Week 1 homework

IMPORTANT NOTE ABOUT WEBASSIGN:

In the WebAssign versions of these problems, various details have been changed, so that the answers will come out differently. The method to find the solution is the same, but you will need to repeat part of the calculation to find out what your answer should have been.

WebAssign Problem 1: The following are dimensions of various physical parameters that will be discussed later on in the text. Here [L], [T], and [M] denote, respectively, dimensions of length, time, and mass.

	Dimension		**Dimension**
Distance (*x*)	[L]	Acceleration (*a*)	$[L]/[T]^2$
Time (*t*)	[T]	Force (*F*)	$[M][L]/[T]^2$
Mass (*m*)	[M]	Energy (*E*)	$[M][L]^2/[T]^2$
Speed (*v*)	[L]/[T]		

Which of the following equations are dimensionally correct?

a. $F = ma$
b. $x = \frac{1}{2}at^3$
c. $E = \frac{1}{2}mv$
d. $E = max$
e. $v = \sqrt{Fx/m}$

REASONING AND SOLUTION

a. $F = [M][L]/[T]^2$; $ma = [M][L]/[T]^2 = [M][L]/[T]^2$ so $F = ma$ is dimensionally correct.

b. $x = [L]$; $at^3 = ([L]/[T]^2)[T]^3 = [L][T]$ so $x = (1/2)at^3$ is *not* dimensionally correct.

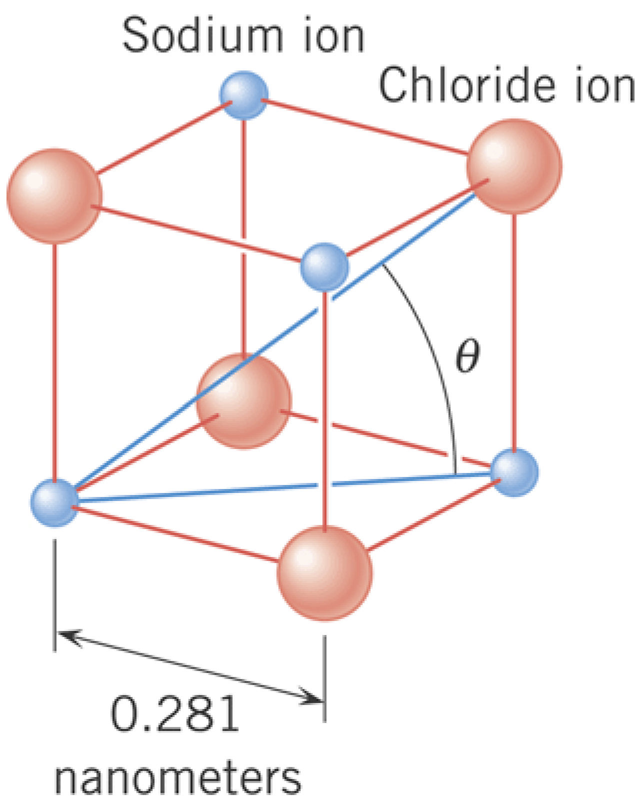
c. $E = [M][L]^2/[T]^2$; $mv = [M][L]/[T]$ so $E = (1/2)mv$ is *not* dimensionally correct.

d. $E = [M][L]^2/[T]^2$; $max = [M]([L]/[T]^2)[L] = [M][L]^2/[T]^2$ so $E = max$ is dimensionally correct.

