch velocity v_{0y} , which, in turn, is proportional to the launch speed v_0 .

c. Since the range is given by $R = v_{0x} t$ and since both v_{0x} and t are proportional to v_0 , the range R is proportional to v_0^2 .

SOLUTION The given range is 23 m. When the launch speed doubles, the range increases by a factor of $2^2 = 4$, since the range is proportional to the square of the speed. Thus, the new range is

$$R = 4(23 \text{ m}) = \boxed{92 \text{ m}}$$

Practice conceptual problems:

3. Is the acceleration of a projectile equal to zero when it reaches the top of its trajectory? If not, why not?

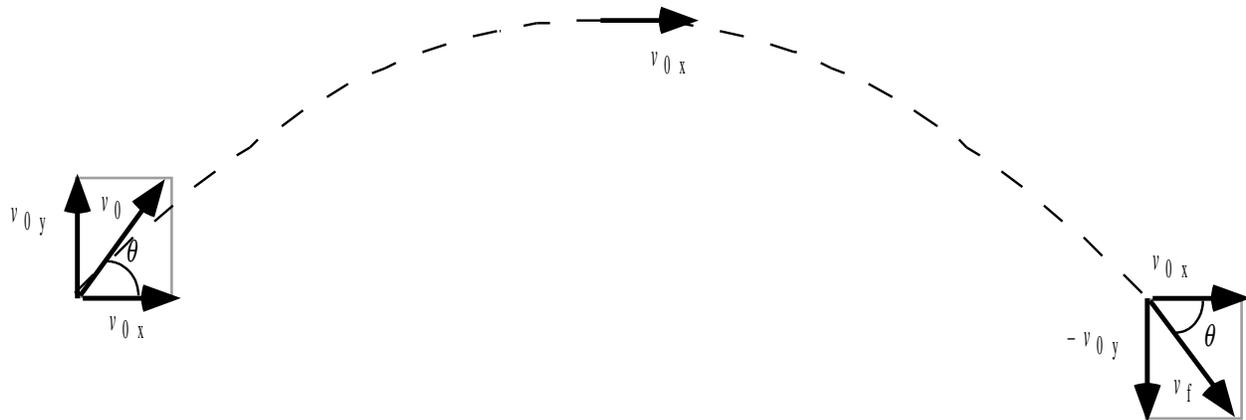
REASONING AND SOLUTION As long as air resistance is negligible, the acceleration of a projectile is constant and equal to the acceleration due to gravity. The acceleration of the projectile, therefore, is the same at every point in its trajectory, and can *never* be zero.

5. A tennis ball is hit upward into the air and moves along an arc. Neglecting air resistance, where along the arc is the speed of the ball (a) a minimum and (b) a maximum? Justify your answers.

REASONING AND SOLUTION The figure below shows the ball's trajectory. The velocity of the ball (along with its x and y components) are indicated at three positions. As long as air resistance is neglected, we know that $a_x = 0$ and a_y is the acceleration due to gravity. Since $a_x = 0$, the x component of the velocity remains the same and is given by v_{0x} . The initial y component of the velocity is v_{0y} and decreases as the ball approaches the highest point, where $v_y = 0$. The magnitude of the y component of the velocity then increases as the ball falls downward. Just before the ball strikes the ground, the y velocity component is equal in magnitude to v_{0y} . The speed is the magnitude of the velocity.

a. Since $v_y = 0$ when the ball is at the highest point in the trajectory, the speed is a minimum there.

b. Similarly, since v_y is a maximum at the initial and final positions of the motion, the speed is a maximum at these positions.



8. A rifle, at a height H above the ground, fires a bullet parallel to the ground. At the same instant and at the same height, a second bullet is dropped from rest. In the absence of air resistance, which bullet strikes the ground first? Explain.

REASONING AND SOLUTION The two bullets differ only in their horizontal motion. One bullet has $v_x = 0$, while the other bullet has $v_x = v_{0x}$. The time of flight, however, is determined only by the vertical motion, and both bullets have the same initial vertical velocity component ($v_{0y} = 0$). Both bullets, therefore, reach the ground at the same time.

13. On a riverboat cruise, a plastic bottle is accidentally dropped overboard. A passenger on the boat estimates that the boat pulls ahead of the bottle by 5 meters each second. Is it possible to conclude that the boat is moving at 5 m/s with respect to the shore? Account for your answer.

REASONING AND SOLUTION Since the plastic bottle moves with the current, the passenger is estimating the velocity of the boat relative to the water. Therefore, the passenger cannot conclude that the boat is moving at 5 m/s with respect to the shore.