Simple Harmonic Motion
What would be the **thermal efficiency** of a Carnot engine if the engine’s hot and cold reservoirs are at the same temperature?

a) 100%
b) 67%
c) 50%
d) 33%
e) 0%
A chunk of rock of mass 100 kg at 300 K falls from a cliff of height 100 m into a large lake, also at 300 K. The change in entropy of the lake, assuming all the rock’s kinetic energy upon entering the lake converts to thermal energy absorbed by the lake, is

a) 333 J
b) 500 J/K
c) 350 kJ
d) 333 J/K
e) 350 K
Two simple pendula are of the same length but with masses $m_1$ and $m_2$ where $m_2 = 2m_1$. The ratio of their periods $T_1: T_2$ is

a) 1:2
b) 2:1
c) 1:4
d) 4:1
e) 1:1
Two masses \( m_1 \) and \( m_2 \) where \( m_2 = 4m_1 \) move with simple harmonic motion while attached to springs of spring constant \( k \). The ratio of their periods \( T_1: T_2 \) is

a) 1:2  
b) 2:1  
c) 1:4  
d) 4:1  
e) 1:1
Hooke’s law:
The force exerted by a spring is proportional to the negative of the displacement of the spring’s end.

\[ F_S = -k \Delta x \]

The constant of proportionality, \( k \), called the spring constant, depends on the spring, and has units \( \text{N/m} \).

Note that the force is NOT constant:
Therefore the acceleration is NOT constant:
The familiar 3 equations in kinematics are no longer true.
So NO \( v = at \) NOR \( x = \frac{1}{2} a t^2 \) for a spring.

What to do?
Hooke’s Law and ENERGY

The spring force is a conservative force.

TOTAL ENERGY IS STILL CONSERVED!

Kinetic energy is still defined the familiar way, $\frac{1}{2} m v^2$. What about potential energy?

Elastic potential energy:
The energy associated with the spring force is given by half the product of the spring constant, $k$, and the displacement-squared, $x^2$.

$$E_P = \frac{1}{2} kx^2$$
Hooke’s Law and ENERGY

TOTAL ENERGY \[ E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2. \]

Kinetic Energy \( E_K \) and Potential Energy \( E_P \) oscillate but their sum is constant.

When \( x \) reaches its maximum \( x = A \), the velocity is zero and \( E = \frac{1}{2} k A^2 \).

When \( x = 0 \), the velocity reaches its maximum value and \( E = \frac{1}{2} m v_{\text{max}}^2 \).
Blocks and springs

A block on a spring undergoes Simple Harmonic Motion (SHM). SHM basically describes simple systems that oscillate back and forth.

From conservation of energy, we can determine the speed of the block as a function of position.

\[ v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)} \]

Simple harmonic motion:
An object undergoes simple harmonic motion when it obeys Hooke’s law.
Question

Let the total mechanical energy of a harmonic oscillator be $E_0$ and the maximum displacement be $A$. When the displacement is $A/2$, what is the kinetic energy?

a) $E_0/2$.
b) $E_0/4$.
c) $3E_0/4$.
d) $E_0$.
e) $E_0/8$. 
Describing oscillations

Amplitude: $A$
Maximum distance, $x_{\text{max}}$, from equilibrium position.

Period: $T$
Time taken for one oscillation.

Frequency: $f$
Number of oscillations per second.

$$f = \frac{1}{T}$$
Frequency has units of s$^{-1}$ or Hertz (Hz), where $1 \text{ Hz} = 1 \text{ s}^{-1}$.

Angular frequency: $\omega$
Number of radians per second.

$$\omega = 2\pi f$$
Angular frequency has units of rad/s.
As the ball rotates like a particle in uniform circular motion...

The $x$-component of the ball’s velocity equals the projection of $\vec{v}_0$ on the $x$-axis.

...the ball’s shadow on the screen moves back and forth with simple harmonic motion.
SHM and uniform circular motion

As the ball rotates like a particle in uniform circular motion...

Piston motion back and forth leads to Rotational Motion of Wheels
Blocks and springs and SHM

A block on a spring undergoes **Simple Harmonic Motion (SHM)**.

**Period of block-spring oscillations**

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

**Frequency of block-spring oscillations**

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

**Angular frequency \( \omega \) of block-spring oscillations**

\[ \omega = \sqrt{\frac{k}{m}} \]
This procedure has actually been used to “weigh” astronauts in space. A 42.5 kg chair is attached to a spring and is allowed to oscillate. When it is empty it takes 1.30 s to make one complete vibration. With an astronaut sitting in it, with her feet off the floor, the chair takes 2.54 s for one cycle. What is the mass of the astronaut?

a) 100 kg
b) 110 kg
c) 120 kg
d) 140 kg
e) 90 kg
Circular motion with angular speed $\omega$ and SHM

This will give the time dependence of $x$, $v$, $a$ for SHM

$x = A \cos \theta$

TIME DEPENDENCE BECAUSE

$\theta = \omega t$

$x = A \cos \omega t$

$v = -\omega A \sin \omega t$

$\omega$ is the angular frequency given in radians per second.
Position, velocity, and acceleration

**Position**
\[ x(t) = A \cos(\omega t) \]

**Velocity**
\[ v(t) = -A\omega \sin(\omega t) \]

**Acceleration**
\[ a(t) = -A\omega^2 \cos(\omega t) \]
Which of the following is true about SHM?

a) When the absolute displacement is maximum, the speed is maximum.
b) When the absolute displacement is minimum, the speed is maximum.
c) When the absolute displacement is maximum, the magnitude of the acceleration is minimum.
d) When the magnitude of the acceleration is minimum, the speed is minimum.
Question

An object of mass 0.40 kg, hanging from a spring with a spring constant of 8.0 N/m, is set into simple harmonic motion. What is the magnitude of the acceleration of the object when the object is at its maximum displacement of 0.10 m?

a) 0 m/s².
b) 0.45 m/s².
c) 1.0 m/s².
d) 2.0 m/s².
e) 2.4 m/s².
In a simple harmonic oscillator with a mass $M$ attached to a spring with spring constant $k$, the mass is decreased to $M/4$ and the spring constant is increased to $4k$. If the original period of the oscillation was $T$, what is the new period of oscillation?

a) $T$.  
b) $4T$.  
c) $T/4$.  
d) $16T$.  
e) $T/16$. 
A mass of 2.0 kg is attached to a spring that has a spring constant of 8.0 N/m. If the mass is pulled a distance of 0.10 m and released, what is the period of the oscillation?

a) 4.0 s.
b) $\pi$ s.
c) $2\pi$ s.
d) $\pi/2$ s.
e) $1/(2\pi)$ s.
Simple pendulum

The restoring force causing the pendulum to oscillate harmonically is the tangential component of the gravity force $-mg \sin \theta$.

For a small enough angles (measured in radians!), we can approximate

$$\sin \theta \simeq \theta$$

The restoring force on the pendulum (was $F = -mg \sin \theta$)

$$F \simeq -m g \theta$$

Therefore, for small angles, the pendulum approximately obeys Hooke’s law.
A simple pendulum undergoes **Simple Harmonic Motion** (SHM).

**Period** of pendulum oscillations

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

**Frequency** of pendulum oscillations

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \]

**Angular frequency** of pendulum oscillations

\[ \omega = \sqrt{\frac{g}{L}} \]
Question

Using a simple pendulum of length 0.170 m, a geologist counts 120 complete swings in a time of 100 s. What is the value of g in this location?

a) 9.73 m/s²  
b) 9.68 m/s²  
c) 9.80 m/s²  
d) 10.00 m/s²  
e) 9.56 m/s²