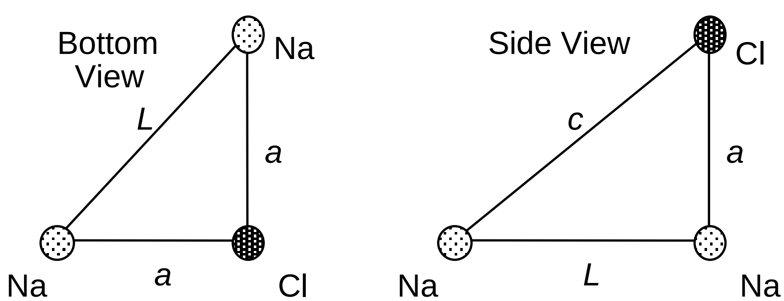
e. v = [L]/[T]; $(Fx/m)^{1/2}$ = $\{([M][L]/[T]^2)([L]/[M])\}^{1/2}$ = $\{[L]^2/[T]^2\}^{1/2}$ = [L]/[T]
so

$v = (Fx/m)^{1/2}$ is dimensionally correct .

WebAssign Problem 2: The drawing shows sodium and chloride ions positioned at the corners of a cube that is part of the crystal structure of sodium chloride (common table salt). The edge of the cube is 0.281 nm $(1 \text{ nm} = 1 \text{ nanometer} = 10^{-9} \text{ m})$ in length. Find the distance (in nanometers) between the sodium ion located at one corner of the cube and the chloride ion located on the diagonal at the opposite corner.



REASONING AND SOLUTION Consider the following views of the cube.



The length, L, of the diagonal of the bottom face of the cube can be found using the Pythagorean theorem to be

$$L^2 = a^2 + a^2 = 2(0.281 \text{ nm})^2 = 0.158 \text{ nm}^2 \quad \text{or} \quad L = 0.397 \text{ nm}$$

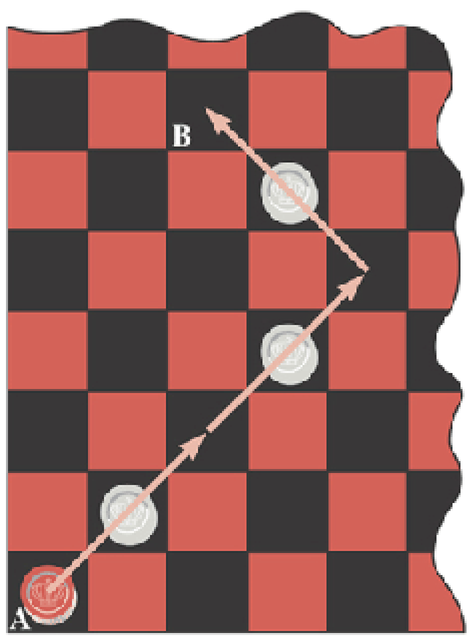
The required distance c is also found using the Pythagorean theorem.

$$c^2 = L^2 + a^2 = (0.397 \text{ nm})^2 + (0.281 \text{ nm})^2 = 0.237 \text{ nm}^2$$

Then,

$$c = \boxed{0.487 \text{ nm}}$$

WebAssign Problem 3: The drawing shows a triple jump on a checkerboard, starting at the center of square A and ending on the center of square B. Each side of a square measures 4.0 cm. What is the magnitude of the displacement of the colored checker during the triple jump?



REASONING The triple jump consists of a double jump (assumed to end on a square that we label C) followed by a single jump. The single jump is perpendicular to the double jump, so that the length ℓ_{AC} of the double jump, the length ℓ_{CB} of the single jump, and the magnitude ℓ_{AB} of the total displacement form a right triangle. Thus, we have

$$\ell_{AB} = \sqrt{\ell_{AC}^2 + \ell_{CB}^2}$$

SOLUTION The diagonal of one square on the checkerboard has a length d of

$$d = \sqrt{(4.0\text{ cm})^2 + (4.0\text{ cm})^2} = 5.66\text{ cm}$$

Since $\ell_{AC} = 4\,d$ and $\ell_{CB} = 2\,d$, it follows that

$$\ell_{AB} = \sqrt{(4d)^2 + (2d)^2} = d\sqrt{20} = (5.66\text{ cm})\sqrt{20} = \boxed{25\text{ cm}}$$

WebAssign Problem 4: An 18-year-old runner can complete a 10.0-km course with an average speed of 4.39 m/s. A 50-year-old runner can cover the same distance with an average speed of 4.27 m/s. How much later (in seconds) should the younger runner start in order to finish the course *at the same time* as the older runner?

