Temperature

Fahrenheit temperature \( T_F = \frac{9}{5}T_C + 32 \) Celsius temperature

Celsius temperature \( T_C = \frac{5}{9}(T_F - 32) \) Fahrenheit temperature

Kelvin temperature \( T_K = T_C + 273.15 \) Celsius temperature

<table>
<thead>
<tr>
<th></th>
<th>K</th>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water boils</td>
<td>373</td>
<td>100</td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>100 K</td>
<td>100 C</td>
<td>180 F</td>
</tr>
<tr>
<td>Water freezes</td>
<td>273</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>( CO_2 ) solidifies</td>
<td>195</td>
<td>-78</td>
<td>-109</td>
</tr>
<tr>
<td>Oxygen liquefies</td>
<td>90</td>
<td>-183</td>
<td>-298</td>
</tr>
<tr>
<td>Absolute zero</td>
<td>0</td>
<td>-273</td>
<td>-460</td>
</tr>
</tbody>
</table>
An Ideal Gas

\[ pV = Nk_B T = \frac{N}{N_A} RT = nRT \]

- \( k_B \): Boltzmann constant \( 1.381 \times 10^{-23} \) J/molecule·K
- \( R \): gas constant \( 8.3145 \) J/mol·K
- \( N_A \): Avogadro’s number \( 6.022 \times 10^{23} \) molecules/mol
- \( n \): the number of moles of the molecule
Microscopic Description of gas

• A real gas consists of a vast number of molecules, each moving randomly and undergoing millions of collisions every second.
  • Despite the apparent chaos, averages, such as the average number of molecules in the speed range 600 to 700 m/s, have precise, predictable values.
  • The “micro/macro” connection is built on the idea that the macroscopic properties of a system, such as temperature or pressure, are related to the average behavior of the atoms and molecules.
An Ideal Gas

• The distribution of molecular speeds in a sample of N\textsubscript{2} gas at 20 C
An Ideal Gas

• Consider a container with volume $V$ containing $N$ particles of mass $m$
  • The particles bounce elastically off the sides of the wall
  • Consider the 1 dimensional case first

• The force acting on the wall when one particle bounces is:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{\Delta t}$$

• To determine what the total force from all of the collisions is, we need to figure out how many such collisions take place
An Ideal Gas

• Consider a small part of the container surface, with area A, against which the particles are bouncing
  • If a particle is going to collide with the wall in a time $\Delta t$, then it must be at least a distance of $v_x \Delta t$ from the wall
• How many particles are in this volume?

$$\frac{N}{V} = \frac{N_{vol}}{v_x \Delta t A} \rightarrow N_{vol} = \frac{N}{V} v_x \Delta t A$$

• Let’s assume that half of them are moving towards the wall, and half away from the wall
  • Therefore the number of collisions against the wall is:

$$N_{col} = \frac{1}{2} N_{vol} = \frac{1}{2} \frac{N}{V} v_x \Delta t A$$
An Ideal Gas

• Let’s put these two pieces together
  • The total force against the wall is the force from one collision times the number of collisions

\[ F_{\text{tot}} = F N_{\text{col}} \]

\[ = \left( \frac{2mv_x}{\Delta t} \right) \left( \frac{1}{2} \frac{N}{V} v_x \Delta t A \right) \]

\[ = mv_x^2 \frac{N}{V} A \]

• Since the pressure is the total force per unit area, we find that

\[ p = \frac{F_{\text{tot}}}{A} = mv_x^2 \frac{N}{V} \]

• Is this starting to look like the ideal gas law…?
An Ideal Gas

- The next step in deriving the ideal gas law is to extrapolate from 1 dimension to 3 dimensions
  - To do this, remember that the particles aren’t all traveling with velocity \( v_x \)
    - If half are traveling with \( v_x \) and half with \(-v_x\), then the average \( v_x \) is 0!
    - Better to think about the average of \( |v_x| \) or \( v_x^2 \), which is non-zero
  - So now, let’s assume that \( v_x, v_y, \) and \( v_z \) should be treated on equal footing
    - that is the average value of \( (v_x)^2, (v_y)^2, \) and \( (v_z)^2 \) are all the same
      \[
      (v^2)_{avg} = (v_x^2)_{avg} + (v_y^2)_{avg} + (v_z^2)_{avg} = 3(v_x^2)_{avg}
      \]
    - Similarly, the average translational kinetic energy is
      \[
      K_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{3}{2}m(v_x^2)_{avg}
      \]
An Ideal Gas

