Analytical Physics 1B Lecture 8: Fluids

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Friday, March 23, 2017

Office hour (Serin 123): 11 am – noon, Mondays
(March 26, The resonance/buoyance workshop set-up in Serin 123)
Now consider a piece of paper in the middle of the yellow packet. Which of the following best describes the forces acting on the one piece of paper shown?

1. \( F_{\text{table on paper}} \)
   \( F_{\text{Earth on paper}} \)

2. \( F_{\text{bottom yellow on paper}} \)
   \( F_{\text{Earth on paper}} \)
   \( F_{\text{top yellow on paper}} \)

3. \( F_{\text{bottom yellow on paper}} \)
   \( F_{\text{table on paper}} \)
   \( F_{\text{Earth on paper}} \)
   \( F_{\text{top yellow on paper}} \)

4. \( F_{\text{bottom yellow on paper}} \)
   \( F_{\text{Earth on paper}} \)
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   $F_{\text{Earth on paper}}$

2. $F_{\text{bottom yellow on paper}}$
   $F_{\text{Earth on paper}}$

3. $F_{\text{bottom yellow on paper}}$
   $F_{\text{table on paper}}$
   $F_{\text{Earth on paper}}$

4. $F_{\text{bottom yellow on paper}}$
   $F_{\text{Earth on paper}}$
Now consider a piece of paper in the middle of the blue packet. Which of the following best describes the forces acting on the one piece of paper shown?

1. $F_{\text{table on paper}}$
   $F_{\text{Earth on paper}}$

2. $F_{\text{table on paper}}$
   $F_{\text{Earth on paper}}$

3. $F_{\text{bottom blue on paper}}$
   $F_{\text{top blue on paper}}$
   $F_{\text{Earth on paper}}$

4. $F_{\text{bottom blue on paper}}$
   $F_{\text{top blue on paper}}$
   $F_{\text{Earth on paper}}$
Now consider a piece of paper in the middle of the blue packet. Which of the following best describes the forces acting on the one piece of paper shown?

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   \( F_{\text{Earth on paper}} \)

2. \( F_{\text{table on paper}} \)
   \( F_{\text{Earth on paper}} \)

3. \( F_{\text{top blue on paper}} \)
   \( F_{\text{bottom blue on paper}} \)
   \( F_{\text{Earth on paper}} \)

4. \( F_{\text{bottom blue on paper}} \)
   \( F_{\text{top blue on paper}} \)
   \( F_{\text{Earth on paper}} \)
As you go deeper down in the stack of paper, which of the following observations is TRUE?

1. The paper gets heavier.
2. The paper gets more dense.
3. The paper above it pushes down harder on it.
4. The paper below it pushes up harder on it.
5. Both 3 and 4 are true.
As you go deeper down in the stack of paper, which of the following observations is TRUE?

1. The paper gets heavier.
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5. Both 3 and 4 are true.
Pressure on Fluid

- Consider an element of fluid at rest
  - Since it is at rest, all of the forces acting on it must cancel out

- Force due to the pressure on top:
  \[ F_{\text{top}} = (p + dp)A \]

- Force due to the pressure on the bottom:
  \[ F_{\text{bottom}} = pA \]

- Force due to gravity:
  \[ F_{\text{grav}} = mg = Ady\rho g \]
Pressure on Fluid

- Balancing the forces, we have:

\[ F_{\text{top}} + F_{\text{grav}} = F_{\text{bottom}} \]

\[ \downarrow \]

\[ (p + dp)A + A dy \rho g = pA \]

\[ \downarrow \]

\[ dp = -dy \rho g \]

\[ \downarrow \]

\[ p_2 - p_1 = -\rho g(y_2 - y_1) \]
\[ p_2 - p_1 = \rho g (y_2 - y_1) \]

\[ p = p_0 + \rho gh \]

**Pascal’s law**: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

\[ F_2 = (A_2/A_1) \cdot F_1 \]
Pressure

1 Pa = 1 N/m\(^2\), 1 bar = 10\(^5\) Pa
1 atm = 1.013 \times 10\(^5\) Pa = 1.013 bar = 14.7 PSI (lb/in\(^2\))

Density

Air (1 atm, 20°C) 1.2 kg/m\(^3\)
Water 1000 kg/m\(^3\) (1 g/cm\(^3\))
Ice 920 kg/m\(^3\)
Sea water 1030 kg/m\(^3\)
Aluminum 2700 kg/m\(^3\)
Mercury Barometer

- Long glass tube filled with mercury then inverted in a dish of mercury
  - near vacuum at the top (consists only of mercury vapor)

- From before:
  \[ p_2 - p_1 = -\rho g (y_2 - y_1) \]
  - \(1\) atm = \(1.013 \times 10^5\) Pa
  - density of Hg = \(1.36 \times 10^4\) kg/m\(^3\)

\[
 h = (y_2 - y_1) = \frac{0 - p_{\text{atm}}}{-\rho g} \approx 0.760 \text{ m}
\]

\(1\) atm = 760 mmHg
Beaker Pressure

In each case a beaker is filled with water to the height shown. The diameters of the beakers are also shown. The cylinders have identical holes in their side at the same height above the base. There are corks in all of the holes.