REASONING The younger (and faster) runner should start the race after the older runner, the delay being the difference between the time required for the older runner to complete the race and that for the younger runner. The time for each runner to complete the race is equal to the distance of the race divided by the average speed of that runner (see Equation 2.1).

SOLUTION The difference in the times for the two runners to complete the race is $t_{50} - t_{18}$, where

$$t_{50} = \frac{\text{Distance}}{(\text{Average Speed})_{\text{50-yr-old}}} \quad \text{and} \quad t_{18} = \frac{\text{Distance}}{(\text{Average Speed})_{\text{18-yr-old}}} \qquad (2.1)$$

The difference in these two times (which is how much later the younger runner should start) is

$$t_{50} - t_{18} = \frac{\text{Distance}}{(\text{Average Speed})_{\text{50-yr-old}}} - \frac{\text{Distance}}{(\text{Average Speed})_{\text{18-yr-old}}}$$

$$= \frac{10.0\times 10^3\text{ m}}{4.27\text{ m/s}} - \frac{10.0\times 10^3\text{ m}}{4.39\text{ m/s}} = \boxed{64\text{ s}}$$

WebAssign Problem 5: A golfer rides in a golf cart at an average speed of 3.10 m/s for 28.0 s. She then gets out of the cart and starts walking at an average speed of 1.30 m/s. For how long (in seconds) must she walk if her average speed for the entire trip, riding and walking, is 1.80 m/s?

REASONING The time t_{trip} to make the entire trip is equal to the time t_{cart} that the golfer rides in the golf cart plus the time t_{walk} that she walks; $t_{\text{trip}} = t_{\text{cart}} + t_{\text{walk}}$. Therefore, the time that she walks is

$$t_{\text{walk}} = t_{\text{trip}} - t_{\text{cart}} \qquad (1)$$

The average speed $\bar{v}_{\text{trip}}$ for the entire trip is equal to the total distance, x_{cart} + x_{walk}, she travels divided by the time to make the entire trip (see Equation 2.1);

$$\bar{v}_{\text{trip}} = \frac{x_{\text{cart}} + x_{\text{walk}}}{t_{\text{trip}}}$$

Solving this equation for t_{trip} and substituting the resulting expression into Equation 1 yields

$$t_{\text{walk}} = \frac{x_{\text{cart}} + x_{\text{walk}}}{\bar{v}_{\text{trip}}} - t_{\text{cart}} \qquad (2)$$

The distance traveled by the cart is $x_{\text{cart}} = \bar{v}_{\text{cart}} t_{\text{cart}}$, and the distance walked by the golfer is $x_{\text{walk}} = \bar{v}_{\text{walk}} t_{\text{walk}}$. Substituting these expressions for x_{cart} and x_{walk} into Equation 2 gives

$$t_{\text{walk}} = \frac{\bar{v}_{\text{cart}} t_{\text{cart}} + \bar{v}_{\text{walk}} t_{\text{walk}}}{\bar{v}_{\text{trip}}} - t_{\text{cart}}$$

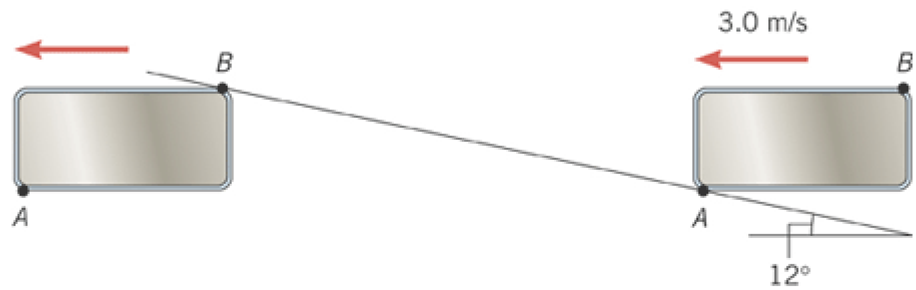
The unknown variable t_{walk} appears on both sides of this equation. Algebraically solving for this variable gives