- Therefore we have:

\[ p = m(v_x^2)_{\text{avg}} \frac{N}{V} = \frac{2}{3} K_{\text{avg}} \frac{N}{V} \rightarrow pV = \frac{2}{3} NK_{\text{avg}} \]

- The final step in “deriving” the ideal gas law is to relate the average kinetic energy to the temperature
  - It turns out that:

\[ \frac{1}{2} m(v^2)_{\text{avg}} = \frac{3}{2} k_B T \]

- where T is the temperature and \( k_B \) is Boltzmann’s constant (\( k_B = 1.381 \times 10^{-23} \text{ J/molecule}\cdot\text{K} \))

- Boltzmann’s constant is the constant of proportionality relating the average kinetic energy to the temperature
An Ideal Gas

• Finally, this means that

\[ pV = Nk_B T \]

• For those used to the “chemistry version” of this equation

\[ R = k_B N_A \]

  • where \( R \) is the “gas constant” 8.3145 J/mol·K
  • and \( N_A \) is Avogadro’s number 6.022x10^{23} \text{ molecules/mol}

• Therefore,

\[ pV = Nk_B T = \frac{N}{N_A} RT = nRT \]

  • where \( n \) is the number of moles of the molecule
Macro-Micro Connection

• Assumptions for ideal gas:
  • # of molecules $N$ is large
  • they obey Newton’s laws
  • thermal equilibrium
  • elastic collisions with walls and each other

• What we call temperature $T$ is a direct measure of the average translational kinetic energy

• What we call pressure is a direct measure of the number density of particles and how fast they are moving

$$T = \frac{2}{3k_B} K_{\text{avg}}$$

$$p = \frac{2}{3} \frac{N}{V} K_{\text{avg}}$$

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle_{\text{avg}}} = \sqrt{\frac{3k_B T}{m}}$$
Ideal Gas in Cylinders

Two cylinders are filled to the same height $H$ with ideal gas. The gases are different, and the cross-sectional areas of the cylinders are different. Both cylinders have pistons that are free to move without friction.

How does temperature of the gas in cylinder A compare to the temperature of the gas in cylinder B?

1. $T_A = \frac{1}{2} T_B$
2. $\frac{1}{2} T_A = T_B$
3. $\frac{1}{4} T_A = T_B$
4. $T_A = \frac{1}{4} T_B$
Cylinders with equal cross-sectional areas contain different volumes of an ideal gas sealed in by pistons. There is a weight sitting on top of each piston. The gas is the same in all four cases and is at the same temperature. The pistons are free to move without friction.

Rank the pressure of the gas in each cylinder.

A. B > C > D > A
B. D > B > A > C
C. D > B = A > C
D. B = C > A = D
E. We need to know how many moles of gas are in each piston to answer this.
Kelvin Scale

• Since we have a picture of temperature as kinetic energy of particles, we now can appreciate Kelvin as a scale
  • The temperature in kelvin is related to the temperature in Celsius by: \( T_K = T_C + 273.15 \)
    • Hint: don’t forget to measure temperature ratios in kelvin (temperature differences in either kelvin or celsius are fine)

• Zero kelvin (aka “absolute zero”) corresponds to the point where the matter has no motion whatsoever
Intermolecular Forces

• We obtained the ideal-gas equation from a simple molecular model
  • This ignored many different factors, among which are the size of the molecules themselves and the attractive forces between them

• van der Waals equation accounts for these effects empirically:

\[ \left( p + \frac{a n^2}{V^2} \right) (V - n b) = n R T \]

• a and b are empirical constants that differ for different gases
  • “b” represents the volume of a mole of molecules so that V-nb represents how much volume there is left of empty space
  • “a” represents the force of attraction between particle species effectively reducing the force of collision against the walls
An Ideal Gas

- The distribution of molecular speeds in a sample of N\textsubscript{2} gas at 20°C

\[
\frac{1}{2} m (v^2)_{\text{avg}} = \frac{3}{2} k_B T
\]

14 x \(m(\text{H}_2)=m(\text{N}_2)\)

Earth’s escape speed = 11,200 m/s