Purpose:
- To understand work, potential energy, & kinetic energy.
- To understand conservation of energy and how energy is converted from one form to the other.

Apparatus: Pasco track, Pasco cart, LabPro interface and cables, motion sensor, pulley, small bucket & long thin string, assorted weights

Background: Reread your textbook to refresh your memory on the concepts of energy, work, kinetic energy, and potential energy. In this lab we will be using four equations, which we summarize here:

$$W = F \Delta x \cos \theta,$$  \hspace{1cm} \text{(work)} \hspace{1cm} (1)

Where $\theta$ is the angle between the force $F$ doing the work and the displacement $\Delta x$ of the object during the time work is being done. Work $W$ can be positive or negative. The kinetic energy of a mass $M$ moving with speed $v$ is:

$$KE = \frac{1}{2} M v^2$$  \hspace{1cm} \text{(kinetic energy)} \hspace{1cm} (2)
The change in gravitational potential energy of a mass $m$ raised a distance $\Delta y$ above the zero-level of potential energy is:

$$PE = m \ g \ \Delta y$$  \hspace{0.5cm} \text{(gravitational potential energy)} \hspace{0.5cm} (3)$$

Since only changes in potential energy are relevant, the point where the potential energy is zero can be arbitrarily chosen. One often chooses the lowest point in height as the zero-level of $PE$.

Conservation of energy can be expressed as:

$$KE_i + PE_i + W_{nc} = KE_f + PE_f,$$  \hspace{0.5cm} \text{(energy conservation)} \hspace{0.5cm} (4)$$

Here $W_{nc}$ is the work done on the system by non-conservative forces (like friction). Any work done by conservative forces (like gravity) is already taken care of by the potential energy due to such forces. Note in the case of friction that the force of friction always opposes the motion ($\theta = 180^\circ$) so that $W_{nc}$ is always negative. Therefore friction always removes energy from the system. Also remember that work is a process that transfers energy into or out of a system. Although a system can contain energy, it cannot contain work.

In other exercises you will also deal with the potential energy of a spring when compressed a distance $\Delta x$,

$$PE = \frac{1}{2} k \ \Delta x^2,$$  \hspace{0.5cm} \text{(potential energy of a spring)} \hspace{0.5cm} (5)$$

where $k$ is the force constant of the spring ($F = -k \ \Delta x$). Note that the potential energy is zero when the spring is not compressed or stretched.

**Experimental Set-up:** You will study a wheeled cart of mass $M$ (mass range 0.2 - 0.5 kg) on a horizontal low friction track. $M$ is attached to a smaller mass $m$ (here 0.020 - 0.080 kg) by means of a string passing over a pulley. The small mass $m$ is allowed to fall vertically to the floor through a distance $\Delta y$. The initial speed of both $m$ and $M$ is $v_i = 0$. You will measure $v_f$, the final speed of $M$, after the small mass $m$ travels a distance $\Delta y$ to the floor and the cart has moved horizontally over an equal distance.

The speeds of $m$ and $M$ are equal as long as the connecting string stays taut.

![Diagram of experimental setup](image)

You will use the same motion sensor as you did in the first two labs. Open the file WorkEnergy.cml, which is in the folder on your computer Desktop: Course Folders -> 161 -> Energy and Work. The velocity at any time or position can be obtained from the Logger Pro program.
The speed \( v_f \) will depend on three variables -- \( M, m, \Delta y \) -- that you can vary to test conservation of energy [eq. (4) above]. You may also examine whether any discrepancy can reasonably be attributed to friction.

**Important checks:**
- Check that the low-friction track is level.
- Be sure to add the mass of the weight-holder (~ 5 gram) to the small masses you attach to get to the total falling mass \( m \).
- Because of friction, there is a minimum value for \( m \) that is needed to produce any acceleration. Determine this value and be sure that you use considerably larger values of \( m \) for your experiment (\( m > 20 \) gram). Otherwise the friction will dominate and give poor results.