$$t_{\text{walk}} = \frac{\bar{v}_{\text{cart}} t_{\text{cart}} - \bar{v}_{\text{trip}} t_{\text{cart}}}{\bar{v}_{\text{trip}} - \bar{v}_{\text{walk}}}$$

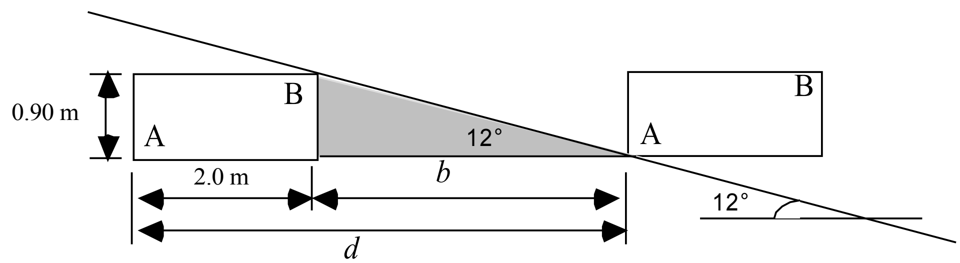
SOLUTION The time that the golfer spends walking is

$$t_{\text{walk}} = \frac{\bar{v}_{\text{cart}} t_{\text{cart}} - \bar{v}_{\text{trip}} t_{\text{cart}}}{\bar{v}_{\text{trip}} - \bar{v}_{\text{walk}}} = \frac{(3.10 \text{ m/s})(28.0 \text{ s}) - (1.80 \text{ m/s})(28.0 \text{ s})}{(1.80 \text{ m/s}) - (1.30 \text{ m/s})} = \boxed{73 \text{ s}}$$

WebAssign Problem 6: You are on a train that is traveling at 3.0 m/s along a level straight track. Very near and parallel to the track is a wall that slopes upward at a 12° angle with the horizontal. As you face the window (0.90 m high, 2.0 m wide) in your compartment, the train is moving to the left, as the drawing indicates. The top edge of the wall first appears at window corner A and eventually disappears at window corner B. How much time passes between appearance and disappearance of the upper edge of the wall?

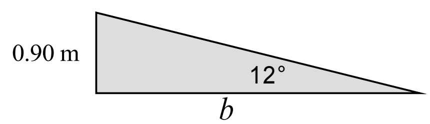


REASONING AND SOLUTION The upper edge of the wall will disappear after the train has traveled the distance d in the figure below.



The distance d is equal to the length of the window plus the base of the 12° right triangle of height 0.90 m.

The base of the triangle is given by



$$b = \frac{0.90 \text{ m}}{\tan 12^\circ} = 4.2 \text{ m}$$

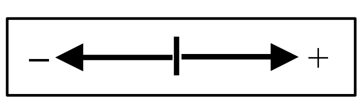
Thus, $d = 2.0 \text{ m} + 4.2 \text{ m} = 6.2 \text{ m}$.

