1. A roller coaster coasts around a vertical, circular loop of radius 10 m. What is the minimum speed that the car must have at the bottom of the loop so that it makes it around the top of the loop without losing contact with the track? (There is no friction nor propulsion, and the car may be treated as a point particle.)

\[ \frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 + 2 m g R \]

26.4 m/s

\[ m v_f^2 = m g \frac{R}{R} \quad \Rightarrow \quad v_f^2 = g R + 4 g R = 5 g R \quad v_f = 22.1 \text{ m/s} \]

2. Four balls of different masses are thrown from the top of a building at different angles, but all with the same initial speed. Ball 2 has the greatest mass, while ball 3 has the smallest mass of the four balls. Neglecting air resistance, which of the balls has the greatest speed just before it hits the ground?

Energy conservation

\[ \frac{(m_1 + m_2) v^2}{2} = \frac{m_1 g h}{2} - \frac{m_2 g h}{2} \quad \Rightarrow \quad v^2 = \frac{m_1 - m_2}{m_1 + m_2} g h \]

3. Two masses are connected by a light string over a light, frictionless pulley. The 5 kg mass is released from rest at a point 4 m above the floor. Determine the speed of each mass when they pass each other. (You may treat the masses as point particles.)

\[ a) \ 7.2 \text{ m/s} \]
\[ b) \ 3.13 \text{ m/s} \]
\[ c) \ 5.02 \text{ m/s} \]
\[ d) \ 6.24 \text{ m/s} \text{ for the 5 kg mass, 4.62 m/s for the 3 kg mass} \]
\[ e) \ 4.62 \text{ m/s} \text{ for the 5 kg mass, 6.24 m/s for the 3 kg mass} \]

4. Two objects are initially at rest on a frictionless, horizontal surface. The mass of object 1 is larger than that of object 2. After they have been acted upon by the same force over the same displacement, which statements about the objects' momenta \( p \) and their kinetic energies \( KE \) are correct?

\[ a) \ p_1 > p_2 \text{ and } KE_1 = KE_2 \]
\[ b) \ p_1 = p_2 \text{ and } KE_1 < KE_2 \]
\[ c) \ p_1 > p_2 \text{ and } KE_1 > KE_2 \]
\[ d) \ p_1 = p_2 \text{ and } KE_1 = KE_2 \]
\[ e) \ p_1 < p_2 \text{ and } KE_1 > KE_2 \]

5. An astronaut of mass \( M \) is floating in her spacecraft and is initially at rest (relative to the spacecraft). She throws away her purse of mass \( m \) with speed \( v \) (relative to the spacecraft). What is her recoil speed (relative to the spacecraft)?

\[ a) \ \frac{mv}{(M - m)} \]
\[ b) \ 0 \]
\[ c) \ \frac{mv}{(M + m)} \]
\[ d) \ \frac{mv}{M} \]
\[ e) \ \frac{2mv}{(M - m)} \]

\[ \text{speed} = \left| \frac{v}{M} \right| \]
6. An object of mass \( m \) moves to the right with a speed \( v \). It collides head-on with an object of mass \( 3m \) moving with speed \( v/3 \) in the opposite direction. If the two objects get stuck to each other in the collision, what is their speed after the collision?

\[
\begin{align*}
\text{(a)} & \quad v/3 \\
\text{(b)} & \quad v/2 \\
\text{(c)} & \quad \frac{3mv}{3} - \frac{3m}{3} = 4m v_f \\
\text{(d)} & \quad 2v \\
\text{(e)} & \quad \Rightarrow v_f = 0
\end{align*}
\]

7. A compact disc in a CD player accelerates from rest up to an angular speed of 31.4 radians/second in a time of 0.892 seconds. What is the (uniform) angular acceleration of the disc and how many rotations does the disc make while coming up to speed?

\[
\begin{align*}
\text{(a)} & \quad 35.2 \text{ rad/s}^2, 2.23 \text{ revs.} \\
\text{(b)} & \quad 32 \text{ rad/s}^2, 2.23 \text{ revs.} \\
\text{(c)} & \quad 35.2 \text{ rad/s}^2, 14.0 \text{ revs.} \\
\text{(d)} & \quad 32 \text{ rad/s}^2, 14.0 \text{ revs.} \\
\text{(e)} & \quad 35.2 \text{ rad/s}^2, 14.0 \text{ revs.}
\end{align*}
\]

8. A uniform, solid disk of radius \( r \) has a circular portion of diameter \( r \) removed as shown in the diagram. What are the \((x, y)\) coordinates of its center of mass?

\[
\begin{align*}
\frac{\pi r^2 - \pi \left( \frac{1}{2} r \right)^2}{2} x_{cm} + \frac{\pi \left( \frac{1}{2} r \right)^2}{2} z = 0 \\
\Rightarrow x_{cm} = -\frac{r}{6}
\end{align*}
\]

\[
\begin{align*}
\text{(a)} & \quad r/6, 0 \\
\text{(b)} & \quad 0, 0 \\
\text{(c)} & \quad -r/6, 0 \\
\text{(d)} & \quad -r/3, 0 \\
\text{(e)} & \quad r/10, 0
\end{align*}
\]