**PART I**

**Activity 1 (60 min): Dependence of the final speed \( v_f \) on \( m \) and \( \Delta y \), and the product \( (m \Delta y) \).**
Use the set-up described above to determine the final speed of the cart for several different values for \( m \) while keeping \( M \) constant. In each case calculate the height \( \Delta y \) above the floor at which \( m \) should be released so that the product \( m \Delta y \) is always the same value (this means that for each run PE changes by the same amount). Then make a second set of measurements where \( m \) is fixed and only \( \Delta y \) varies. Answer the questions on the lab report sheet.

**Activity 2 (40 min): Relationship between kinetic energy and total potential energy.**
Using the data from Activity 1 (second set) for which \( m \) is a constant, make a plot of total potential energy versus \( v_f \) and fit the data to curves that vary as \( v_f, v_f^2, \) and \( v_f^3 \) to see which gives the best fit. Answer the questions on the lab report sheet.

**Activity 3 (20 min): Conservation of energy.**
Use your data from Activity 1 to determine whether conservation of energy holds. Discuss any discrepancies as indicated on the lab report sheet.

**PART II**

**Activity 4 (60 min): Work.**
Draw free-body diagrams for \( m \) and \( M \) for the experimental set-up used above. Calculate the work each of these forces does during the time mass \( m \) falls to the floor. Include friction. Use symbols rather than numbers and be careful about the signs.
ENERGY and WORK (Part I) (preliminary questions)

Name: __________________________________________________ Section: ________
Partner: __________________________________________________

1. A 130 gram arrow is shot straight up with an initial velocity of 20 m/s.
   a) What is its initial kinetic energy? To what height does the arrow go?

   b) At what height does the arrow have maximum potential energy? What is the value of the maximum potential energy?

   c) At what height does the kinetic energy equal the potential energy? What is the value of its speed at this height?
PART I (try to be very accurate and record 3 significant figures if possible)

**Activity 1: Dependence of final speed \( v_f \) on \( m \) (weights + holder) and \( \Delta y \) and their product \( m \Delta y \).** Determine the final speed at the floor for several different values for \( m \) (20, 30, 40, 50, 60 gram weights and the 5 gram holder) while keeping \( M \) constant. Before you start a run calculate the height \( \Delta y \) above the floor at which \( m \) should be released, so that the product \( m \Delta y \) always has the same value for all data for Table 1. Calculate the corresponding change in potential energy \( PE \). To improve your accuracy repeat each measurement (for given \( M, m, \Delta y \) ) THREE times and record the average final speed \( v_f \) with 3 significant figures. Use values of \( \Delta y \) between 0.2 and 0.8 m.

For your first good run, include the graph of \( x - t \) and \( v - t \). In the \( v - t \) graph identify the three points: 1. Start of run, 2. When weight hits the floor, 3. When the cart hits the gate and stops.

**Table 1**, fixed \( M \), vary \( m \), choose \( y \) so that product \( m \Delta y \) is constant, (so initial \( PE \) is constant for each run in this table)

<table>
<thead>
<tr>
<th>M(kg) fixed</th>
<th>m(kg) + holder</th>
<th>( \Delta y )(m) calculate</th>
<th>( m \Delta y )(kg.m) constant</th>
<th>( PE_m ) (J)</th>
<th>( v_i = 0 )</th>
<th>3 runs, find ( v_f )</th>
<th>average ( v_f )</th>
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\[ PE_m \text{ is initial gravitational potential energy of } m \text{ at height } y \text{ above the floor, Eq. (3).}\]

Comment on the values of the final speed in the last column of Table1. Should they really all have the same value? (This is kind of a trick question. They should decrease very slightly with increasing \( m \). Why?)
Now repeat the experiment with **the same fixed values for M and m as in the first line of data in Table 1.** In the next 4 lines vary \( \Delta y \) (now the change in \( PE_m \) is not constant).