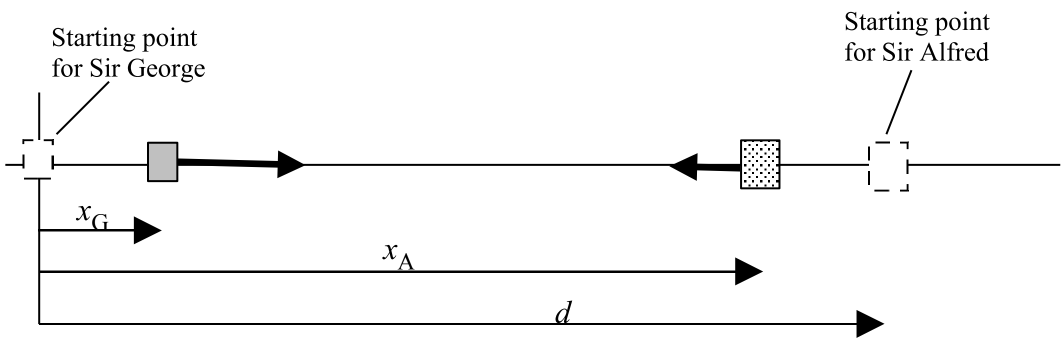
The time required for the train to travel 6.2 m is, from the definition of average speed,

$$t = \frac{x}{v} = \frac{6.2 \text{ m}}{3.0 \text{ m/s}} = \boxed{2.1 \text{ s}}$$

WebAssign Problem 7: In a historical movie, two knights on horseback start from rest 88.0 m apart and ride directly toward each other to do battle. Sir George's acceleration has a magnitude of 0.300 m/s^2, while Sir Alfred's has a magnitude of 0.200 m/s^2. Relative to Sir George's starting point, where do the knights collide?

REASONING The drawing shows the two knights, initially separated by the displacement d, traveling toward each other. At any moment, Sir George's displacement is x_G and that of Sir Alfred is x_A. When they meet, their displacements are the same, so $x_G = x_A$.





According to Equation 2.8, Sir George's displacement as a function of time is

$$x_G = v_{0,G}t + \tfrac{1}{2}a_G t^2 = (0\text{ m/s})\,t + \tfrac{1}{2}a_G t^2 = \tfrac{1}{2}a_G t^2 \qquad (1)$$

where we have used the fact that Sir George starts from rest $\left(v_{0,G} = 0\text{ m/s}\right)$.

Since Sir Alfred starts from rest at $x = d$ at $t = 0$ s, we can write his displacement as (again, employing Equation 2.8)

$$x_A = d + v_{0,A}t + \tfrac{1}{2}a_A t^2 = d + (0\text{ m/s})\,t + \tfrac{1}{2}a_A t^2 = d + \tfrac{1}{2}a_A t^2 \qquad (2)$$

Solving Equation 1 for t^2 $\left(t^2 = 2x_G / a_G\right)$ and substituting this expression into Equation 2 yields

$$x_A = d + \tfrac{1}{2}a_A\left(\frac{2x_G}{a_G}\right) = d + a_A\left(\frac{x_G}{a_G}\right) \qquad (3)$$

Noting that $x_A = x_G$ when the two riders collide, we see that Equation 3 becomes

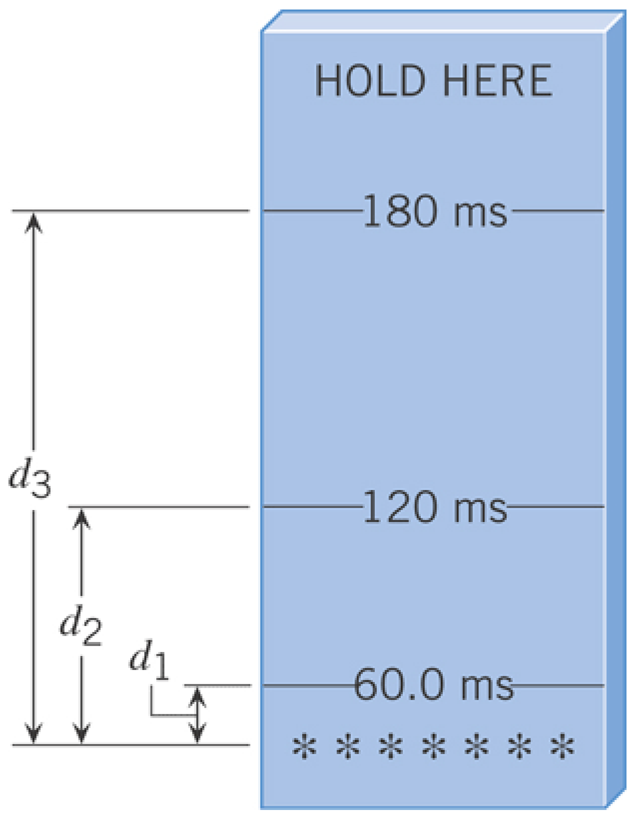
$$x_G = d + a_A\left(\frac{x_G}{a_G}\right)$$

