PHYSICS 161 FLUID DYNAMICS

Apparatus: Large graduated cylinder (A= 52.4 cm$^2$) with small hole (A=0.126 cm$^2$) at bottom. Cylinder is placed in bucket. Computer needs stopwatch in Lab App.

ACTIVITY 1.

In this workshop you will study the rate at which water flows through a small hole in the bottom of a tall cylinder. You will measure how fast the water level falls with time. Then, you will compare the measured rate with the one predicted from Bernoulli's equation.

Let's say the top (T) surface of the water column has surface area $A_T$ and drops with velocity $v_T$ as the water flows through the hole at the bottom (B). In the hole, which has a cross sectional area $A_B$, water flows with velocity $v_B$. We know that the volume flow rates at the top and bottom are the same, i.e.,

$$A_T v_T = A_B v_B. \quad (1)$$

Thus, by measuring $v_T$ we can calculate $v_B$.

ACTIVITY 2.

We can also take another approach to determine $v_B$. If there is no friction and no turbulence, the pressure $P_T$ and the velocity $v_T$ at the top surface, which is at height $y_T$, are related to the corresponding pressure $P_B$, velocity $v_B$ and height $y_B$ of the water in the hole by Bernoulli's equation

$$P_T + \frac{1}{2} \rho v_T^2 + \rho g y_T = P_B + \frac{1}{2} \rho v_B^2 + \rho g y_B, \quad (2)$$

where $g$ is the gravitational acceleration and $\rho$ is the density of water. The top surface and the bottom hole are open to atmospheric pressure, so $P_T = P_B$. (Note that $v_B$ and $P_B$ refer to the fluid just outside the cylinder.)

We can eliminate $v_T$ by replacing it with $(A_B / A_T)v_B$, and write $h$ for the height difference $y_T - y_B$:

Now can find the Bernoulli prediction for the flow velocity through the hole at the bottom.
SUMMARY.

Your task is first in Activity 1 to measure the time $\Delta t$ it takes for the water level to drop by a small distance $\Delta h$. From this, you can calculate the average velocity $\bar{v}_T$ at the top, which through Eq. (1) gives you the average flow velocity $\bar{v}_B$ through the hole during that time interval.

Subsequently in Activity 2 you will find the outflow velocity $v_B$ without any time measurement, using Bernoulli’s equation. This method only uses the measurement of heights.
A horizontal tube has a wide part where the air velocity is 0.35 m/s. It narrows to a section where the air velocity is 0.70 m/s.

In which section is the pressure the largest?

What is the change in air pressure from wide to narrow section?

(The density of air is 1.28 $\frac{kg}{m^3}$. Assume here that air is an incompressible fluid.)
PROCEDURE

Locate on the computer the Stopwatch in Lab Apps., which will allow to measure time.

The apparatus consists of a graduated cylinder (cross sectional area \( A_T = 52.4 \text{ cm}^2 \) or \( 52.4 \times 10^{-4} \text{m}^2 \)) filled with water.

A small hole (area \( A_B = 0.126 \text{ cm}^2 \) or \( 0.126 \times 10^{-4} \text{m}^2 \) ) at the bottom will release the water into a bucket.

Activity 1.

With your finger covering the hole, fill the cylinder close to the maximum graduated level, 2000 ml. Then, place the filled cylinder onto the catch bucket, removing your finger from the hole. Try not to shake the cylinder, to minimize sloshing the water around!

When the water level passes the height 32 cm (0.32 m), start the timer, and stop it when the level passes 28 cm (0.28 m). Enter the result for \( \Delta t \) into the table below (remember the timer displays milliseconds).

Then push the "reset" button, and measure the time it takes for the water level to drop from 11 cm (0.11 m) to 9 cm (0.09 m). Again enter the result in the table.

Now complete the table below. (USE MKS UNITS)

OUTFLOW VELOCITY FROM THE CONTINUITY EQUATION:
<table>
<thead>
<tr>
<th></th>
<th>$h_1$</th>
<th>32 cm = 0.32 m</th>
<th>11 cm = 0.11 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_2$</td>
<td>28 cm = 0.28 m</td>
<td>9 cm = 0.09 m</td>
<td></td>
</tr>
<tr>
<td>$\Delta h = h_1 - h_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{h} = \frac{1}{2} (h_1 + h_2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{v}_T = \Delta h / \Delta t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{v}_B = (A_T / A_B) \bar{v}_T$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Activity 2.**

Now use Bernoulli’s relation to determine $v_B$.

**OUTFLOW VELOCITY FROM BERNOULLI’S EQUATION:**

<table>
<thead>
<tr>
<th></th>
<th>$h_1$</th>
<th>32 cm = 0.32 m</th>
<th>11 cm = 0.11 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_2$</td>
<td>28 cm = 0.28 m</td>
<td>9 cm = 0.09 m</td>
<td></td>
</tr>
<tr>
<td>$\bar{h} = \frac{1}{2} (h_1 + h_2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_T$ at $\bar{h} = \frac{1}{2} (h_1 + h_2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_B$ (just outside cylinder)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_T = (A_B / A_T) v_B$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_B$ (Bernoulli)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
QUESTIONS:

1. How well do the measured values for the flow velocity \( \vec{v}_B \) agree with the values, derived from Bernoulli's equation?

2. For the cylinder filled with water to \( h = 30 \text{ cm} \) find the pressure \( P_C \) in the fluid at a point \( C \) at \( h = 0 \text{ cm} \) near the bottom but still inside the cylinder.

3. What is the flow velocity \( v_C \) at point \( C \)?

4. Calculate the velocity of a water droplet after free-falling (from rest) through a vertical distance \( h = 30 \text{ cm} \). Compare this velocity with the outflow velocity \( v_B \) for a water column of height \( h \) that you found from your experiment?