Rank the pressure on the corks.
A. B > A > C > D
B. B > A > C = D
C. A = B = C = D
D. B > C > A > D
E. We don’t know the relationship between diameter and pressure.
Buoyancy

- **Archimedes’s principle:** When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.
Buoyant Force

- Density of object < density of the fluid
  
  \[ \text{Weight of object} = \text{Weight of water displaced} = \text{buoyant force} \]

- Density of object = density of the fluid

  \[ \rho_{\text{object}} = \rho_{\text{fluid}} \]

  \[ \rho = 920 \text{ kg/m}^3 \]

Some fish can remain at a fixed depth without moving by storing gas in their bladder. Submarines take on or discharge water into their ballast tanks.
Suspended Blocks

Blocks that have different masses and volumes are suspended by strings in water. The blocks are at two different depths below the surface as shown.

![Diagram of suspended blocks]

Rank the buoyancy force exerted on the blocks by the water.

A. $D > A > B = C$
B. $D = A > B = C$
C. $D = A > B > C$
D. $D > A > B > C$
E. $A = B = C = D$
Floating Blocks

Wood blocks that have different masses and different volumes are floating in water. On top of these blocks are additional masses as shown.

A. $A > D > C > B$
B. $D > B > A = C$
C. $A = D > B = C$
D. $B = D > A = C$
E. $D > A = B > C$
Suspended Blocks II

Four blocks are suspended from strings in water. Cubes A and C are at the same depth, as are cubes B and D. Hint: the density of water is 1 g/cm³.

Rank the tension in the strings.

A. D > A > B > C
B. D = A > B = C
C. B = D > A = C
D. B > D = C > A
E. D > B > A > C
Fluid Dynamics

• Consider an **ideal fluid**
  • Fluid motion is very complicated. However, by making some assumptions, we can develop a useful model of fluid behavior.

• An ideal fluid is:
  • **Incompressible** – the density is constant
  • **Irrotational** – the flow is smooth, no turbulence
  • **Non-viscous** – fluid has no internal friction
  • **Steady flow** – the velocity of the fluid at each point is constant in time.
Fluid Dynamics

- mass flowing into the pipe equals mass flowing out of the pipe

\[ m_1 = m_2 \rightarrow \rho V_1 = \rho V_2 \]

- Since \( v = \frac{dx}{dt} \):

\[ V = A dx = Av \, dt \]

- Since \( dt_1 = dt_2 \), we have:

\[ A_1 v_1 = A_2 v_2 \]

Continuity equation
Bernoulli’s Equation

• Accounts for fluid flow

\[ p_2 - p_1 = \rho g (y_2 - y_1) \]

\[ p + \rho g y + \frac{1}{2} \rho v^2 = \text{constant} \]

• Consider points 1 and 2
  • they have the same vertical coordinate so…

\[ p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 \]

• And from continuity equation

\[ v_2 = \frac{A_1}{A_2} v_1 \]

\[ p_1 - p_2 = \frac{1}{2} \rho v_1^2 \left( \left( \frac{A_1}{A_2} \right)^2 - 1 \right) \]
Bernoulli’s Equation

• Notice also that the pressure difference $p_1 - p_2$ is also related to the height difference!

\[ p_1 - p_2 = \rho gh \]

• We can plug this into our previous equation to find

\[ v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}} \]

• Therefore the height differences tells us the flow of velocity!
Storms

In a storm how does a house lose its roof?

Air flow is disturbed by the house. The "streamlines" crowd around the top of the roof.

⇒ faster flow above house

⇒ reduced pressure above roof to that inside the house

⇒ roof lifted off because of pressure difference.
Rabbits

Why do rabbits not suffocate in the burrows?

Air must circulate. The burrows must have two entrances. Air flows across the two holes is usually slightly different ⇒ slight pressure difference ⇒ forces flow of air through burrow. One hole is usually higher than the other and the a small mound is built around the holes to increase the pressure difference.