Solving this equation for x_G gives $x_G = \dfrac{d}{1 - \dfrac{a_A}{a_G}}$.

SOLUTION Sir George's acceleration is positive $\left(a_G = +0.300\text{ m/s}^2\right)$ since he starts from rest and moves to the right (the positive direction). Sir Alfred's acceleration is negative $\left(a_A = -0.200\text{ m/s}^2\right)$ since he starts from rest and moves to the left (the negative direction). The displacement of Sir George is, then,

$$x_G = \frac{d}{1 - \frac{a_A}{a_G}} = \frac{88.0 \text{ m}}{1 - \frac{(-0.200 \text{ m/s}^2)}{(+0.300 \text{ m/s}^2)}} = \boxed{52.8 \text{ m}}$$

WebAssign Problem 8: The drawing shows a device that you can make with a piece of cardboard, which can be used to measure a person's reaction time. Hold the card at the top and suddenly drop it. Ask a friend to try to catch the card between his or her thumb and index finger. Initially, your friend's fingers must be level with the asterisks at the bottom. By noting where your friend catches the card, you can determine his or her reaction time in milliseconds (ms). Calculate the distances d_1, d_2, and d_3.



REASONING AND SOLUTION In a time t the card will undergo a vertical displacement y given by

$$y = \tfrac{1}{2}at^2$$

where $a = -9.80 \text{ m/s}^2$. When $t = 60.0 \text{ ms} = 6.0 \times 10^{-2}$ s, the displacement of the card is 0.018 m, and the distance is the magnitude of this value or $\boxed{d_1 = 0.018 \text{ m}}$.

Similarly, when t = 120 ms, $\boxed{d_2 = 0.071\text{ m}}$, and when t = 180 ms, $\boxed{d_3 = 0.16\text{ m}}$.

WebAssign Problem 9: Before working this problem, review Conceptual Example 15. A pellet gun is fired straight downward from the edge of a cliff that is 15 m above the ground. The pellet strikes the ground with a speed of 27 m/s. How far above the cliff edge would the pellet have gone had the gun been fired straight upward?

REASONING Equation 2.9 $\left(v^2 = v_0^2 + 2ay\right)$ can be used to find out how far above the cliff's edge the pellet would have gone if the gun had been fired straight upward, provided that we can determine the initial speed imparted to the pellet by the gun. This initial speed can be found by applying Equation 2.9 to the downward motion of the pellet described in the problem statement.

SOLUTION If we assume that upward is the positive direction, the initial speed of the pellet is, from Equation 2.9,

$$v_0 = \sqrt{v^2 - 2ay} = \sqrt{(-27\text{ m/s})^2 - 2(-9.80\text{ m/s}^2)(-15\text{ m})} = 20.9\text{ m/s}$$

Equation 2.9 can again be used to find the maximum height of the pellet if it were fired straight up. At its maximum height, v = 0 m/s, and Equation 2.9 gives

$$y = \frac{-v_0^2}{2a} = \frac{-(20.9\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = \boxed{22\text{ m}}$$

WebAssign Problem 10: While standing on a bridge 15.0 m above the ground, you drop a stone from rest. When the stone has fallen 3.20 m, you throw a second stone straight down. What initial velocity must you give the second stone if they are both to reach the ground at the same instant? Take the downward direction to be the negative direction.