**Table 2**, vary \( y \) (while keeping both M and m fixed), so initial PE will vary.

<table>
<thead>
<tr>
<th>M(kg) fixed</th>
<th>m(kg) + holder</th>
<th>( \Delta y )(m)</th>
<th>m ( \Delta y )(kg m)</th>
<th>( PE_m )(J)</th>
<th>( v_i = 0 )</th>
<th>3 runs, find ( v_f )</th>
<th>average ( v_f )</th>
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What conclusion can you draw from your data in Table 2 about the relationship between \( v_f^2 \) and \( PE_m \)? (Later you will specify this relationship in more detail)

**Activity 2: Relationship between kinetic energy and total potential energy.**
In Tables 1 and 2 you calculated \( PE_m \). Where did you choose the zero-level of potential energy \( PE_m \)?

You now want to calculate the total potential energy of m and M, \( PE_{M+m} = PE_m + PE_M \). Notice that \( PE_m \) changes but \( PE_M \) does not. (The height of cart M does not change during a run).

Do you have to choose the zero-level for \( PE_m \) and \( PE_M \) to be at the same height?
Fill in Table 3 using all your previous data which have the same value of m (see Table 2 and Table 1).

Table 3 (for fixed m = kg, and fixed M = kg)

<table>
<thead>
<tr>
<th>Δy(m)</th>
<th>PEₘ</th>
<th>PEₘ₊ₘ</th>
<th>vₙ</th>
<th>vₙ²</th>
<th>vₙ³</th>
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Using Logger Pro, make a plot of PEₘ₊ₘ versus vₙ for the data given in Table 3. Make a linear fit to the data (which would mean that PEₘ₊ₘ is proportional to vₙ).

Also plot PEₘ₊ₘ versus vₙ², then try a linear fit of those data.

Similarly plot PEₘ₊ₘ versus vₙ³, followed by a linear fit.

For each of the three fits record the value of the correlation coefficient R. Make a copy of your graphs showing all three fits and attach it to your report. [Pay attention to the proper title and labeling in the graphs.]

Which power of vₙ provides a better fit to your data? What is the value of the correlation coefficient in each plot?

In view of the definition of kinetic energy given in Eq.(2), the linear fit to vₙ² would be proof that the change in potential energy is related to the change in kinetic energy. Is there enough evidence for such a proof from your fits?
Activity 3: Check of conservation of energy expressed in Joules.
Use your data from first line of Table 2 to determine whether conservation of energy holds. Use that data to fill in Table 4:

Table 4 (kinetic and potential energies and total energy)

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<tr>
<td>Initial K.E. of M (in Joule)</td>
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<tr>
<td>Initial K.E. of m</td>
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<td>Initial P.E. of M (w.r.t. track)</td>
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<td>Initial P.E. of m (w.r.t. floor)</td>
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<td>Total initial energy of m + M</td>
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<td>Final K.E. of M</td>
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<td>Final K.E. of m</td>
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<td>Final P.E. of M (w.r.t. track)</td>
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<tr>
<td>Final P.E. of m (w.r.t. floor)</td>
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<tr>
<td>Total final energy of m + M</td>
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<td>Difference between initial and final total energies</td>
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From this table what can you conclude about total energy of m + M before and after the fall?

You did this experiment to verify that energy is conserved, but your data show a discrepancy between the initial and final total energies. To what do you attribute this discrepancy? List the energy losses that may have occurred along the way in order of importance.
Report -- ENERGY and WORK  (PART II)

Name: ________________________________  Section: __________
Partner: ______________________________ Date: __________
Partner: ______________________________

Activity 4: Work.
Draw separate free-body diagrams for m and M for the experimental set-up used above. Calculate the work each of these forces does during the time m falls to the floor. Include friction. Use symbols rather than numbers and be careful about the signs.

What is the total work done on M during the fall?

What is the total work done on m during the fall?

What is the total work done on M+ m during the fall?

What is wrong with this statement?
*During the time that m falls through a distance \( \Delta y \) the force of friction \( F_f \) removes an amount of work \( W_f = - \Delta y F_f \) from the system of \( (M + m) \).*