9. A uniform, 10 m-long ladder weighing 50 N leans against a frictionless, vertical wall. It is just about to slip when it makes an angle of 50° with the ground. What is the coefficient of static friction between the ladder and ground?

\[
\begin{align*}
\sum F_x = 0 & \quad \Rightarrow F_x = F_n \\
\sum F_y = 0 & \quad \Rightarrow F_N = W \\
\sum \tau = 0 & \quad (\text{torques } \omega r \text{ to } \theta) \\
F_h \left( \sin 50° \right) & = \frac{W}{2} \sin 40° \\
\cos 50° & = C = 10 \text{ m}
\end{align*}
\]

\[
\begin{align*}
\text{(a)} & \quad 0.51 \\
\text{(b)} & \quad 0.18 \\
\text{(c)} & \quad 0.21 \\
\text{(d)} & \quad 0.42 \\
\text{(e)} & \quad 0.64
\end{align*}
\]

10. Two children, Nathaniel and Christina are playing on a hill, rolling automobile tires down the slope. Nathaniel claims that the tire will roll faster if he curls up in the tire opening and rides down inside it. Christina claims this will cause the tire to roll more slowly. Which child is correct?

\[
\begin{align*}
\text{(a)} & \quad \text{Nathaniel} \\
\text{(b)} & \quad \text{Christina} \\
\text{(c)} & \quad \text{Nathaniel, if his mass less than that of the tire} \\
\text{(d)} & \quad \text{Christina, if Nathaniel’s moment of inertia is greater than that of the tire} \\
\text{(e)} & \quad \text{Neither – the tire will roll at the same speed in either case.}
\end{align*}
\]

\[
\begin{align*}
\text{mg} h & = \frac{1}{2} m v^2 + \frac{1}{2} I \frac{v}{r}^2 \\
gh & = \frac{v^2}{2} \left( 1 + \frac{I}{m \, r^2} \right)
\end{align*}
\]

\[
\begin{align*}
v & = \frac{2gh}{1 + \frac{I}{m \, r^2}} \\
\frac{I}{m \, r^2} & \text{ is smaller with child in tire than without} \\
& \Rightarrow v \text{ is greater!}
\end{align*}
\]
11. A horizontal disk with moment of inertia $I_1$ rotates with angular speed $\omega_0$ about a vertical, frictionless axle. A second horizontal disk, with moment of inertia $I_2$ and initially not rotating, drops onto the first disk. Because of friction between the two disks, they eventually reach the same angular speed $\omega$. What is $\omega/\omega_0$?

   a) $I_1/I_2$
   b) $I_2/I_1$
   c) $I_1/(I_1 + I_2)$
   d) $I_2/(I_1 + I_2)$
   e) $(I_1 + I_2)/I_1$

   angular momentum is conserved

   \[ I_1 \omega_0 = (I_1 + I_2) \omega \]

   \[ \frac{\omega}{\omega_0} = \frac{I_1}{I_1 + I_2} \]

12. The force law for springs was discovered by

   a) Isaac Newton
   b) Robert Hooke
   c) Johannes Kepler
   d) Galileo
   e) Copernicus

13. A 10.1 kg uniform board is wedged into a corner and held by a spring at a 50° angle, as shown. The spring has a spring constant of 176 N/m and is parallel to the floor. How much is the spring stretched from its unstretched length?

   \[ \sum F = 0 \]

   a) 0.0 m
   b) 0.314 m
   c) 0.118 m
   d) 0.472 m
   e) 0.236 m

   \[ \frac{W}{2} \sin 40° + Fl \sin 50° = 0 \]

   \[ F = \frac{W}{2} \tan 40° \]

   \[ \Delta x = \frac{E}{k} = \frac{W}{2k} \tan 40° = 0.236 \text{ m} \]

14. A mass $m$ on a frictionless, horizontal floor is attached to a massless, horizontal spring with spring constant $k$, whose other end is fixed. The mass is now used to compress the spring a distance $d$ and then released. What is the maximum speed achieved by this mass?

   a) $2d\sqrt{k/m}$
   b) $d\sqrt{k/m}$
   c) $d(m/k)^2$
   d) $dk/m$
   e) $dm/kd$

   \[ \frac{1}{2} kd^2 = \frac{1}{2} mv^2 \]

   \[ \Rightarrow v = \sqrt{\frac{k}{m}}d \]

15. A block on the end of a spring is pulled to a position $x = A$, from the equilibrium position $x = 0$, and released. Through what total distance does it travel in one full oscillation cycle?

   a) $A/2$
   b) $A$
   c) $2A$
   d) $4A$
   e) $6A$