REASONING To find the initial velocity $v_{0,2}$ of the second stone, we will employ Equation 2.8, $y = v_{0,2}t_2 + \frac{1}{2}at_2^{\,2}$. In this expression t_2 is the time that the second stone is in the air, and it is equal to the time t_1 that the first stone is in the air minus the time $t_{3.20}$ it takes for the first stone to fall 3.20 m:

$$t_2 = t_1 - t_{3.20}$$

We can find t_1 and $t_{3.20}$ by applying Equation 2.8 to the first stone.

SOLUTION To find the initial velocity $v_{0,2}$ of the second stone, we employ Equation 2.8, $y = v_{0,2}t_2 + \frac{1}{2}at_2^2$. Solving this equation for $v_{0,2}$ yields

$$v_{0,2} = \frac{y - \frac{1}{2}at_2^2}{t_2}$$

The time t_1 for the first stone to strike the ground can be obtained from Equation 2.8, $y = v_{0,1}t_1 + \frac{1}{2}at_1^2$. Noting that $v_{0,1}$ = 0 m/s since the stone is dropped from rest and solving this equation for t_1, we have

$$t_1 = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-15.0\text{ m})}{-9.80\text{ m/s}^2}} = 1.75\text{ s} \qquad (1)$$

Note that the stone is falling down, so its displacement is negative ($y = -15.0$ m). Also, its acceleration a is that due to gravity, so $a = -9.80\text{ m/s}^2$.

The time $t_{3.20}$ for the first stone to fall 3.20 m can also be obtained from Equation 1:

$$t_{3.20} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-3.20\text{ m})}{-9.80\text{ m/s}^2}} = 0.808\text{ s}$$

The time t_2 that the second stone is in the air is

$$t_2 = t_1 - t_{3.20} = 1.75\text{ s} - 0.808\text{ s} = 0.94\text{ s}$$

The initial velocity of the second stone is

$$v_{0,2} = \frac{y - \frac{1}{2}at_2^2}{t_2} = \frac{(-15.0\text{ m}) - \frac{1}{2}(-9.80\text{ m/s}^2)(0.94\text{ s})^2}{0.94\text{ s}} = \boxed{-11\text{ m/s}}$$

WebAssign Problem 11: Concept Questions Concept Simulation 2.3 at www.wiley.com/college/cutnell provides some background for this problem. A ball is thrown vertically upward, which is the positive direction. A little later it returns to its point of release. (a) Does the acceleration of the ball reverse direction when the ball starts its downward trip? (b) What is the displacement of the ball when it returns to its point of release? Explain your answers.

Problem If the ball is in the air for a total time of 8.0 s, what is its initial velocity?

CONCEPT QUESTIONS
a. The acceleration of the ball does not reverse direction on the downward part of the trip. The acceleration is the same for both the upward and downward parts, namely –9.80 m/s^2.

b. The displacement is $y = 0$ m, since the final and initial positions of the ball are the same.

SOLUTION The displacement of the ball, the acceleration due to gravity, and the elapsed time are known. We may use Equation 2.8, $y = v_0 t + \frac{1}{2}at^2$, to find the initial velocity of the ball. Solving this equation for the initial velocity gives

$$v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{0\text{ m} - \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(8.0\text{ s}\right)^2}{8.0\text{ s}} = \boxed{+39\text{ m/s}}$$

Practice conceptual problem:

Chapter 1

15. In preparation for this question, review Conceptual Example 7. Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ satisfy the vector equation $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$. (a) How does the magnitude of $\vec{\mathbf{B}}$ compare with the magnitude of $\vec{\mathbf{A}}$? (b) How does the direction of $\vec{\mathbf{B}}$ compare with the direction of $\vec{\mathbf{A}}$? Give your reasoning.

