Lecture 4: learning objectives

You will be able to describe projectile motion graphically and apply two-dimensional kinematics equations to motion with constant acceleration.

You will be able to solve problems in two dimensions involving relative velocity.
Projectile motion:
Motion in two dimensions, near the surface of the Earth, under the influence of gravity alone.

Horizontal and vertical motions are completely independent of each other.

\[
x = v_0 \cos \theta_0 t
\]
\[
y = y_0 + v_0 \sin \theta_0 t - \frac{1}{2} gt^2
\]
\[
v_y = v_0 \sin \theta_0 - gt
\]
\[
v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)
\]
\[
R = v_0 \cos \theta_0 T
\]
\[
T = \frac{2}{g} \frac{v_0 \sin \theta_0}{g}
\]

These only apply when vertical displacement is zero!
Projectile motion

The acceleration vector for projectile motion has components (x is horizontal, y vertical):

\[ a_x = 0 \quad a_y = -g \]

The y-component of velocity is zero at the peak of the path.

The x-component of velocity remains constant in time.
Projectile motion

Trajectory:
Path followed by projectile in projectile motion.

Time of flight:
Total time taken by projectile to travel from point of projection to landing point.

Range:
Total horizontal distance from point of projection to landing point.

The maximum height reached occurs when the vertical speed is zero.

The horizontal component of the velocity is constant.
Complementary angles

\[ v_i = 50 \text{ m/s} \]

Complementary values of the initial angle \( \theta \) result in the same value of the range, \( R \).
Relative Velocity

The motion of object A may be viewed from two different reference frames. One is from the stationary frame on the Earth (call it E) and the other is from a moving frame (in this example, car B).

\[ \vec{r}_{AB} = \vec{r}_{AE} - \vec{r}_{BE} \]

\[ \vec{v}_{AB} = \vec{v}_{AE} - \vec{v}_{BE} \]