REASONING AND SOLUTION The vector equation **A** + **B** = 0, implies that **A** = –**B**.

a. The magnitude of **A** must be equal to the magnitude of **B**.

b. The vectors **A** and **B** must point in opposite directions, as indicated by the minus sign in **A** = –**B**.

The same conclusions can be reached from a geometric argument. If **A** + **B** = 0, then, when **A** and **B** are placed tail-to-head, the head of **B** must touch the tail of **A**. Thus, the vectors **A** and **B** must have the same length (i.e., the same magnitude) and point in opposite directions.

16. Vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ satisfy the vector equation $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$, and their magnitudes are related by the scalar equation $A^2 + B^2 = C^2$. How is vector $\vec{\mathbf{A}}$ oriented with respect to vector $\vec{\mathbf{B}}$? Account for your answer.

REASONING AND SOLUTION The equation **A** + **B** = **C** tells us that the vector **C** is the resultant of the vectors **A** and **B**. The magnitudes of the vectors **A**, **B**, and **C** are related by $A^2 + B^2 = C^2$. This has the same form as the Pythagorean theorem that relates the length of the two sides of a right triangle and the length of the hypotenuse. Thus, the vectors **A** and **B** must be at right angles (or perpendicular) to each other.

17. Vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$satisfy the vector equation $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$, and their magnitudes are related by the scalar equation $A + B = C$. How is vector $\vec{\mathbf{A}}$oriented with respect to vector $\vec{\mathbf{B}}$? Explain your reasoning.

REASONING AND SOLUTION The equation **A** + **B** = **C** tells us that the vector **C** is the resultant of the vectors **A** and **B**. The magnitudes of the vectors **A**, **B**, and **C** are related by $A + B = C$. In other words, the length of the vector **C** is equal to the combined lengths of vectors **A** and **B**. Therefore, the vectors **A** and **B** must point in the same direction.

Chapter 2

1. A honeybee leaves the hive and travels 2 km before returning. Is the displacement for the trip the same as the distance traveled? If not, why not?

REASONING AND SOLUTION The displacement of the honeybee for the trip is *not* the same as the distance traveled by the honeybee. As stated in the question, the distance traveled by the honeybee is 2 km. The displacement for the trip is the shortest distance between the initial and final positions of the honeybee. Since the honeybee returns to the hive, its initial position and final position are the same; therefore, the displacement of the honeybee is zero.

6. Give an example from your own experience in which the velocity of an object is zero for just an instant of time, but its acceleration is not zero.

REASONING AND SOLUTION Answers will vary. One example is a ball that is thrown straight up in the air. When the ball is at its highest point, its velocity is momentarily zero. Since the ball is close to the surface of the earth, its acceleration is nearly constant and is equal to the acceleration due to gravity. Thus, the velocity of the ball is momentarily zero, but since the ball is accelerating, the velocity is changing. An instant later, the velocity of the ball is nonzero as the ball begins to fall. Another example is a swimmer in a race, reversing directions at the end of the pool.

12. A ball is dropped from rest from the top of a building and strikes the ground with a speed v_f. From ground level, a second ball is thrown straight upward at the same instant that the first ball is dropped. The initial speed of the second ball is $v_0 = v_f$, the same speed with which the first ball will eventually strike the ground. Ignoring air resistance, decide whether the balls cross paths at half the height of the building, above the halfway point, or below the halfway point. Give your reasoning.

REASONING AND SOLUTION The first ball has an initial speed of zero, since it is dropped from rest. It picks up speed on the way down, striking the ground at a speed v_f. The second ball has a motion that is the reverse of that of the first ball. The second ball starts out with a speed v_f and loses speed on the way up. By symmetry, the second ball will come to a halt at the top of the building. Thus, in approaching the crossing point, the second ball travels faster than the first ball. Correspondingly, the second ball must travel farther on its way to the crossing point than the first ball does. Thus, the crossing point must be located in the upper half of the building.